Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 19 (1974)

Artikel: A method of analysis of reinforced or prestressed concrete structures under triaxial stress

Autor: Janda, Lubor

DOI: https://doi.org/10.5169/seals-17532

IABSE
AIPC
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SEMINAR on:
«CONCRETE STRUCTURES SUBJECTED TO TRIAXIAL STRESSES»
17th-19th MAY, 1974 - ISMES - BERGAMO (ITALY)

A method of analysis of reinforced or prestressed concrete structures under triaxial stress

Une méthode de calcul des constructions en béton armé ou en béton précontrainte sous étrainte triaxiale

Eine Dimensionierungsmethode von Stahl- oder Spannbetonkonstruktionen unter dreiaxialer Spannung

LUBOR JANDA, Doc., Ing. C. Sc. - Associate professor
Chair of Concrete Structures, The Faculty of Civil Engineering
Technical University, Prague

1. Principles of the Method

Extensive laboratory works, particularly those carried out in France by Launay, Gachon, Poitevin, Magnas, Audibert /1/,/2/, /3/ have revealed that very approximately the elastic and/or visco-elastic (for long term loads) behaviour of concrete is very closely related to the state of stress in which internal cracks develop. The laboratory works carried out so far do not yet enable direct reliable determination of the surface of internal crack initiation which would include, in a system of ordinates with the axes in the direction of principal stresses σ_1, σ_2, σ_3, the whole field of triaxial stresses; they do not enable either, to determine more accurately the changes of its form under various types of loading, e.g. under repeated or long term loads. An analysis given in the work by Magnas and Audibert /3/ reveals that in the field of biaxial and triaxial stresses, before the minor or mean main compressive stress has exceeded about 1/5 of the direct compressive strength value, a surface homothetic with the surface of rupture of concrete with a homothethy coefficient of about 2.5 can be satisfactorily considered as the surface of usable i.e. permissible stresses which ensures, together with the required safety, also the elastic behaviour of concrete. In the fields in which the mean principal stress is higher, i.e. in the fields with a higher compression of concrete, the homothethy coefficient should be higher, and for the point situated on the axis of the surface in the direction of compression it should even grow to infinity. The surface of crack initiation, from which the surfaces of permissible stresses must be derived, is - namely - closed in the direction of its compression axis (Fig.1, meridians A and B), in contradistinction from the surface of strength (rupture) which can be considered theoretically as open in the direction of its axis in the field of compression (Fig.1, meridians Γ). So far the shape of both surfaces in the field when one or several principal stresses are tensile stresses is uncertain. The shape of the surface of rupture and particularly the shape of the surface of internal crack initiation is also influenced by the type of load, e.g. long term or repeated load, composition of concrete (i.e.mechanical properties and grading of aggregates, pore size) and reinforcement which are the sources of local stress irregularities in the structure, as well as the conditions of the placing of concrete during its hardening and the quality of cement to which the initial disturbance of the unloaded concrete body by hair cracks, primarily because of the shrinkage of concrete, are due. Since the contemporary state of laboratory research does not make it possible to take these influences into account more accurately, it seems most acceptable at present to base the practical design of the structure on the shape of the surface of rupture of concrete deduced in the conditions of the simplest way of stressing, i.e. under unrepeated, short term load, which has been investi-

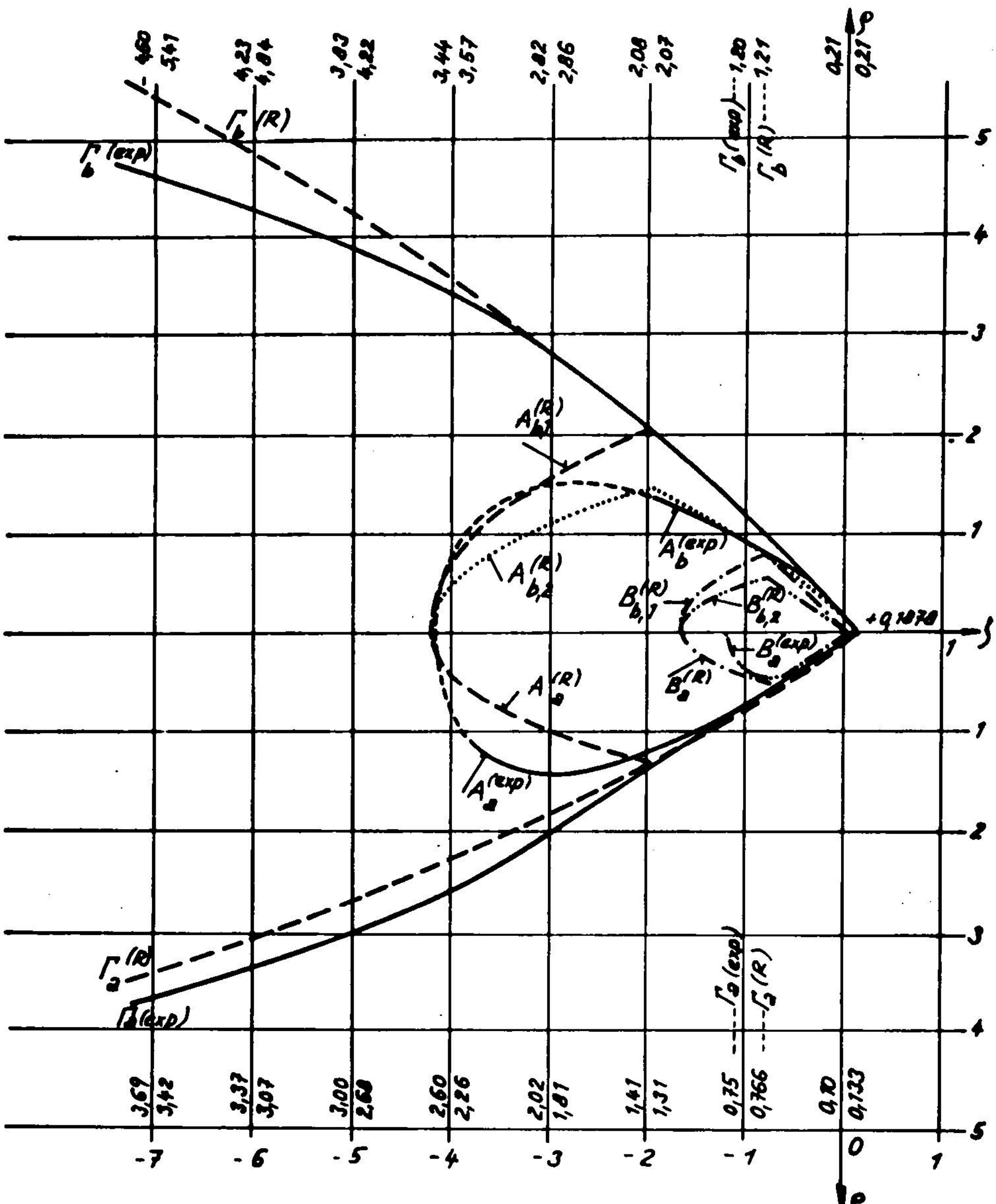

FIG. 1

Meridians of the surface of rupture Γ_a, Γ_b, surfaces of crack initiation A_a, A_b and surfaces of quasi-elastic limit B_a, B_b for the tensile ($\sigma_2 = \sigma_3$, $\sigma_1 > \sigma_2$) and the compressive ($\sigma_1 = \sigma_2$, $\sigma_3 < \sigma_1$) character of stresses according to test results of Launay, Gachon /2/ (index "exp") and according to the idealization based on Reimann's proposal /4/ (index "R").

Les méridians de la surface de résistance Γ_a, Γ_b, de la surface de l' amorce d' une fissuration interne A_a, A_b et de la surface du seuil approx. de l' élasticité B_a, B_b pour la contrainte de caractère de tension ($\sigma_2 = \sigma_3$, $\sigma_1 > \sigma_2$), et de compression ($\sigma_1 = \sigma_2$, $\sigma_3 < \sigma_1$) après des expériments de Launay, Gachon /2/ (index "exp") et après l' idéalisation de Reimann /4/ (index "R").

Meridiane der Festigkeitsfläche Γ_a, Γ_b, der Fläche des Begins von Bildung der Innenrissen A_a, A_b und der Fläche der approximativen Elastizitätsgrenze B_a, B_b für den Zug- ($\sigma_2 = \sigma_3$, $\sigma_1 > \sigma_2$) und Druck- ($\sigma_1 = \sigma_2$, $\sigma_3 < \sigma_1$) beanspruchungscharakter, nach den Experimenten von Launay, Gachon /2/ (Index "exp"), und nach der Idealisation nach Reimann /4/ (Index "R").

gated and verified in greatest detail with laboratory works. Partial tests have further verified that the state of stress at the crack initiation limit under unrepeated short-term load is closely related to the strength under repeated long-term load. From this form it is further possible to deduce an idealized shape of the surface of the internal crack initiation and from that, in turn, the surface of permissible stresses.

In respect of the fields in which one or more principal stresses of the concrete body are tensile stresses, it cannot be assumed in the majority of cases that concrete alone would be capable of transferring these stresses reliably; therefore, their transfer is entrusted to suitably arranged reinforcement. However, to ensure that the approximate simple assumption used for the calculation of stresses, viz. that the material of the structure is elastic and homogeneous, can be used also in these fields, it is necessary that the tensile strains be small, i.e. that the originating fissures in concrete be of limited widths only. As the failure of concrete under tensile stresses is brittle, i.e. proceeds practically without major plastic deformations, it can be assumed that the criterion of the elastic behaviour of the structure will be comparatively well complied with until the state of stress has been reached which will cause the failure of concrete due to tension. The experience with the design of reinforced concrete structures has shown that the proportioning can be safely based on the stresses calculated under the assumption of elasticity even in the case when the ideal tensile stress of concrete has considerably exceeded the tensile strength of concrete. When dimensioning triaxially stressed structures it is necessary to proceed more carefully in such cases as the fact that under greater strain the tensioned zone of concrete is excluded from action and the tension is transferred by reinforcement only, often changes considerably the assumption under which the state of stress has been calculated. In spite of this, however, even in these cases it is sometimes possible to remain satisfied with an approximate calculation of the state of stress on the assumption of elasticity.

To be able to suggest a practically applicable and realistic method of assessment of the state of stress of the generally triaxially stressed concrete structure in the elastic field, it is necessary to deal with the selection of a suitable mathematical interpretation of the surface of rupture of concrete and suggest suitable reduction coefficients for an approximate determination of the surface of permissible (or usable) stresses.

2. Surface of Rupture of Concrete

The surface of rupture of concrete deduced theoretically and from test results, i.e. phenomenologically, has the shape of a non-rotary surface enclosed in the field of all-directional tension and open in the field of all-directional uniform (hydrostatic) pressure. Its axis is identical with the spatial diagonal of the system of axes σ_1, σ_2, σ_3, its meridians have the form approaching parabolas and in the planes perpendicular to the axis of the surface the sections have the form of triangles with rounded corners and convex sides.

This shape can be described analytically most characteristically by means of a transformation of the original system of axes σ_1, σ_2, σ_3, into a system whose one axis (z) is identical with the spatial diagonal of the original system of axes σ_1, σ_2, σ_3 and introduce special ordinates r and in the plane perpendicular to this axis (z). For the transformation the relations

$$z = \frac{1}{\sqrt{3}} (\sigma_1 + \sigma_2 + \sigma_3) = \frac{J_1}{\sqrt{3}} = \sqrt{3}\ \sigma_{oct} \tag{1}$$

$$r = \frac{1}{\sqrt{3}} \sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2} = \frac{\sqrt{2}}{\sqrt{3}} J_2 = \sqrt{3}\ \tau_{oct} \tag{2}$$

$$\cos\delta = \frac{1}{r\sqrt{6}} (\sigma_1 + \sigma_2 - 2\sigma_3) = \varphi(J_2, J_3) \tag{3}$$

are valid, where

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$J_2 = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}$$

$$J_3 = J_2 \sqrt[3]{-\frac{1}{4}(3\cos\delta - 4\cos^3\delta)} = J_2 \sqrt[3]{-\frac{1}{4}\cos 3\delta}$$

J_1, J_2, J_3 are the invariants of the tensor of stress and σ_{oct} and τ_{oct} and the normal and the shearing components respectively of the stress applied to the planes of the octahedron.

Thus the surface of rupture of concrete can be described either in the system of ordinates of σ_1, σ_2, σ_3, or in the system of z, r, δ in which simultaneously the dependence on the invariants of the tensors of stress J_1, J_2, J_3 is expressed. Thus generally

$$f_1(\sigma_1,\sigma_2,\sigma_3) \equiv f_2(z,r,\delta) \equiv f_3(J_1,J_2,J_3) \equiv f_4(\sigma_{oct}, \tau_{oct}, \delta) = 0$$

It is obvious that the introduction of the third invariant J_3 into the equation of the surface expresses its non-rotary character. Simultaneously it describes also the influence of the mean principal stress σ_2 on strength. At the same time the ratio of the ordinates r_a of the points on the meridian Γ_a corresponding with the state of stress of tensile character (when $\sigma_2 = \sigma_3$, $\sigma_1 > \sigma_2$) and the ordinates r_b of the points on the meridian Γ_b corresponding with the state of stress of compressive character (when $\sigma_1 = \sigma_2$, $\sigma_3 < \sigma_1$) varies, according to the tests carried out by Launay, Gachon and Poitevin /1/ within the limits of

$$\frac{r_a}{r_b} = 0.48 \text{ to } 0.8$$

A number of authors has proposed analytical, idealized expressions characterizing the shape of the surface of rupture of concrete which approach more or less the shape based on tests. The functions contain, in the capacity of initial constants, the strengths of concrete under simple cases of loading, i.e. mostly in simple compression and tension, sometimes also pure shear, and are of the second to the fourth order.

Analytically simplest is the replacement of a non-rotary surface with a rotary surface, most frequently with a rotary paraboloid, which means that the dependence on the third invariant of the tensor of stress J_3 has been neglected. The deviations are - as it has been already described - considerable; for this reason this expression cannot be considered entirely satisfactory.

Departing from the principle that the mathematical formulation should be as simple as possible and that the deviations ascertained by a comparison with the surface of rupture determined by the tests shoud lie within acceptable limits the expression proposed by Reimann /4/ seems satisfactory. Recently several even better proposals of the analytical formulation of the surface of rupture have been presented, from the number of which let us name, for example, the general formulation by Launay, Gachon and Poitevin /1/ enabling the introduction of a larger number of constants, further the "lemniscate criterion" introduced by Magnas-Audibert /3/ with a surface of the 4th order introducing the simple compressive and simple tensile strength as initial constants, and finally the general formulation of Geniyev and Kisyukov /5/ with the surface of the 2nd order introducing the strengths in simple compression, simple tension and pure shear as initial constants. From the last form Geniyev /6/ has deduced also the basic relation of the deformation theory of the plasticity of concrete.

In one of the proposed variants Reimann /4/ selects a surface of the 2nd order with the meridians $\Gamma^{(R)}$ in the form of quadratic parabolas and with a cross section $\Omega^{(R)}$ consisting of straigth lines and segments of circle (Fig.1,2). He introduces uniformly into the constants of the function of the surface the average values of the ratios of the strength of concrete under biaxial equal compression R_u and the strength in simple tension R_z to the strength in simple compression R_D, viz. $R_u/R_D = 1.3$ and $R_z/R_D = 0.1$. The surface can be then defined in relative ordinates

$$\zeta = \frac{z}{R_D}, \quad \varrho = \frac{r}{R_D} \quad \text{and} \quad \delta$$

as follows:

$$\varrho = \varkappa\sqrt{28.969000 - 11.915007\zeta} - 5.170418 \text{ x)} \tag{4}$$

For $\cos\delta \leq \lambda$ is $\varkappa = \lambda$ (5a)

for $1 > \cos\delta > \lambda$ is $\varkappa = \dfrac{1}{\cos\delta + \sqrt{(\frac{1}{\lambda^2} - 1)(1 - \cos^2\delta)}}$ (5b)

The coefficient λ expresses the ratio r_a/r_b. It can be introduced with one average value only in the whole extent of the surface; Reimann recommends the value of $\lambda = 0.635$.

x) In /4/ Reimann gives $\varrho = \varkappa\sqrt{29.06 - 11.93\zeta} - 5.18$. With these approximate values of constants the points representing the strengths in simple compression, simple tension and biaxial equal compression do not coincide with sufficient accuracy with the surface as it is necessary in accordance with the introduced assumption.

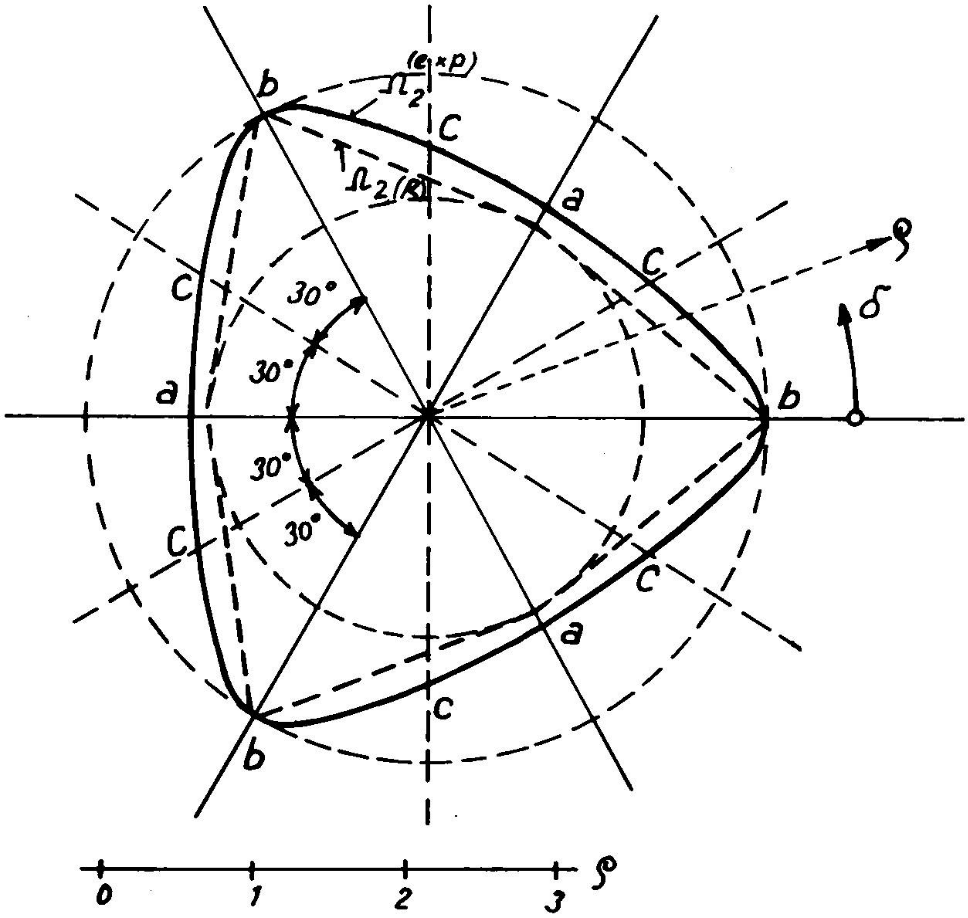

FIG. 2

Cross section of the surface of rupture in the point of $\zeta = 2$. The form of $\Omega_2^{(exp)}$ corresponds with the results of tests carried out by Launay, Gachon and Poitevin /1/, the form of $\Omega_2^{(R)}$ is the idealization according to Reimann /4/.

Le coup transversale de la surface de résistance ultime en point $\zeta = 2$. La forme $\Omega_2^{(exp)}$ appartiens aux résultats des expériments de Launay, Gachon et Poitevin /1/, la forme $\Omega_2^{(R)}$ est idéalisée après Reimann /4/.

Querschnitt durch die Festigkeitsfläche im Punkte $\zeta = 2$. Die Form von $\Omega_2^{(exp)}$ entspricht den experimentellen Resultaten von Launay, Gachon und Poitevin /1/, die Form $\Omega_2^{(R)}$ ist die idealisierte Form nach Reimann /4/.

By means of comparisons with the latest test results it can be proved /Janda /7/, /8/ / that the introduction of this simple expression always remains on the safe side in the sphere currently occuring in the structures. Only in the case of higher stresses, and only in the vicinity of the compressive character of the state of stress, this idealization leads to minor over-estimation of strength.

To be able to deduce the approximate shape of the surface of internal crack initiation from the surface of rupture, we can depart, for the time being, only from incomplete experimental data given, for instance, in the work by Launay and Gachon /2/ which reveal that on the meridian $\Gamma_a^{(exp)}$ (of tensile character), approximately up to the value of $\zeta \doteq -1.2$, the surface of internal crack initiation generally approaches the surface of rupture (i.e.brittle, sudden failure); for higher negative values of ζ the deviation are greater, the greatest negative ordinate being approximately $\zeta \doteq -4.2$ (Fig.1). So far we are justified, within this sphere, to select the analytically simplest possible shape of the idealized surface of crack initiation, although there will be certain minor deviation as compared with the shape assessed on the basis of hitherto executed tests.

Let us introduce, for example, that until the ordinate $\zeta = -2.0$ the course will follow that defined by Eq.(4), and from this ordinate in the direction of the negative axis the surface will have parabolic meridians whose apex will be at the point of $\zeta = -4.20$. The shape of the meridians $A_a^{(exp)}$ and $A_b^{(exp)}$ derived from these tests, and the idealized shapes of $A_a^{(z)}$ and $A_{b_1}^{(R)}$ are compared in Fig.1. This part of the surface has the equation

$$\rho = 2.095870\,\varkappa\,\sqrt{1.909091 + 0.454545\,\zeta} \qquad (6)$$

which holds if $\zeta < -0.2$.

Better approximation can be obtained by an adjustment of Eq.(4) and (6) in the sphere of the meridians $A_b^{(exp)}$ to the form

for $\zeta \geqq -2.0$

$$\rho = \alpha \varkappa \sqrt{28.969000 - 11.915007\zeta} - 5.170418 \qquad (4')$$

and for $\zeta < -2$

$$= 2.095870 \alpha \varkappa \sqrt{1.909091 + 0.454545\zeta} \qquad (6)$$

The satisfactory values of the coefficient α and the coefficient λ which we insert in Eq.(5a) and (5b) are

$\alpha = 0.72$ $\qquad\qquad$ $\lambda = 0.88$

The form of these meridians is marked as $A_a^{(R)}$ and $A_{b,2}^{(R)}$ in Fig.1.

From the surface of crack initiation idealized in this way we deduce the surfaces of permissible stresses as homothetic surfaces with suitably graded homothethy coefficients.

3. Sphere of Compressive Stresses

In the sphere of stresses in which all principal stresses are compressive stresses, use can be made of a single homothethy coefficient which expresses the safety and whose suitable magnitude varies about $m = 2.5$ [x]. As it can be seen from the course of the approximately experimentally determined surface bordering the elastic behaviour of concrete (Fig.1, meridian $B_a^{(exp)}$,

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x) In the French instruction "Circulaire No.44 du 12 août 1965, Ministére des Travaux Publics et des Transports, which is based on Caquot's envelope of Mohr's circles, the safety coefficient of $m = 2.38$ for the compressive sphere and that of $m' = 4.76$ for the tensile sphere are used.

according to Launay and Gachon /21/ in comparison with the idealized course $B_a^{(R)}$ and $B_{b,1}^{(R)}$ and/or $B_a^{(R)}$ and $B_{b,2}^{(R)}$ the introduction of this coefficient simultaneously ensures satisfactorily the elastic behaviour of concrete in the sphere of currently occuring stresses. For the states of stress when $\zeta \dot{<} 1$ the idealized sphere of permissible stresses is somewhat larger than the experimentally determined sphere of elasticity.

In a practical analysis we determine, for the given state of stress $\sigma_1 \geqq \sigma_2 \geqq \sigma_3$, the angle δ from Eq.(3). For this angle we then calculate, from Eq.(5a) or (5b), the coefficient $\varkappa$ for the respective meridian Γ_m of the surface of crack initiation (Fig.3) whose equations are given by the expressions (4) and (6) or (4') and (6'). The given state of stress is represented in the plane of this meridian by point A_1 whose coordinates $\zeta = \zeta_1$, $\rho = \rho_1$ are given by the relation (1), (2) with the introduction of

$$\zeta = \frac{z}{R_D} \qquad\qquad \rho = \frac{r}{R_D}$$

The ratio of the coordinates ζ_m, ρ_m of the intersection point A_m of the vector OA (Fig.3) departing from the origin towards the point A, with the meridian Γ_m, and the coordinates ζ_1, ρ_1 of point A determines the attained safety margin m_1. Thus

$$m_1 = \frac{\zeta_m}{\zeta_1} = \frac{\rho_m}{\rho_1}$$

The coordinate ζ_m is calculated as the root of the quadratic equation

$$\zeta_m^2 + (10.340836 \frac{j}{\varkappa} + 11.915007) \frac{\varkappa^2}{j^2} \zeta_m - 2.235778 \frac{\varkappa^2}{j^2} = 0$$

where $j = \dfrac{\rho_1}{\zeta_1}$. Of physical importance is the root

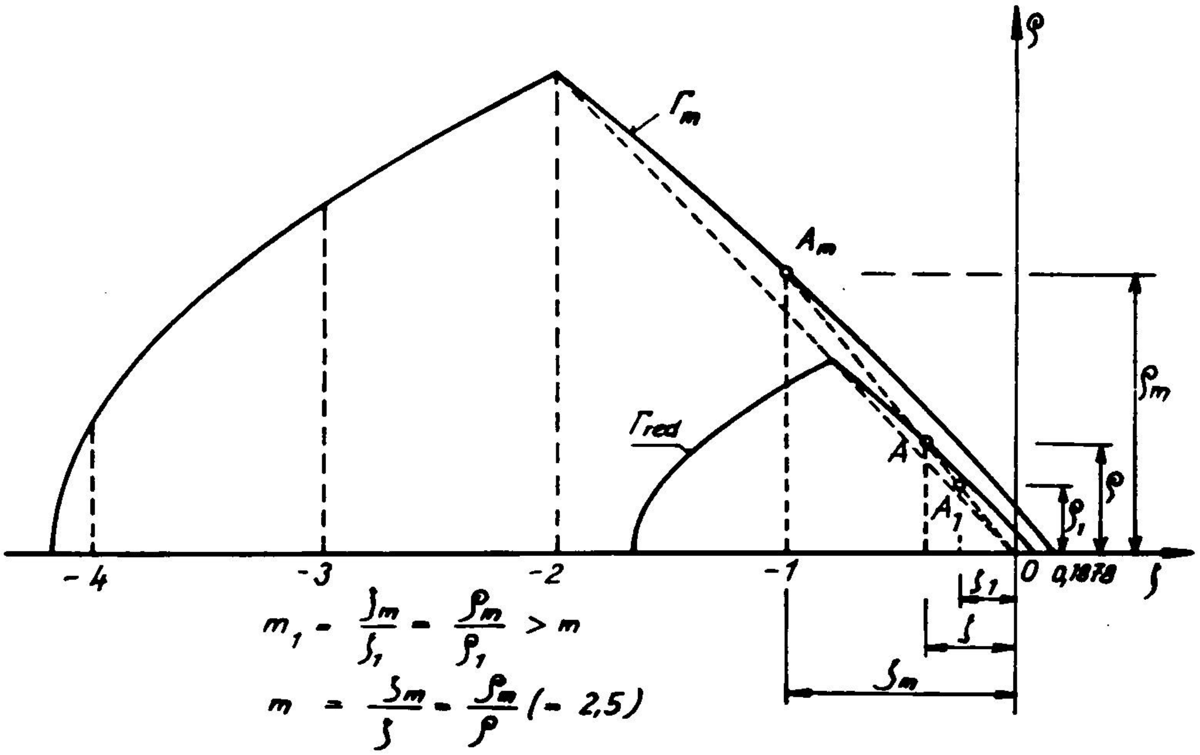

FIG. 3

Homothethy of the meridians of the surface of crack initiation and the surface of permissible stresses. Determination of the attained safety margin m_1 .

La homothétie des méridians de la surface de l' amorce d' une fissuration interne et de la surface des contraintes permissibles. La détermination du coefficient de sécurité m_1 .

Die Omhothetie der Meridiane der Fläche des Beginns von Bildung der Innenrissen und der Fläche des zulässigen Spannungen. Die Feststellung vom Sicherheitsgrad m_1 .

whose sign is identical with the sign of the ratio j.

If $\int_m < -2$, it is necessary to use, instead of (7), the equation

$$\int_m^2 - 1.996645 \frac{\varkappa^2}{j^2} \int_m - 8.386004 \frac{\varkappa^2}{j^2} = 0 \quad (8)$$

The design is satisfactory, if $m_1 \geqq m$.

4. Sphere of Tensile Stresses, Reinforcement

In the sphere in which one or several principal stresses are tensile stresses it is possible, as we have already said, to proceed analogously with the exception that the magnitude of the coefficient m is selected in accordance with whether the structure is to be made of plain, reinforced or prestressed concrete. This calculation, however, is burdened by a number of uncertainties. In spite of this it appears acceptable so far, being a simple process to check up the validity of the assumption of the plastic behaviour of the structure and can form also a suitable basis for the design of the arrangement of reinforcement.

According to experience with the design of concrete structures, which are projected into regulations, the following safety margins m' can be approximately recommended with regard to the idealized surface of rupture in the tensile sphere:

for plain concrete structures	$m' \geqq 6$
for fully prestressed concrete structures	$m' \geqq 4.3$
for partly prestressed concrete structures	$m' \geqq 1.7$
for reinforced concrete structures	$m' \geqq 0.5$ to 0.3 .

That part of the structure in which tensile stresses occur must be provided with reinforcement. The most effective arrangement of reinforcement is a rectangular skeleton whose one system of bars deviates only very little from the direction of the maximum principal tensile stress. When determining the tensile forces to be sustained by the reinforcement it is possible to introduce the customary simplifying assumptions, viz. that concrete is not capable of transferring any tensile stresses and that the shearing stress of concrete in the planes perpendicular to the axes of the bars are resolved into stresses in the direction of the bars and into additional compressive stresses of concrete in the direction deviating by certain selected angles φ_{xy}, φ_{zx}, φ_{yz}, from these planes. The most economic utilization of the reinforcement occurs when these angles are selected at 45°; the value which would be optimal from the view point of the transfer of additional compressions of concrete would be about 30°. The dowel effect of reinforcement is neglected in favour of safety. These assumptions agree with the assumptions introduced, for example, by Kuyt /9/ for the design of the reinforcement of reinforced concrete slabs.

If the reinforcement bars are placed in the directions of the axes x, y, z, the forces transferred by these bars from unit areas of the planes perpendicular to the axes x, y, z are determined by the equations (Fig.4):

$$p_x = \sigma_x + \tau_{xy}\,\mathrm{tg}\,\varphi_{xy} + \tau_{xz}\,\mathrm{cotg}\,\varphi_{zx} \tag{9a}$$

$$p_y = \sigma_y + \tau_{yz}\,\mathrm{cotg}\,\varphi_{xy} + \tau_{yz}\,\mathrm{tg}\,\varphi_{yz} \tag{9b}$$

$$p_z = \sigma_z + \tau_{zx}\,\mathrm{tg}\,\varphi_{zx} + \tau_{zy}\,\mathrm{cotg}\,\varphi_{yz} \tag{9c}$$

where

σ_x, σ_y, σ_z, τ_{xy}, τ_{xz}, τ_{zy} are normal and tangential stresses of concrete respectively.

Additional compressive stresses of concrete applied in the planes of xy, yz and zx are

$$\nu_{xy} = \nu_{yx} = -\frac{\tau_{xy}}{\sin\varphi_{xy}\cos\varphi_{xy}} \tag{10a}$$

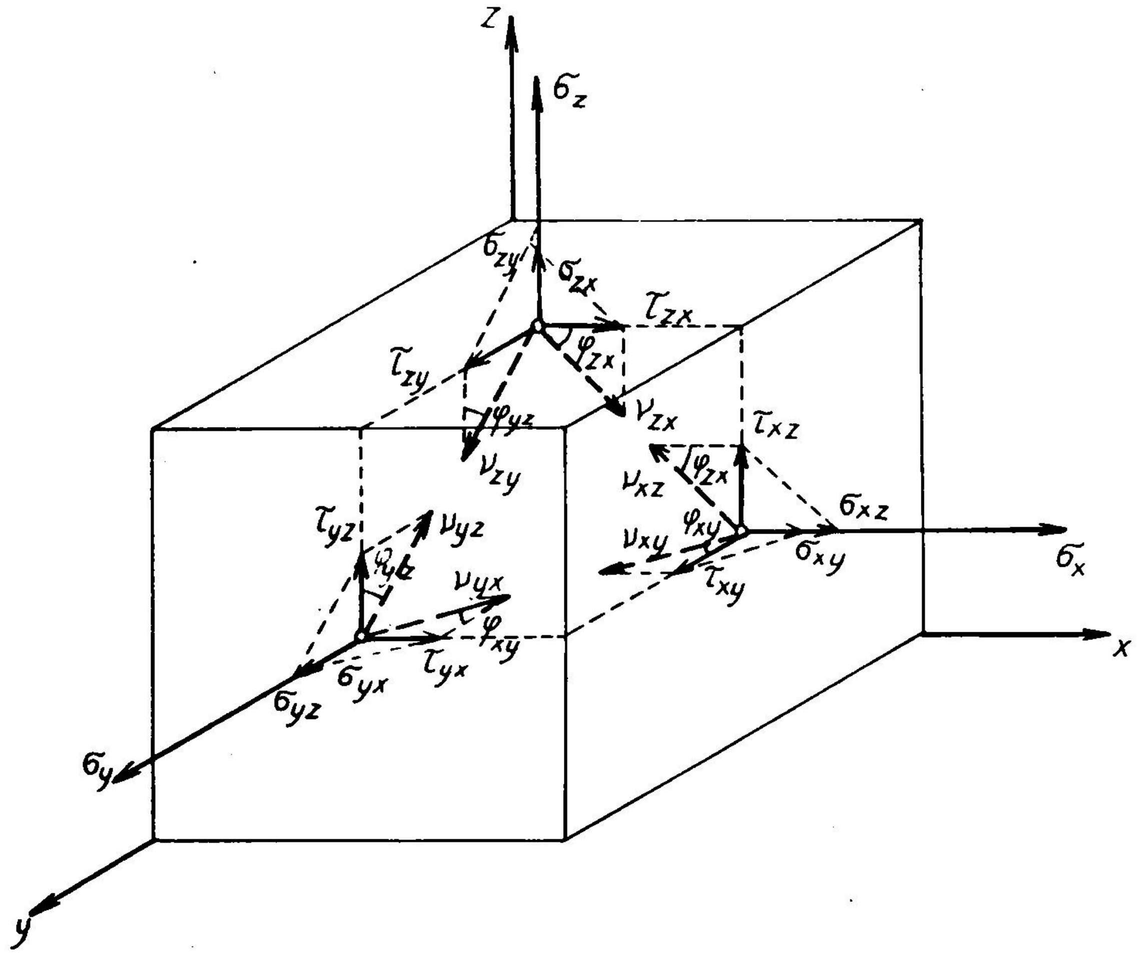

FIG. 4

Resolution of forces for the assessment of reinforcement stress.

La décomposition des efforts pour examiner la tension de l' armature.

Das Zulegen der Kräfte für Spannungsbeurteilung der Bewehrung.

$$\nu_{yz} = \nu_{zy} = - \frac{\tau_{yz}}{\sin \varphi_{yz} \cos \varphi_{yz}} \tag{10b}$$

$$\nu_{zx} = \nu_{xz} = - \frac{\tau_{zx}}{\sin \varphi_{zx} \cos \varphi_{zx}} \tag{10c}$$

In conclusion mention should be made of the fact that the reinforcement designed under these assumptions, i.e. under the assumption of an elastic, homogeneous structure, is often somewhat heavier than it would be if the design were based on the assumption of the ultimate load and wide opening of cracks. However, to be able to recommend a certain reduction of the quantity of reinforcement, it would be necessary, particularly in the case of structures from which watertightness or the limitation of crack width are required, and in the case of structures exposed to axial stresses, to verify this method experimentally.

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Summary

The theoretical basis of the calculation of the state of stress of the concrete structure stressed triaxially under performance loading conditions is the elastic or visco-elastic behaviour of concrete. If this condition should be fulfilled it is necessary that the stress of concrete does not exceed the boundaries of the elastic or visco-elastic behaviour. In the proposal of the method of analysis, therefore, the permissible stresses surface of concrete is derived from the crack initiation surface.

The form of both surfaces is idealized and based on the results of laboratory tests hitherto executed, above all in France, (Launay, Gachon, Poitevin). To achieve the simplest analytical expression possible the formulas are based on the form suggested by Reimann amended in the field of big all-directional compressions. For the fields in which tensile stresses transferred by reinforcement occur, it is recommended to base the considerations also on the assumption of elastic, homogeneous material. The paper suggests variable safety coefficients with regard to the permissible stresses surface, differentiated for the structures of plain, reinforced and prestressed concrete, and presents the formulas for the calculation of reinforcement stresses.

Résumé

C'est le comportement élastique respectivement visqueux-élastique du béton qui constitue la base théorique pour le calcul de l'état de tension des construction en béton soumises aux efforts tridirectionnels pendant l'état de service de la charge. Pour accorder cette condition, il faut que le taux de travail du béton ne dépasse pas la zone du comportement élastique respectivement visqueux-élastique. En proposant le mode de l'analyse, la surface des contraintes admissibles du béton dérive pour cette raison de la surface du début de développement des fissures intérieures. La forme de deux surfaces est bien idéalisée, sortant des résultats obtenus jusqu'ici par des essais de laboratoire, ceux des français avant tout (Launay, Gachon, Poitevin). Pour arriver à une expression analytique la plus simple que possible, c'est la forme projetée par Reimann et adaptée dans la zone de grandes pressions de tous sens qui représente le point de départ. Pour des zones comportant les contraintes de traction transmises à l'aide de l'armature, on recommande de partir également de la conjecture d'une matière homogène élastique. En raison de la surface des contraintes admissibles ainsi que de la différenciation des constructions en béton ordinaire, armé et précontraint on propose les coefficients variables de sécurité. On indique les formules pour le calcul de la contrainte de l'armature.

Zusammenfassung

Den theoretischen Grund für die Bemessung der dem dreiachsiagen Spannungsrustand unterworfenen Stahl-und Spannbetonkonstruktionen unter Betriebsbelastung bildet das elastische, resp. visko-elastische Benehmen von Beton. Wenn diese Bedingung erfüllt werden soll ist es notwendig, dass die Betonspannung nicht den Raum des elastischen, resp. visko-elastischen, Benehmens überschreitet. Bei dem Entwurf des Bemessungsfortganges wird deshalb die Fläche der zulässigen Spannungen von der Fläche des Beginns von Bildung der Innenrissen abgeleitet. Die Form beider Flächen ist idealisiert. Es wird von der Resultate der bisherigen Laboratorprüfungen, vor allem von den französischen (Launay, Gachon, Poitevin), herausgegangen. Zwecks maximaler analytischer Vereinfachung benützt man die von Reimann entworfene Form, die in dem Gebiet von grossen allseitigen Drücken angepasst wurde. In den Gebieten, wo mit Stahleinlagen übernommene Zugspannungen erscheinen, wird es empfohlen, die Voraussetzung vom elastischen und homogenen Material zu bewahren. Es werden veränderliche Sicherheitskoeffiziente im Bezug zur Fläche der Zulässigen Spannungen, für den Fall von Beton, Stahl-und Spannbeton, entworfen. Es werden Formeln für die Berechnung von Stahlspannungen angegeben.