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II

A Simple Design Criterion for Biaxially Loaded Columns

Un critère simple pour le dimensionnement de colonnes chargées de façon biaxiale

Ein einfaches Kriterium für zweiaxig exzentrisch belastete Stützen

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OBJECT.

The purpose of the criterion is to check the safety against biaxial failure of reinforced concrete slender columns subject to general loadings and restraints. Conditions for applicability are: - the columns must have two major planes, - limit load factors can be determined for loadings acting separately on each major plane. The criterion provides only a simplified check, not an exact determination, of the limit load factor when both loadings act together.

DESCRIPTION.

Given is a column, with different restraints in the two major planes, subject to one fixed system of axial loads: \bar{Z} , and two systems of lateral loads: \bar{X} and \bar{Y} (Fig. 1).

First, the values of the multipliers $\alpha_x \gamma_{lim}$ and $\alpha_y \gamma_{lim}$ which separately determine the limit state are calculated.

The present criterion considers admissible a combination of the two load systems: $\alpha_x \cdot \bar{X} + \alpha_y \cdot \bar{Y}$, such that the following relationship is satisfied:

$$\bar{\alpha}_x < 1 - \bar{\alpha}_y \quad \text{with:} \quad \bar{\alpha}_x = \alpha_x / \alpha_x \gamma_{lim}$$
$$\bar{\alpha}_y = \alpha_y / \alpha_y \gamma_{lim}$$

or equivalently:

$$\bar{\alpha}_y < 1 - \bar{\alpha}_x$$

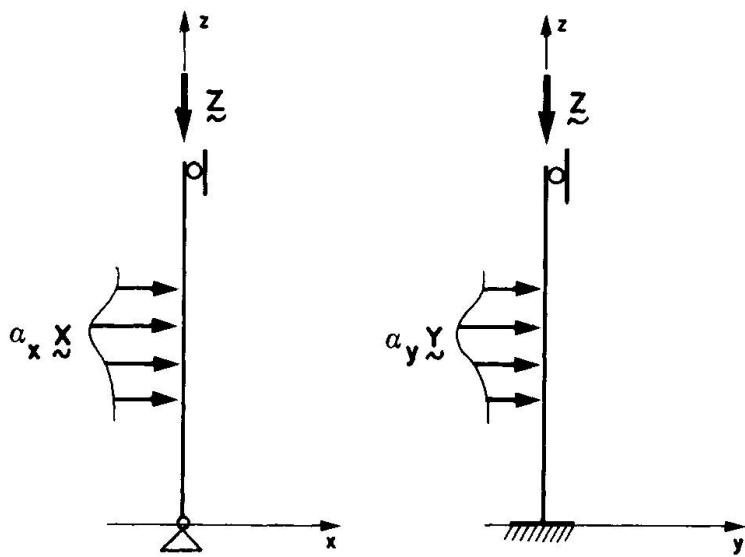


Fig. 1 - Column with different restraints on the two planes

The inequalities above, when plotted as in Fig. 2, imply that the point $C (\bar{\alpha}_x, \bar{\alpha}_y)$ be internal to the triangle 0-1-1.

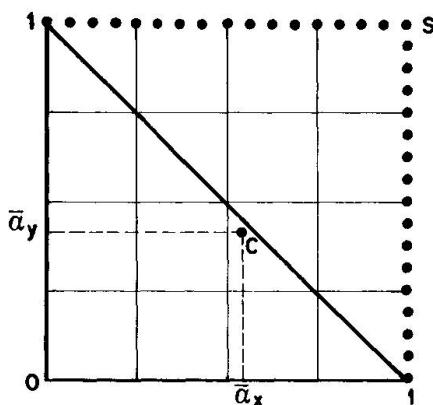


Fig. 2 - Admissible loading domain

The criterion itself doesn't specify by what means the load factors of major planes are determined: either by an "exact" or by an approximate analysis, or by a code formula.

In fact, it just extends to biaxial cases any kind of uniaxial check of a column.

COMMENTS.

Generally, in the analysis for the two separate directions, the respective critical sections are not the same.

When these sections coincide, as for example in a biaxially loaded cantilever column, the application of the proposed criterion is equivalent to a linearization of the reduced interaction diagram of the section itself (Fig. 3).

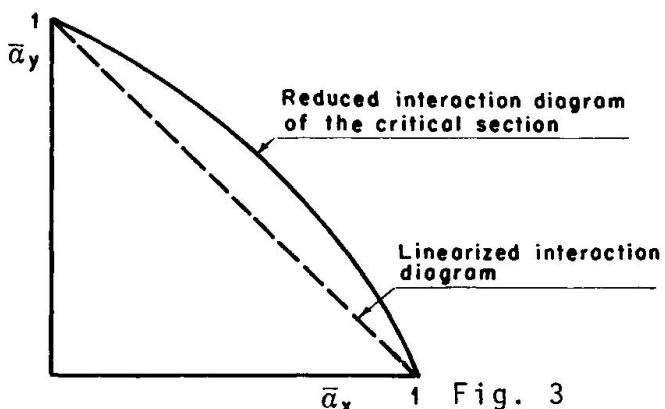


Fig. 3

In fact, several approximate expressions for reduced interaction diagrams, including the linear, are in use today (|1|, |4|, |5|).

When, instead, the "critical" sections of the two uniaxial loadings do not coincide, the criterion no longer refers to the interaction diagram of one section.

For these cases, no adequate practical method or formulas exist to date.

The following may be noted about the proposed criterion.

It would provide very restrictive conditions for those linear elastic cases in which the effects of the two orthogonal deformations do not interfere: for example, if one considers deformed shapes shifted by a quarter of wave, the actual limit load diagrams would approach the dotted rectangle in Fig. 2.

For materials of non-linear behavior and shift of critical sections other than a quarter, the actual diagram ordinarily lies between the triangle and the rectangle, so that the criterion is on the safe side.

COMPARISONS.

The empirical origin of the criterion requires numerical and experimental checks, for evaluating the degree of safety in different cases. A systematic investigation is currently under way, which makes use of rigorous methods of analysis recently set up ([2], [3]).

As a sample of the first results, cases are presented of columns with coincidental critical sections in the two planes, and with critical sections wide apart.

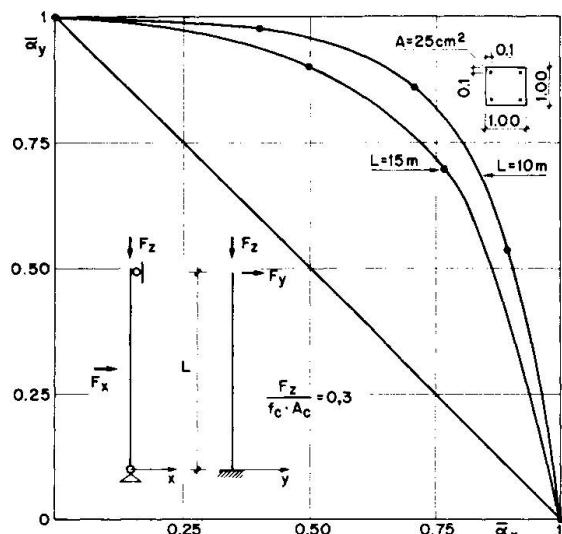


Fig. 4

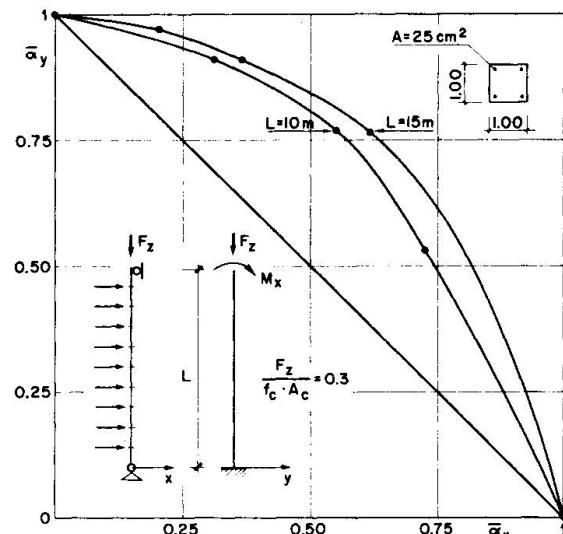


Fig. 5

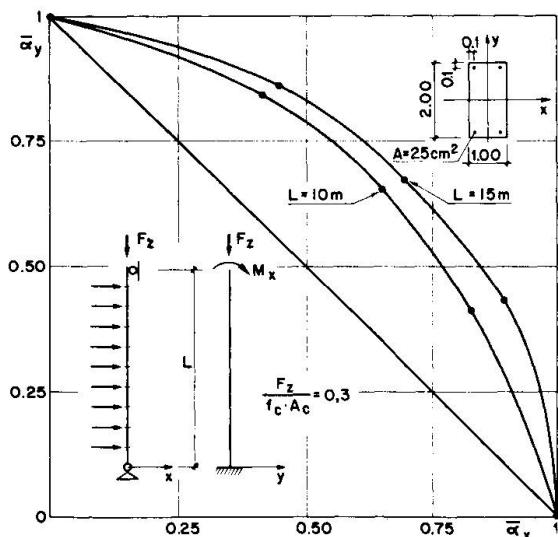


Fig. 6

Figs. 4, 5, 6

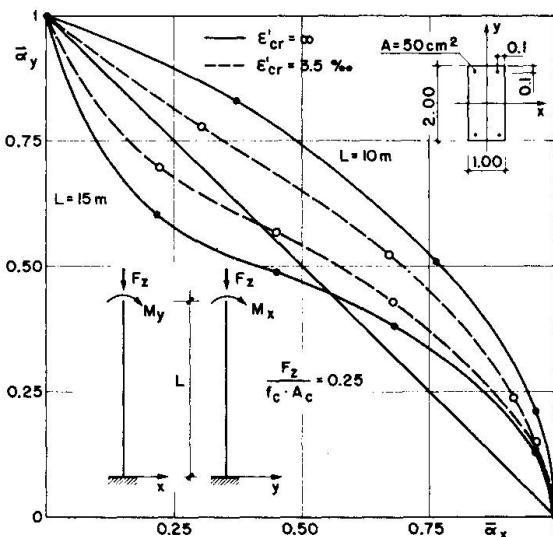


Fig. 7

Fig. 7

The figures 4, 5 and 6 refer to columns simply supported in one plane and cantilevered in the other.

The column in Fig. 4 is acted by concentrated loads, while for the other two examples the loading consists of a distributed load in one plane and a couple in the other.

The diagram in Fig. 7 shows a rectangular cantilever column with two different heights. The solid line diagrams refer to failures calculated with unlimited compressive concrete strain, whereas for the dotted lines the strain was limited to 3.5%.

The diagram relative to $L = 15$ enters partially into the triangular area. This can occur when the slendernesses in the major planes are such that the column collapses for instability in one direction - with the materials still in the linear field -, while it undergoes a clear "material failure" in the other major plane.

A trend can be caught from the examples presented: the approximation improves the more the "critical zones" in the two planes spread into each other, as a consequence of the way of loading or of the restraints, or both.

CONCLUSIONS.

The advantage of the proposed criterion is that a very simple operation in a single diagram checks the capacity of all the sections including the second order effects, once the uniaxial problems have been solved.

On the other hand, the criterion provides sometimes too restrictive values, namely for cases of linear materials and distant critical sections in the two principal planes.

For standard columns, the criterion could undoubtedly be refined by means of extensive numerical checks. It is felt, however, that in the present form it can be of use for those non standard cases whose importance does not call for rigorous analysis.

ACKNOWLEDGEMENT.

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SUMMARY

A simplified criterion is illustrated for checking the biaxial ultimate load of reinforced concrete slender columns, based on a linearization of the interaction diagram.

RESUME

Un simple critère permet la vérification de la charge ultime déviée de colonnes élancées en béton armé; ce critère est basé sur une linéarisation du diagramme d'interaction.

ZUSAMMENFASSUNG

Es wird ein vereinfachtes Kriterium zur Nachprüfung der Tragfähigkeit schlanker Stahlbetonstützen unter zweiaxig exzentrischer Last gegeben, welches auf einer Linearisierung des Interaktionsdiagramms beruht.