

**Zeitschrift:** IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

**Band:** 19 (1974)

**Artikel:** Reliability of small-scale models in predicting behavior of concrete structures subjected to multi-axial stresses

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**DOI:** <https://doi.org/10.5169/seals-17529>

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## Reliability of small-scale models in predicting behavior of concrete structures subjected to multi-axial stresses

*Crédibilité des maquettes en échelle réduite dans la prévision du comportement  
de structures en béton soumises à des contraintes multiaxiales*

*Zuverlässigkeit von Arbeitsmodellen in kleinem Mass-Stab  
zur Vorausbestimmung des Verhaltens von Mehrachsigen  
Spannbeton-Konstruktionen*

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### INTRODUCTION

The reliability of physical models for predicting the post-cracking behavior of complex reinforced concrete structures is not well established in the minds of many engineers, particularly in the U.S.A. A surprisingly large number of engineers are skeptical of test results from models, with their skepticism increasing as the size of the model decreases. Discrepancies between behavior of full size structures (or large models) and that of smaller scaled models are often attributed to "size effects" that simply arise without explanation. The fact is that such discrepancies are most often caused by a lack of satisfying known similitude conditions. It is the opinion of the author that reliable modeling of concrete structures carrying complex multiaxial stress states can be done at very small scales, and that the limiting factor on model sizes is difficulty met in fabrication and instrumentation rather than any inherent problems generated by the smallness of the model. Several examples will be explored in this paper to document recent experience in utilizing small models to predict elastic, inelastic, and failure behavior of complex reinforced concrete structures. In each case comparison is made with similar behavior measured on either large models or full-size specimens.

The crucial factor in achieving high reliability is modeling is the satisfaction of similitude requirements for materials properties. True modeling is the preferred approach, where model and prototype strengths and failure strains are identical. Modeling the tensile strength and failure criterion for model concrete is perhaps the most important factor of all. The post-

yield behavior of the model reinforcing steel is extremely important in high-intensity cyclic loading effects. The model materials requirements and properties will be discussed with each example rather than as a separate item.

Any given model being built in a given laboratory has an optimum scale factor. Very small models require light loads but can present rather formidable problems in fabrication. Large models are easier to build but require much heavier loading and handling equipment. The loading equipment is not a serious problem in a laboratory that is fully equipped to conduct tests on large structures, but it is a severe handicap in a smaller laboratory. Typical scale factors for several classes of structures are:

<u>Type of Structure</u>	<u>Scale factors</u>	
	<u>Elastic model</u>	<u>Ultimate strength model</u>
Shell roof	1/200 to 1/50	1/30 to 1/10
Highway bridge	1/25	1/20 to 1/4
PCRV's	1/100 to 1/40	1/20 to 1/4
Slab structures	1/25	1/10 to 1/4
Dams	1/400	1/75
Wind effects	1/300 to 1/30	not applicable

The emphasis in the Cornell University Structural Models Laboratory has been on very small scale inelastic models. Having reliable modeling techniques at this scale enables one to conveniently and economically explore behavior of complex structural geometries that are prohibitively expensive and space-consuming at larger scales. Recent research studies at Cornell have included cylindrical shell behavior, hyperbolic paraboloid shell behavior, portal frames under cyclic sway loads, infilled shear wall frames, multi-story, multi-bay reinforced concrete frames subjected to combined gravity loads and reversing simulated seismic forces, prestressed concrete pressure vessels loaded to failure, and the factors influencing compressive and tensile strength values of small gypsum concrete cylinders.

With very small scale models, one needs only a modest facility to explore structural behavior problems that can never be tested on full-scale structures. While this modeling philosophy is not necessarily appropriate for laboratories engaged in significant amounts of commercial model studies, it seems to work well in educational institutions where the models are for research and instructional purposes and the available space and resources are not extensive.

#### RESPONSE OF REINFORCED CONCRETE FRAMES TO SEVERE REVERSING LOADS

The initial part of this program included tests on 1/10 scale models of full-size beam-column joints subjected to reversing moments (Refs. 1,2). The full scale test were conducted by Hanson and Conner at the Portland Cement Association Laboratories. The model tests were done to fully substantiate the ability of small doubly reinforced concrete model elements to portray all levels of the complex behavior encountered in multi-story frames.

The specimen geometry, which represents the exterior beam-column joint of a high-rise building between the inflection points normally found under transverse loading conditions, is shown in Fig. 1. Loading for the four prototypes and seven models was full reversal of beam bending according to the following schedule:

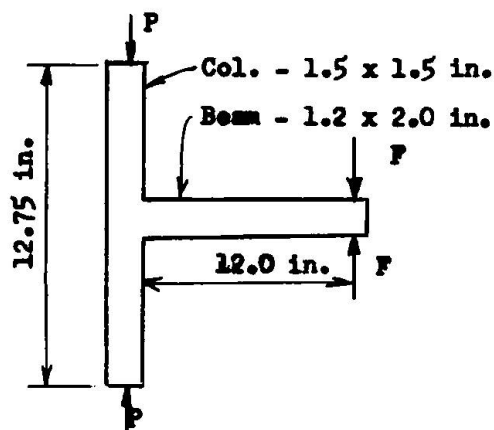


Fig. 1 - Beam-Column Joint

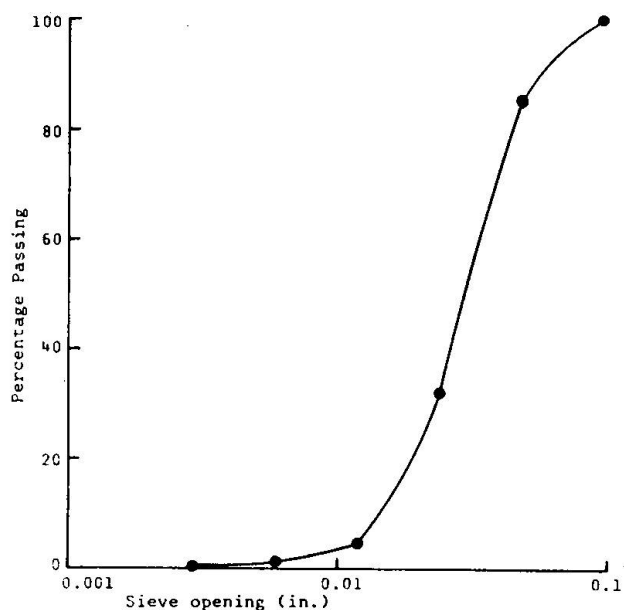


Fig. 2 - Gradation of Filter Sand

Cycle:                      1      2      3      4      5      6      7      8      9  
 Ductility factor: 0.75   2.5   4.0   0.75   0.75   0.75   5.0   5.0   5.0

where the ductility factor is defined as the ratio of rotation in the beam at the prescribed load to that at yield load, measured over a length of one-half the beam depth from the critical section (face of column). The loading is intended to represent two major earthquake shocks. The column of each specimen was also subjected to simultaneous axial load.

The concrete in the column, at the column-to-beam junction, is subjected to reversing combinations of direct stress and shear; modeling this behavior places severe demands on the failure properties of the model concrete. The model concrete was composed of Type III high early strength cement, filter sand, and water in the ratio of 1:5:0.8. The combination of filter sand particle shape (round and smooth) and gradation that minimizes the very fine particle content (Fig. 2) produces a model concrete that has a ratio of tensile to compressive strength ranging from 0.12 to 0.15, which is very similar to that of structural concrete. The uniaxial compressive stress-strain curves for a number of model concretes developed at Cornell and MIT are given in Fig. 3. Test cylinders are either 1.5 by 3.0 in., or 2 by 4 in.

The model reinforcing consisted of deformed wires, 0.113 in. diameter in the beam and a mixture of 0.159 in. and 0.113 in. diameter in the column. 0.041 in. wire beam stirrups and 0.054 in. wire column ties were used. For a typical model-prototype pair of specimens, the material properties were:

	Concrete, psi		Main steel yield, ksi		Hoop steel yield, ksi
	Beam	Col.	Beam	Col.	
Prototype	3200	5325	47.8	69.8	52.8
Model	3150	4703	45.1	68.2	54.5

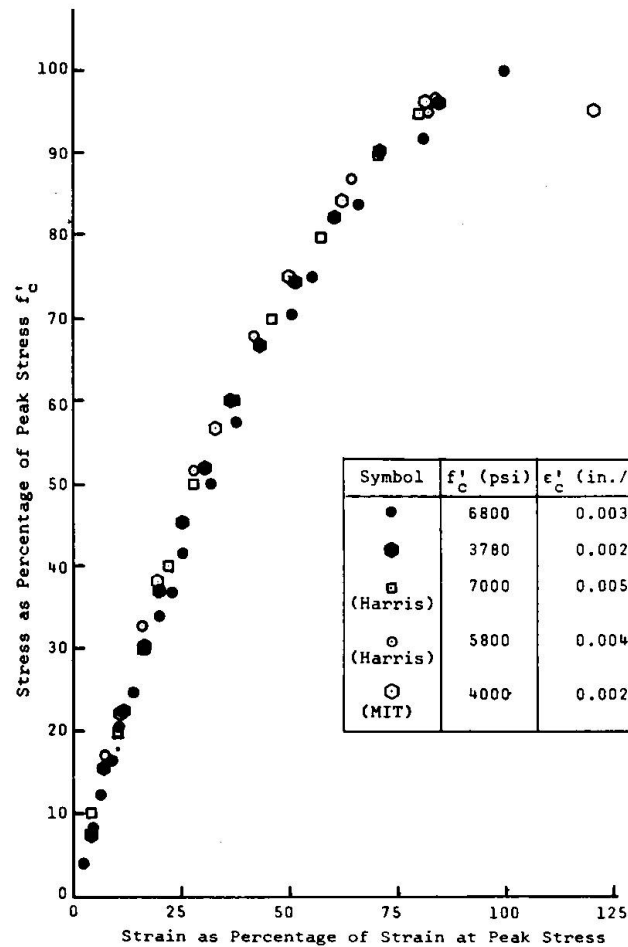


Fig. 3 - Compressive Stress-Strain Curves for Microconcretes

While these corresponding model and prototype quantities may appear to be quite close to each other, the model beam reinforcement strength was 6% low, which would obviously affect the modeling of initiation of yield and post-yield deformations. Thus a stress scale factor of 1.06 was included in scaling the prototype loads down to model values.

Some of the early models had overstrength concrete and did not faithfully reproduce joint cracking patterns. However, with proper strength concrete, the crack similitude at various loads, as shown in Fig. 4, was remarkably good. Moment-rotation response of model and prototype was also nearly identical (Fig. 5). Beam deflection correlations are given in Fig. 6.

Two additional identical models were tested to check repeatability of the load-deformation characteristics of the beams. At the seventh load cycle, for example, the ratios of peak values of beam rotation (model to scaled prototype) were (0.94, 1.20) on the down cycle and (0.92, 0.99) on the up cycle. These values are representative of the accuracies achieved.

Model predictions for beam and column reinforcing stresses and for other prototype specimen designs were also good to excellent. Having established the reliability of modeling the joint behavior under severe cyclic loads, the same techniques were used in studying the response of two-bay, three-story reinforced concrete frames under gravity loads and statically applied seismic forces (Ref.

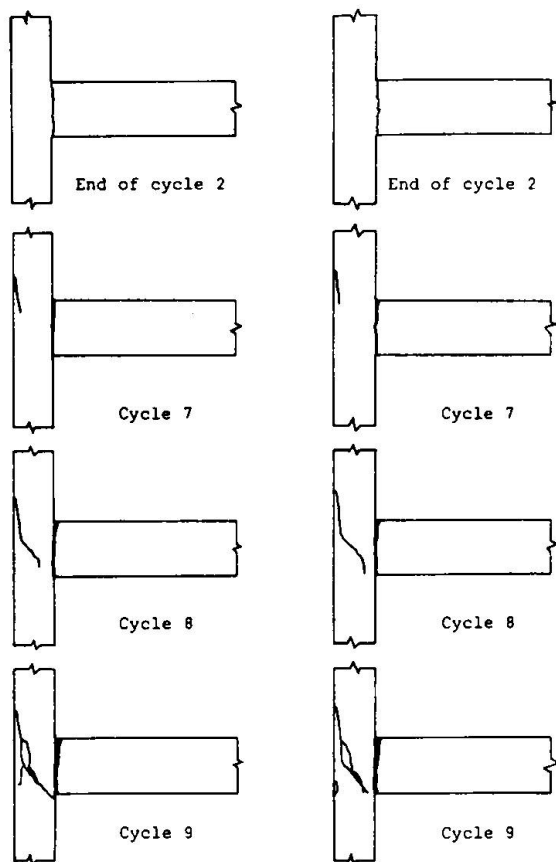


Fig. 4 - Major crack patterns-- Model A3 to left, Prototype I-A to right, for peak downward loads at different cycles.

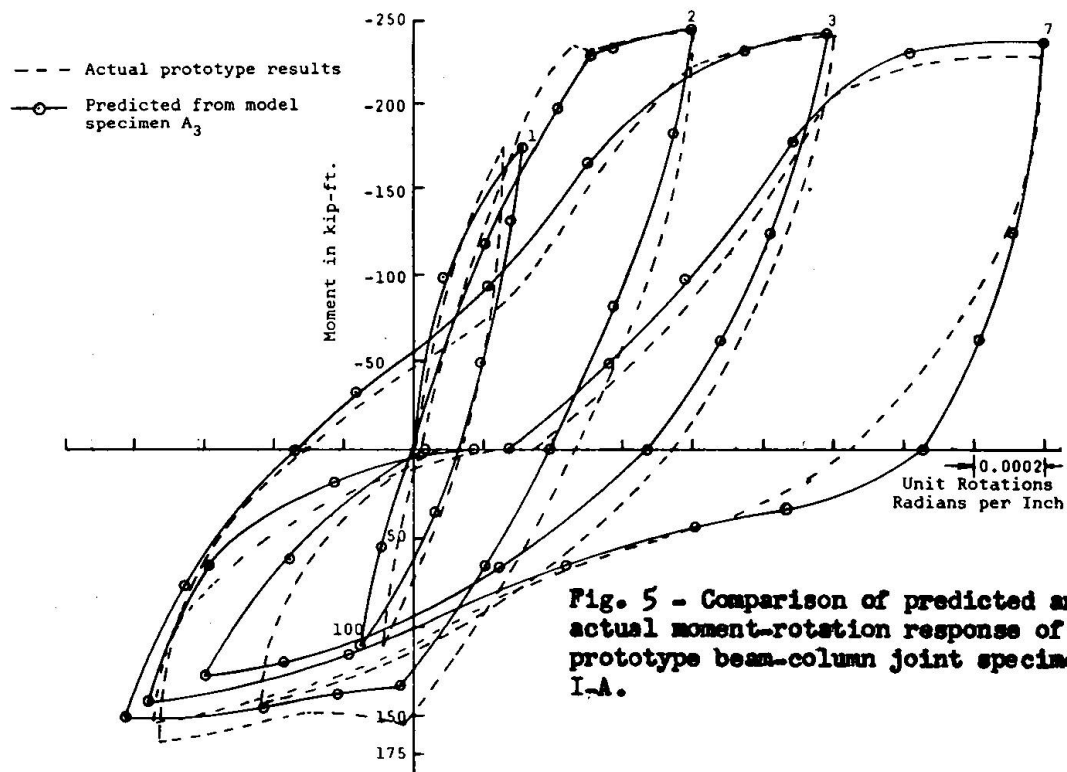


Fig. 5 - Comparison of predicted and actual moment-rotation response of prototype beam-column joint specimen I-A.

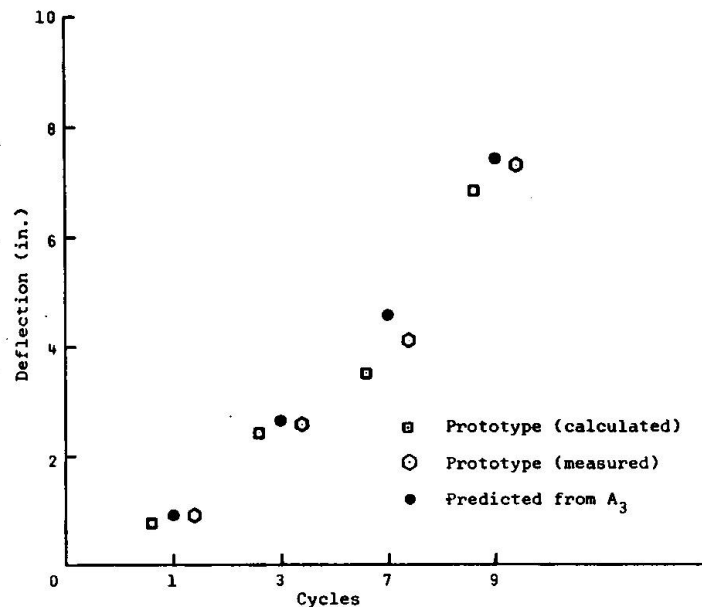


Fig. 6 - Prototype and predicted peak deflections, from Model A<sub>3</sub>, down-load.

2 and 3). These results are not discussed here because there are no prototypes for comparison. Three-dimensional frame behavior is the next logical step in this research program.

#### PRESTRESSED CONCRETE REACTOR VESSELS

A study was undertaken jointly by Cornell University and the Oak Ridge National Laboratory (ORNL) to investigate the suitability and reliability of small scale models for determining certain behavioral aspects of a typical PCRV (Refs. 4 and 5). Modeling objectives included response to elastic loadings (prestressing, working pressures) and inelastic behavior (crack initiation and development, failure mode, and ultimate strength). Thermal effects and long-time inelastic behavior (creep) were considered to be not feasible at the small scales utilized.

The "prototype" structure for this study was in reality a small prestressed concrete vessel, as shown in Fig. 7, with dimensions on the order of 1/10 those of a full size PCRV. Two very small microconcrete models were done at a geometrical scale of 1/2.75, which produces a model size at or near the minimum feasible size. Smaller, 1/5 scale epoxy models were also tested to give elastic response only. The design details for the prototype are given in Fig. 8.

The most critical material property in this type of modeling is concrete tensile strength since the concrete is going from a state of general triaxial compression to a state of triaxial tension, or combinations of smaller compression and tension. Lacking a fully defined failure criterion for either concrete or mortars, the basic requirements specified for the model concrete were that it should match the compressive strength, the split cylinder strength, the modulus of elasticity, and the ultimate compressive failure strain of the prototype concrete. Strength values were to be determined at 28 days and after 2 months of room temperature drying since the 28 day strength properties are not necessarily



The model concrete was made with normal Portland cement, glacially deposited natural sand, and crushed rock coarse aggregate in the ratio of 1:3:1 with a water/cement ratio of 0.52. Stress-strain curves for the prototype and model concretes are given in Figs. 9 and 10, respectively. The prototype concrete had an unusually low modulus, however, and it was not well reproduced in the model concrete. Properties are summarized as follows, for the 90 day dry condition:



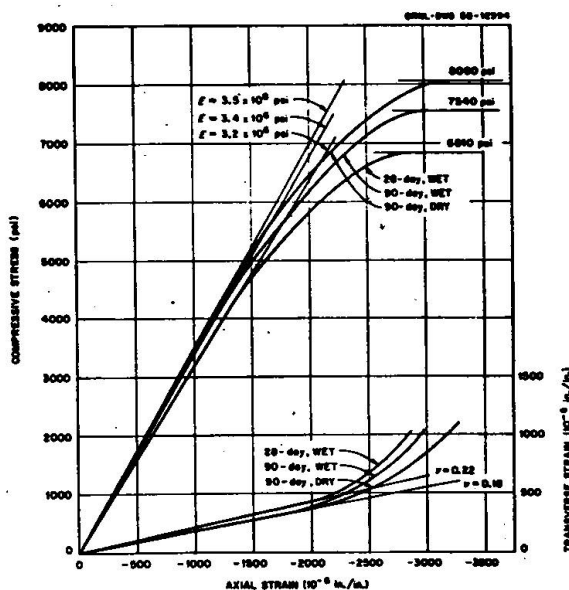


Fig. 9 - PCRV Concrete

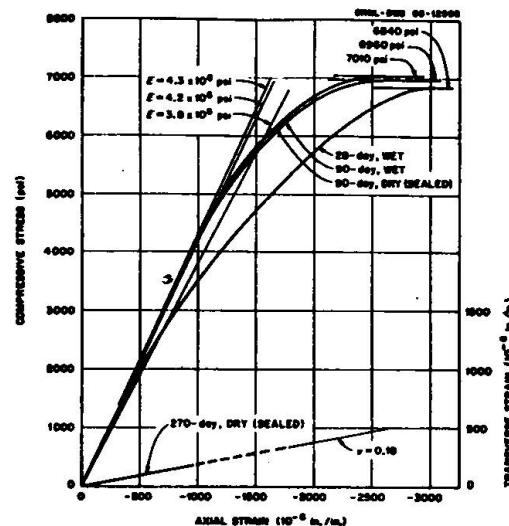


Fig. 10 - Model microconcrete

	Prototype	Model
Compressive strength, psi	8060	7010
Split cylinder strength, psi	635	762
Modulus of elasticity, psi	$3.5 \times 10^6$	$4.3 \times 10^6$
Poisson's ratio	0.18	0.18

The prototype concrete was substantially stronger in compression than the specified 6000 psi design strength, while the tensile strength was lower than that of the model concrete.

Stressteel prestressing bars were used in the prototype and models, with yield strengths of about 152 and 162 ksi, respectively. Conventional steel was used only in the base of the structures and did not influence behavior.

Instrumentation consisted of the usual surface, tendon, and reinforcing bar strain gages, embedded three-dimensional rosettes in the concrete, internal crack detection strips, etc. The prototype and one model had 375 electrical instrumentation points plus dial gages and photoelastic coatings, while the other model had only 70 points. This amount of data gathering points enabled an unusually detailed comparison of response. The internal rosettes had four legs; each leg was an epoxy dowel with four ordinary foil strain gages embedded in it. The final form of the dowel was identical to a threaded machine screw, 1/4 in. diameter by 1.5 in. long. These gages were manufactured by ORNL.

Behavior was measured at all stages of loading plus during the prestressing operation. Only a very small sample of results can be given here. The pressure-test histories of the three concrete structures are summarized in Fig. 11. The nominal design pressure was 500 psi for all vessels, and typical surfaces stresses at the design loading are compared in Fig. 12. The analytical results are obtained from a two-dimensional finite element analysis with some plane analysis modifications in the head penetration region. At all gage locations the models gave elastic results very close to the prototype values.

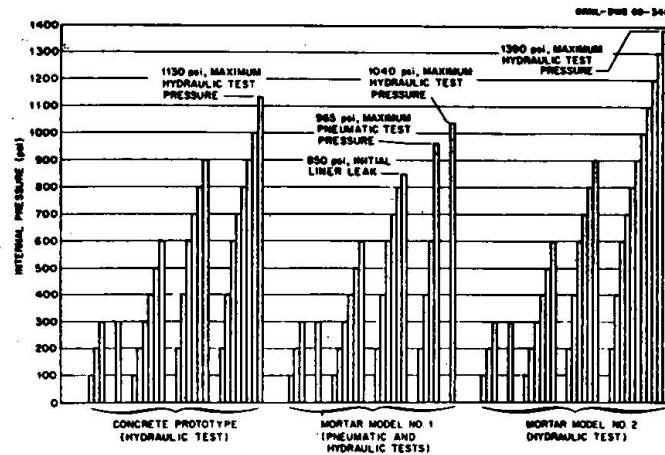


Fig. 11 - Pressure test histories of concrete prototype, first mortar model, and second mortar model

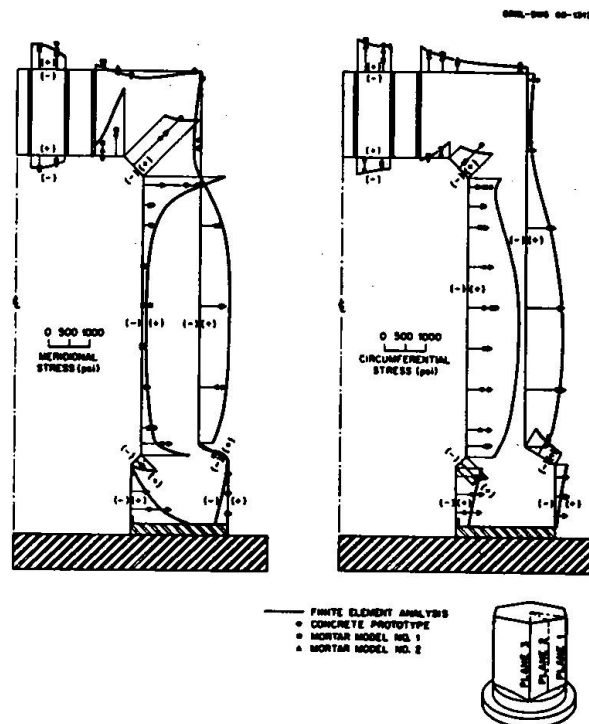


Fig. 12 - Comparison of meridional and circumferential surface stresses in plane 2 for prototype and models subjected to internal pressure of 500 psi. Predictions based on two-dimensional analysis.

The cracking behavior of the prototype and two models was remarkably similar. At about 600 psi, cracking began at the upper head internal haunch area and grew steadily with increasing pressure. At 900-1000 psi, major vertical cracking occurred in the walls, and at 1100-1200 psi, these cracks were fully developed and symmetrically distributed around the vessel walls. Finally, cir-

cumferential cracks began to form in the vessel walls at midheight. This behavior is summarized in Fig. 13. The location and number of major cracks in the prototype and models were essentially the same.

Haunch region strains for prototype and models are given in Fig. 14. External wall strains are shown in Fig. 15, and typical embedded gage results are

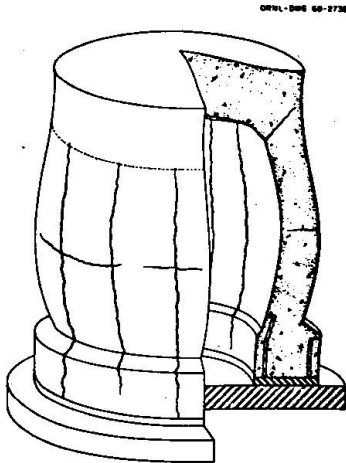


Fig. 13 - Cracking and deformation patterns

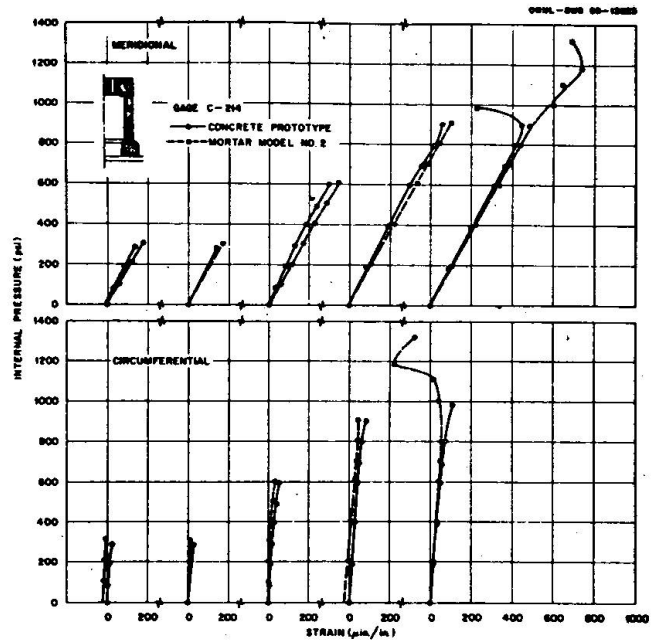


Fig. 14 - Measured meridional and circumferential strains in haunch area for prototype and second mortar model.

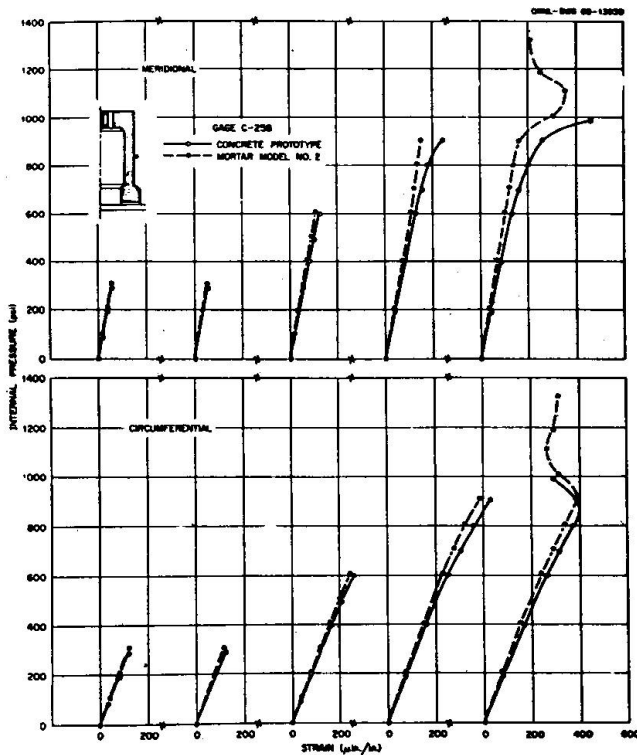


Fig. 15 - Measured meridional and circumferential strains on wall of prototype and mortar model.

given in Fig. 16. Haunch area crack detecting strip output is summarized in Fig. 17.

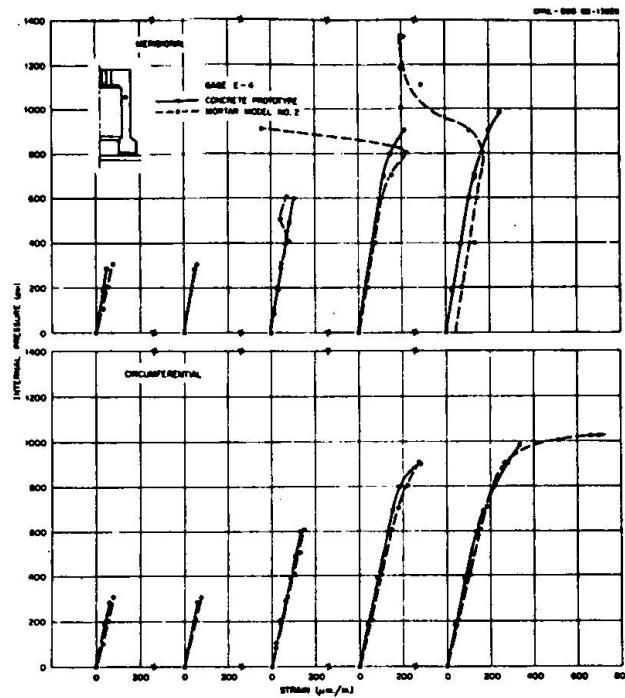


Fig. 16 - Measured meridional and circumferential strains for embedded gage in prototype vessel and second microconcrete model.

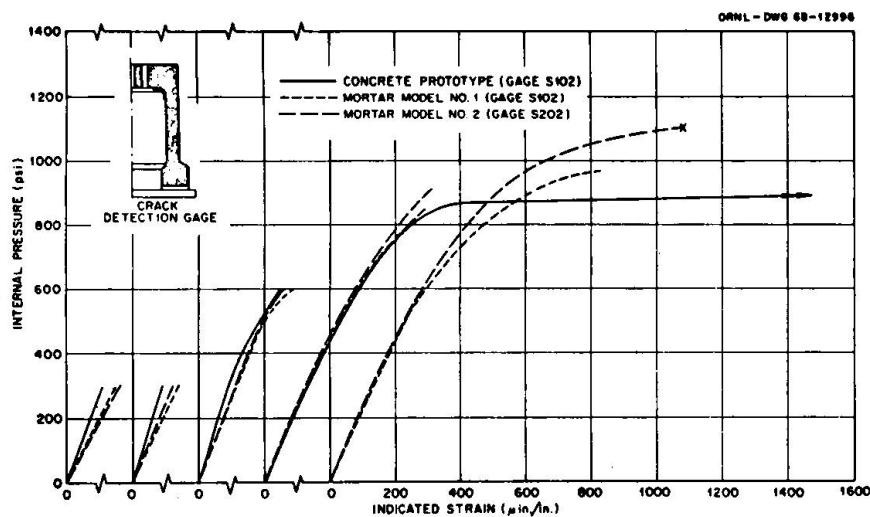


Fig. 17 - Comparison of indicated behavior for meridional crack-detecting strip across haunch of concrete prototype and microconcrete model.

Short-term time-dependent behavior was also compared for the time after prestressing and before testing. Model results ranged from poor to excellent in terms of predicting prototype behavior. This is not surprising considering the many variables involved in creep deformations. The models are useful here only for assessing general behavior trends.

The overpressure levels reached in the prototype and two models were 1130 psi, 1390 psi, and 965 psi. These differences are primarily due to the different quality of epoxy liners and pressurization systems (pumping volume) used for each structure. There is no uniquely defined failure level for this design because it was impossible to prevent leakage after the extensive cracking had occurred. Most important was that the pressure levels defining major cracking were essentially the same for all three structures.

The prestressed epoxy model gave reasonable predictions of prototype elastic stresses, but the high Poisson's ratio of 0.38 produced some discrepancies, particularly near the haunch area. It is felt that a strain-gaged epoxy model has no advantage over a microconcrete model, even for studies restricted to elastic stress determination.

In summary, the small microconcrete models provided basically the same information as the prototype. This type of detailed comparison on geometrically scaled structures lends further support to the reliability of the modeling technique.

#### A GYPSUM CONCRETE FOR MODELS

A gypsum concrete for models with the potential for essentially no "size effects" has been developed at Cornell. It may be used advantageously in modeling multiaxially stressed concrete prototypes, particularly where one must ensure that the tensile strength of the model concrete does not become too high. Mix proportions for a one-day, 3000 psi compressive strength are Ultracal 60 (a product of U.S. Gypsum Co.), high quality washed filter sand, and water in a 1:1:0.32 ratio. The mix is extremely workable because of its high gypsum content and can be easily placed in small models with very low clearances between bars. It also reaches design strength very quickly, thus enabling one to cast models one day and load the next. Since normally one must spend a fair amount of time instrumenting models, tests must often be delayed for a week or so. In this case the model surfaces can be sealed and the strength properties will remain essentially the same as at the time of sealing. It is essential to seal the surface if testing is not done in the first day or two because the mix dries out excessively and becomes too brittle, with a nearly linear stress strain curve. At a moisture content of around 4% the mix has the same stress-strain curve as shown for Portland cement mortars in Fig. 3..

The purpose of the study was to determine the critical parameters influencing the effects of specimen size on compressive and tensile strengths of gypsum mortars, and to attempt to reduce these effects as much as possible. Earlier experimental investigations at Cornell University (Ref. 6) indicated the strong dependence of gypsum mortar properties on moisture content. In the present study the control on moisture content was tightened to eliminate moisture as a variable. Time vs. average moisture content curves were established for all specimen sizes; typical curves are given in Fig. 18.

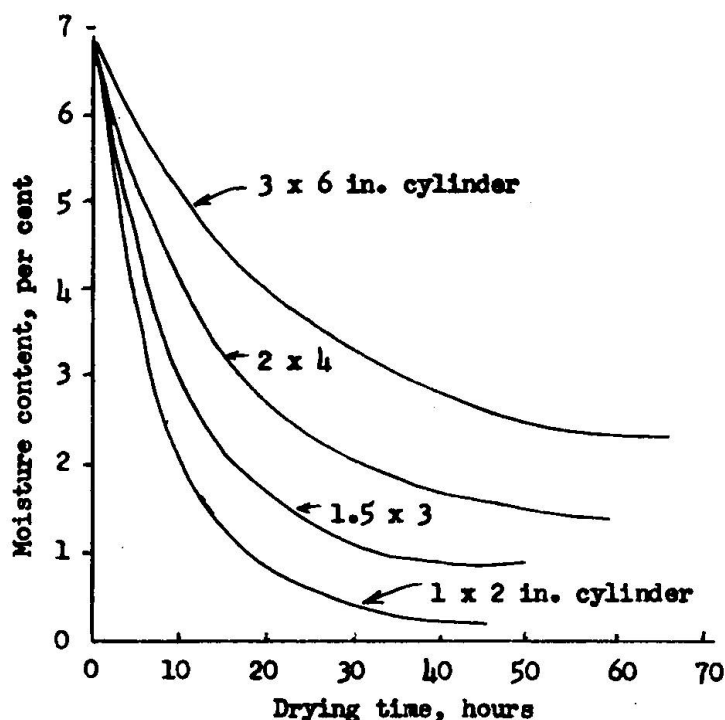


Fig. 18 - Drying time vs. moisture content, gypsum model concrete cylinders

Density measurements showed a distinct difference in density of material as cast in different size forms when a normal placement procedure was followed. Thus a casting technique that utilized both rodding and external vibration was evolved to produce uniform densities in each specimen size. All loading devices were scaled geometrically with the cylinder and beam size. Tempered masonite loading strips were utilized for the split cylinder tests, and loading rates were scaled to maintain constant strain rates. Moisture content was 4% for all sizes.

Six separate test series were conducted. All series had four different sizes of cylinders (as shown in Fig. 18) for both compressive and split cylinder tests, with 3 to 6 cylinders for each size in each series. Two beam sizes were also tested to measure the modulus of rupture. Test results are:

Specimen size	Compressive cylinders		Split cylinder strengths		Modulus of rupture		
	No. of spec.	Ave. strength	No. of spec.	Ave. Strength	Spec. size	No. of spec.	Value
3" by 6"	17	3015 psi	18	312 psi	2" by 3"	18	531 psi
2" by 4"	23	3021	29	312	1" by 1½"	38	574
1.5 by 3"	26	3061	27	306			
1" by 2"	36	3033	36	315			

Coefficients of variation for the three types of tests were: Compression, 0.005 to 0.064 (average = 0.0340), split cylinder, 0.004 to 0.102 (average = 0.048), and modulus of rupture, 0.022 to 0.059 (average = 0.034).

It is concluded that within the size of specimens tested, there is no measurable size effect in compressive or split cylinder tensile strength for

this particular gypsum mortar when both moisture content and density are held constant. There is a definite size effect in modulus of rupture; the most probable reason for this is the different strain gradient across the specimens of differing depths. The potential for use of this material in structures subjected to multiaxial stresses, particularly those involving tensile components, is rather high.

#### CONCLUDING REMARKS

1. This paper has focussed on specific examples relating to the reliability of physical models for predicting the inelastic behavior of reinforced concrete structures. There is considerably additional information from North American laboratories on the accuracy of models (Refs. 8,9,10, and 11, for example). An even larger documentation exists in European literature.
2. The tendency of small mortar specimens to be overstrength in tension can be reduced to tolerable limits by appropriate mix design, including minimization of fines in the aggregate and use of an aggregate with smooth surfaces. Strength values should be based on small specimens that are more representative of model element sizes than the conventional large cylinders or cubes are. It is possible to produce a gypsum model concrete with very small size effects.
3. Many triaxially stressed structures are going from a state of general compression to a state of general tension as they are loaded. Thus the true tensile strength of the concrete, rather than the modulus of rupture with its high strain gradient, is being reached in the concrete. It is felt that there is less "size effect" in true tensile strength than in the modulus of rupture, which is a beneficial factor for modeling triaxially stressed structures.
4. Additional research is needed in developing failure criteria for model concretes and in comparing the criteria with full-scale concrete behavior.
5. A properly executed physical model study can reveal behavior modes that are simply impossible to model mathematically. Thus the physical model approach and the mathematical model approach should be viewed as complementary in the structural design process. If the mathematical approach can give the necessary answers with acceptable reliability, then it should be used in most instances, but if there is doubt about an analytical solution, the structural engineer should not hesitate to turn to the physical approach. The need for models to formulate behavioral modes and to serve as experimental evidence for the corroboration of theory is rather obvious.

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#### ACKNOWLEDGEMENTS

The projects reported in this paper were supported by grants from the National Science Foundation and the Oak Ridge National Laboratory. The prototype PCRV was built and tested by ORNL, who also conducted the analytical studies for the PCRV models and had general charge of the test program and interpretation of test results. The material on reinforced concrete frames is based on the work of Dr. Chowdhury (Refs. 1 and 3) who completed his thesis research under the direction of the author. Mr. George Loh was responsible for the experiments on size effects in gypsum mortars.

#### SUMMARY

Small scale models are shown to have high reliability for predicting the nonlinear behavior of reinforced concrete structures subjected to complex stress histories, provided that the model materials meet the necessary similitude requirements. Examples given include frames under simulated seismic forces and prestressed concrete pressure vessel models of different scales, loaded to failure with internal pressure. Size effects in gypsum model concretes are also explored.



## RESUME

On montre que les maquettes en échelle réduite ont une grande crédibilité dans la prévision du comportement non-linéaire de structures en béton armé soumises à des états de contraintes complexes, pourvu que les matériaux de la maquette satisfaisent les conditions nécessaires pour la similitude.

Les exemples présentés comprennent des cadres soumis à des forces sismiques simulées et des maquettes de caissons en différentes échelles, chargées jusqu' à rupture avec pression à l' intérieur.

On examine enfin les effets de l' échelle sur les bétons en plâtre pour les maquettes.

## ZUSAMMENFASSUNG

Es wird gezeigt, dass Kleinmodelle zur Vorausbestimmung des Verhaltens von nicht-linearen Beton-Konstruktionen unter komplexen Belastungen sehr zuverlässig sind, vorausgesetzt es besteht genügend Ähnlichkeit zwischen dem Modell-Material und dem der zu untersuchenden Konstruktion.

Zu den angeführten Beispielen gehören Strukturen unter simulierten seismischen Belastungen und Modelle von Spannbeton-druckbehältern in verschiedenen Grössen, die innerem Druck bis zum Bersten ausgesetzt wurden.

Ferner werden die Wirkungen der Abmessungen von Beton-Konstruktionen an Hand von Gipsmodellen untersucht.