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## New Prospects for Evaluating the Degree of Safety in Concrete Structures Subjected to Multiaxial Stresses

*Nouvelles perspectives pour l'évaluation du degré de sûreté dans les  
structures en béton soumises à des sollicitations multiaxiales*

*Neue Aussichte für die Bewertung des Sicherheitsgrades in den multiaxialen  
Spannungszuständen untergezogenen Betonstrukturen*

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**SUMMARY.** New prospects are discussed for safety criteria based on failure surface and on a set of inside surfaces limiting different physico-mechanical states of concrete before failure. These surfaces have been deduced by systematic tests performed on full scale concrete and have been checked for microconcrete used in the PCPV models.

**RESUME'.** Ce rapport traite de nouvelles perspectives pour des critères de sûreté basés sur la surface de rupture et sur une série de surfaces internes limitant des états physico-mécaniques différents du béton avant la rupture. Ces surfaces ont été obtenues par des essais systématiques effectués sur béton réel et ont été vérifiées aussi pour le micro-béton employé dans les modèles PCPV.

**ZUSAMMENFASSUNG.** Der Beitrag handelt von neuen Aussichten für Sicherheitskriterien auf Basis von Bruchflächen und von einer Innenflächen-Reihe, die die verschiedenen physikalisch-mechanischen Betonzustände vor dem Bruch abgrenzen. Diese Flächen wurden durch systematische auf naturgrossem Beton durchgeführten Versuche bestimmt und wurden auch für in den PCPV - Modellen benutzten Mikrobeton geprüft.

## INTRODUCTION

The ever-growing need to work with multiaxial stresses, exploiting to the maximum the possibilities of material without forgoing a high degree of safety, has led many researchers to study in depth the behaviour of material in the field of multiaxial stresses.

On the other hand, the lack of sufficiently accurate and complete theories, valid for all possible real situations, has induced designers and civil engineers concerned with structures of great importance and involving great responsibility to take more and more into account the results of experimental research, including that on models.

Moreover, the most recent calculation methods (such as the finite element methods), which are more and more refined and complex owing to the ever-growing capacity of modern computers, also call for more accurate and complete "constitutive equations" in order better to describe any structure in any operational condition.

Bearing in mind all the complex problems associated with the realization of a concrete structure, particular importance must be ascribed to the evaluation of its safety, not only as regards final collapse, but also in respect of loss of efficiency, as a function of the task that the structure has to perform.

This paper aims at a critical appraisal, in the light of the most recent experimental results, of the aforementioned problem, and at explaining the point of view of the experimental researcher in connection and compared with the requirements of the computing and designing engineers of concrete structures.

## ANALYSIS AND DISCUSSION ON THE MOST RECENT EXPERIMENTAL RESULTS IN RELATION TO THE EVALUATION OF THE DEGREE OF SAFETY.

It is well known that the experimental results so far obtained by the best-qualified research laboratories in the world make it possible to build up, with sufficient accuracy, in the principal stress space, the failure surface, through which it is possible to determine whether a given stress state can cause failure. This fact is of fundamental importance, since it clearly establishes the extreme limits of use of the material; and on this experimental fact are based the up-to-date design criteria for concrete failure.

However, when we are sure we are in a domain which is "far" from a condition of failure (namely when stress states are very "internal" to the failure surface), it is very important to know how the material behaves in the actual stress condition, and above all the degree of safety

associated with such a stress condition, not only as regards the failure, but especially in relation to the loss of efficiency, if any, of the structure itself. To this end, knowledge of the failure surface only is no longer sufficient, also because the loss of efficiency of the structure, or of any part of it, does not generally coincide with the condition of collapse.

Neither may be considered completely satisfactory, for establishing the degree of safety, definitions that lead to the determination of coefficients as "values" which somehow or other "map" or "compare" the actual stress condition with the failure stress condition.

Indeed, the failure stress state to which refer cannot definitely be determined from among the infinite points of the failure surface; neither it is always permissible or possible to fix a priori the "stress path" along which the failure should be reached.

Moreover, the failure surface itself, as will be seen later, cannot be univocally determined, apart from the indetermination stemming from dispersion of the experimental results, since the failure states depend intrinsically on the "stress path". This fact has led certain authors to look for failure criteria which are as far as possible "consistent" with all available experimental results, in the sense that these criteria must, to a certain extent, guarantee safety under the worst conditions. However, the definition of "worst condition" is itself in this sense questionable; it is, in fact, undetermined, in the same way as the actual mechanical state is undetermined.

To remove the indetermination, we have inseparably to associate with each point of a given stress path the strain variables, and a certain number of parameters able to fully describe the physical state of the material; these parameters are not generally point functions, but depend on the whole history of the material.

From what has been said, it follows that the failure problem cannot be considered separately from the general problem of knowledge of the "constitutive relations" of a given material, unless this be done arbitrarily and in order to simplify research, study and practical applications.

To avoid the complexity of the problem, and to arrive at satisfactory practical results just the same, it remains, in our opinion, to emphasize the experimental research in order to investigate and locate those factors that are, as far as possible, characteristic and determinant of the occurrence of fundamental phenomena considerably affecting the "constitutive relations". Of these phenomena we may mention: plastic, viscous and anisotropic phenomena; and variations of the inner structure of concrete (microcracks, cracks, deterioration, ageing, stress hardening).

Indeed, although concrete is considered a brittle material, it shows very pronounced elasto-plastic behaviour, with subsequent changes in its internal structure long before the failure: namely, at stress states well within the failure surface.

This has led experimental researchers to concentrate much more of their attention on microcrack initiation and propagation, and on the connections, if any, with characteristic variations in the material, and with changes in the physical state of the concrete's inner structure.

J. B. Newman and K. Newman introduced the expression "discontinuity limit" to indicate the limit at which the first phenomena of the irreversible type (in particular, microcracks) appear, and the expression "working stress" for all the stress states "internal" to such a limit. Thus, a practical safety criterion was already established, based on the evidence of experimental results, in the sense that once it was certain that stress states are and remain inside the "discontinuity limit", there is no question of failure occurring, irrespective of the loading regime to which the structure is subjected.

The study and experimental research of irreversible phenomena has been greatly furthered at the Niguarda Laboratories of the Hydraulic and Structural Research Centre - Research and Development Department of the Italian State Electricity Board (ENEL), confirming on the one hand, and improving the results already obtained, and, on the other, identifying and characterizing the evolution of irreversible phenomena, such as microcracks and cracks, as the stress condition varies.

At the present stage of research it is already possible to have available results of fundamental interest, albeit approximate, and, in our opinion, to foresee the possibility of practical utilization of these results, with particular reference to evaluation of the reliability and degree of safety of a structure, both with regard to failure and to its efficiency.

The process of microcrack formation and propagation is to be considered a decisive cause of concrete fracture and failure, and is characterized by three fundamental stages.

The first stage is marked by the appearance of microcracks which are located at stress-concentration points, favoured by the heterogeneous composition of the concrete. When these phenomena take place, the beginning of "non-linearity" in the stress-strain curve may be found, with the occurrence of plastic deformation and departure from the linear creep assumption.

There then begins a second stage, during which the previously located microcracks start propagating, although the separate formation of

new stable microcracks continues. This process of microcrack propagation is characterized by the fact that, should the stress be "frozen" in the present state, the propagation stops: this is referred to as "stable propagation".

In the third and last stage, the propagation process becomes unstable, that is, the microcracks and cracks that occurred previously, are selfpropagating. The large amount of energy accumulated at the numerous stress concentration points can cause and maintain the process of disintegration, even when the stress condition is "frozen".

Since failure is unavoidable, as soon as this "stress level" is reached, the latter can be considered either as an upper-bound criterion for failure, or as the true ultimate strength.

Experiments carried out at the Niguarda Laboratories have made it possible to locate, with sufficient accuracy, the separation limits to which the previously-described stages correspond, with regard to compression-compression-compression zone, and to obtain, therefore, a subdivision of the principal stress space, within the failure surface, into four domains (Fig. 1).

#### CONSIDERATIONS ON THE EVALUATION OF THE DEGREE OF RELIABILITY AND SAFETY OF STRUCTURES.

At the present stage of research, it seems advisable that any consideration concerning the degree of reliability and safety should be related to the physical condition of concrete represented by the above-described four fundamental domains. This may be done by laying it down that all the stress states included within one of the four domains are to be considered as belonging to one and same group. Thus, the stress states are subdivided into four fundamental groups, in relation to the ensemble of the physico-mechanical phenomena that they can induce in the material (Fig. 1).

Therefore, both general and particular (valid for a given structure or parts of it) safety criteria can be established by demanding that suitable stress states belong to one or more predetermined groups, depending on the efficiency and safety requirements of the whole structure, or of its special parts.

Processes of this type, once the rules for determining general or particular criteria have been fixed, can be directly and easily applied, both at the checking stage of an existing structure, and at the design stage,

and can easily be included in complex calculation programs, such as those with finite elements.

### INFLUENCE OF TIME AND STRESS-PATHS

As regards the time effect, experience shows that the failure surface undergoes changes.

As concrete improves with time (ageing), strength limits become greater, whereas deterioration of concrete over a period of time involves a reduction of these limits. This type of variation can be evaluated by means of suitable experiments which allow for exact way through which these effects appear. The effect of concrete curing does not generally occur, since the structure starts "working" when the material has reached an optimum curing condition; whereas the deterioration effect may be evaluated through suitable quality controls on concrete samples obtained from the structure itself.

However, apart from the aforementioned phenomena, experience shows that owing to the existence of loads at certain levels or due to the way in which they may vary with time, the failure surface can "reduce" itself (Fig. 2).

For instance, tests have been conducted under monoaxial compression conditions at the Niguarda Laboratories on highly plastic, low-strength concrete (about  $100 \text{ kg/cm}^2$ ) in order to make the test machine much more "rigid", compared with the sample, and therefore to reduce to a minimum the potential energy accumulated by the machine, which is discharged on to the sample near the breaking point. The results obtained have shown that there is a significant ultimate strength loss between a test lasting 2 minutes and a test lasting 45 minutes, and a marked and progressive increase in deformation, mainly due to creep.

Although experiments to ascertain whether limit surfaces other than the failure surface diminish have not yet been completed, the previous subdivision of the principal stress space into the four fundamental domains remains entirely valid.

Therefore, the aforementioned surfaces can ideally be shown as surfaces at the real time  $t_0$ , all previous considerations remaining valid.

As regards stress paths, it is clear that, especially for the evaluation of the structure's safety, their influence cannot be disregarded, unless these paths are inside the domain bounded by the discontinuity surface (1<sup>st</sup> domain). Beyond such a limit, as first approximation and for reasons of greater safety, in our opinion, a stress state lying within a

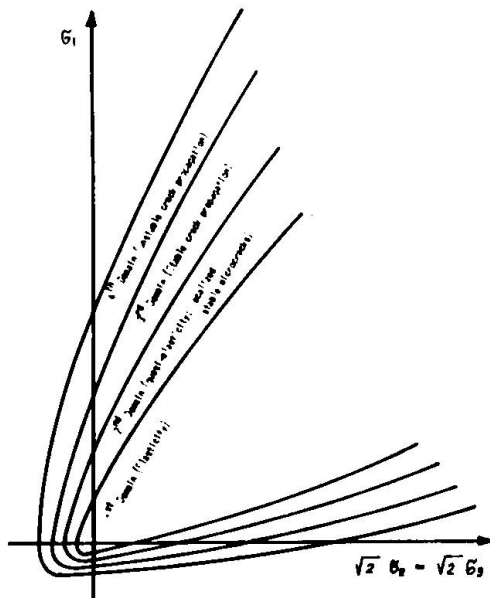


FIG. 1

- Domain limiting curves in Rendulic plane representation
- Représentation dans le plan de Rendulic des courbes délimitant les différentes zones
- Abgrenzungskurven der Gebiete in der Rendulic-Ebene

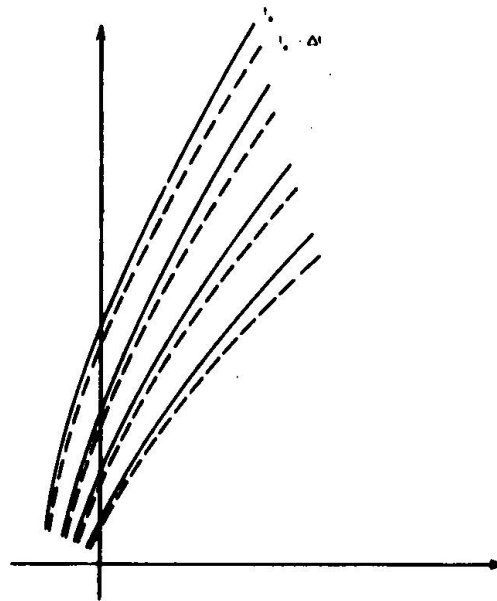


FIG. 2

- Surface "Reduction" in Rendulic plane representation
- Réduction des surfaces dans la représentation dans le plan de Rendulic
- Flächen-Verminderung in der Rendulic-Ebene

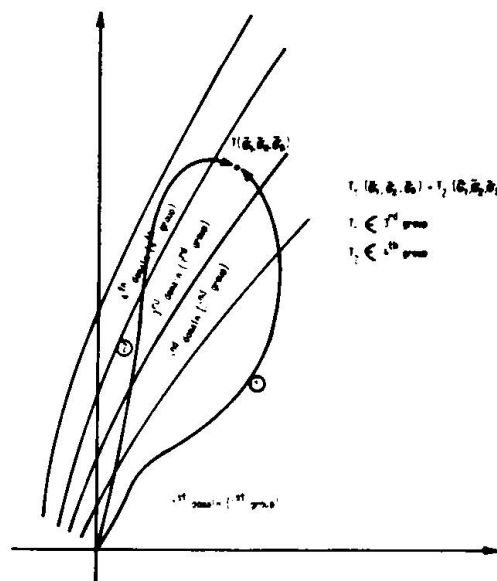


FIG. 3

- Different stress-paths in Rendulic plane
- Différents parcours de sollicitation dans le plan de Rendulic
- Verschiedene Spannungswege in der Rendulic-Ebene



given domain, but which has been reached along a stress path passing through one of upper domains, is to be considered as not belonging to the group corresponding to the domain in which the actual stress state lies, but to the group corresponding to the domain crossed in its upper part by the stress path (Fig. 3). In such a way a subdivision of stress states is implicitly introduced, also as a function of the stress history.

## APPENDIX

The experimental results referred to in this paper have been obtained from tests carried out on different types of concrete, produced in the laboratory with aggregate of a maximum of 30 mm in size. Moreover, experiments have been conducted on specimens prepared with micro concrete (maximum size of aggregate: 8 mm), used at the Experimental Institute for Models and Structures - ISMES, Bergamo, for the construction of models for nuclear reactor vessels. (PCPV).

These latter tests, besides furnishing the elastic and mechanical characteristics used for the calculation and the interpretation of the results concerning experiments carried out on models, have shown that this particular type of concrete behaves under multiaxial stresses like full-scale concrete, especially as regards the considerations developed in this paper.

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List of papers concerning the experimental research at the Niguarda Laboratories of the "Research and Development Department"(ENEL):

- 1) BERTACCHI (P) "Behaviour of concrete under combined loads; a comparison of the concrete shearing strength values obtained from direct tests with the values determined from triaxial tests"  
Eighth International Congress on Large Dams -  
Edinburgh 1964 - Vol. III - 279 p.
- 2) BERTACCHI (P.) and BELLOTTI (R.) "Experimental Research on deformation and failure of concrete under triaxial loads" and complementary paper  
RILEM International Symposium  
I3, Vol. I - p. 37-52, Vol. IV - p. 54-65, October 1972, Cannes
- 3) ROSSI (P.) and BELLOTTI (R.) "Comportamento del calcestruzzo sottoposto a sollecitazioni pluriassiali: indagini sperimentali per la determinazione della superficie di rottura (Failure surface)"  
Relazione n° 167 Serie Verde ENEL, Giugno 1973
- 4) BELLOTTI (R.) and ROSSI (P.) "Comportamento del calcestruzzo sottoposto a sollecitazioni pluriassiali: - Ulteriori ricerche sulla superficie di rottura. - Prime indagini per la determinazione sperimentale degli stati fisici del calcestruzzo precedenti alla rottura"  
Relazione n° 168 Serie Verde ENEL, Settembre 1973