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## Final Summary to Theme I

Résumé final au thème I

Schlusswort zum Thema I

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### 1. General

Attention has been concentrated in Theme I of this Symposium on the more theoretical aspects of reinforced concrete column behaviour. Three main topic areas can be identified in the contributions. These are: load-deformation characteristics of reinforced-concrete cross sections; performance and load-carrying capacity of isolated columns; non-linear analysis of reinforced concrete frames.

In most of the contributions theoretical, computer-oriented methods of analysis are developed with the objective of providing accurate and realistic predictions of column behaviour. However, in several instances simple physical-mathematical models are used to explain and illustrate behaviour on a qualitative rather than a quantitative basis. It will be convenient in the present summary to deal separately with these two approaches.

### 2. Theoretical Considerations

#### Cross-Sections

The effect of various parameters on the extreme fibre compressive strain at failure,  $\epsilon_{cu}$ , is investigated by MUGURUMA and TANAKA for column sections subjected to combined uniaxial bending and axial thrust. With "failure" defined as peak load-carrying capacity, it is shown that  $\epsilon_{cu}$  is a variable which is very much dependent on the load eccentricity, ie on the moment-to-thrust ratio. These results

are of practical interest, because simplified strength calculations are based on an assumed constant value for  $\epsilon_{cu}$ . Before final conclusions can be drawn from this work, comparisons need to be made with test data, and sensitivity analyses are required to determine the effect on computed load-carrying capacity of variations in  $\epsilon_{cu}$  for various moment-to-thrust ratios.

HSU and MIRZA present an analysis of the load-deformation characteristics of a rectangular section in biaxial bending and compression. The analysis is based on a partitioning of the cross section into small elemental areas which can be considered to be in a state of uniaxial stress. Equilibrium equations are used to determine the moments and axial force in the section for an assumed strain distribution. A simplifying assumption regarding the deflected column shape allows the analysis to be extended to treat the behaviour of a column of finite length with simple end conditions.

The effect of a time-varying sustained loading is studied by SPAROWITZ, who uses an iterative step-by-step analysis to determine stresses and strains in the cross section at a sequence of time instants. The section is again partitioned into small areas, and in each time interval the concrete areas are allowed to deform freely under the effects of creep and shrinkage. The instantaneous stresses and strains in steel and concrete are adjusted at the end of the time interval so that requirements of equilibrium and compatibility are satisfied at the next time instant.

The basic assumption in all of these analyses is that the strain distribution in the section is planar. Detailed numerical calculations are carried out by computer. These and other similar cross-sectional analyses allow the stresses and deformations to be calculated in a section of any arbitrary shape for any given loading history, and provide the starting point for the analysis of structural behaviour.

Questions regarding the adequacy of cross-sectional analyses for reinforced concrete are not simple to answer. Whenever the load eccentricity is large enough to cause tensile cracking, the plane strain hypothesis obviously becomes a gross idealization. Local concrete tensile strains then vary from infinity at the crack to almost zero on the surface between adjacent cracks, while tensile

stresses in both concrete and steel in this same region show considerable variation along the member. On the other hand, the assumption of plane distribution of deformations is probably reasonable if applied to a segment of member which is long enough to contain a typical pattern of cracks. Nevertheless, regions of tensile concrete between cracks can contribute significantly to bending stiffness and need to be taken account of if the analysis is to be "realistic". In such circumstances, an analysis of the load-deformation characteristics of a finite segment could be used either to replace the sectional analysis or to confirm the accuracy of the sectional analysis. Segmental analyses have apparently not yet been undertaken, except for the special case of unloading in local regions of a frame as the collapse load is approached.

#### Isolated Columns and Frames

An incremental stiffness analysis with a beam-type finite element is used by KULICKI and KOSTEM to treat both geometric and material non-linearities in a beam column subjected to combined transverse loading and end thrust. A finite-element method is also used by ALDSTEDT and BERGAN for the non-linear analysis of plane reinforced-concrete frames. In both of these contributions, the finite element is assumed to undergo continuous beam-type deformations for which the standard cross-sectional analysis is applicable. Although attention is restricted to short-term loading conditions, the work in each case could probably be extended without conceptual difficulty to cover problems of sustained loading (creep and shrinkage) and cyclic loading (separate loading and unloading curves for concrete and steel). Numerical results obtained in these two studies show good agreement with other calculation methods and with column test results.

OBERNDORFER and FISCHER reverse the usual analysis procedure and take up the design question of providing sufficient steel in the section to ensure stability of a slender column for a given loading condition. The basis for the calculations is a slender column analysis in which a more-or-less standard moment-thrust-curvature calculation is carried out at a number of cross sections. The treatment is extended to one-storey assemblages of columns by providing for compatibility of end displacements at the column heads.

WAKABAYASHI and his co-workers present a simplified analysis of steel-reinforced columns and portal frames subjected to constant vertical loading and cyclic horizontal loading. The concept of a unique moment-thrust-curvature relation is invalid for this type of loading, and an incremental analysis has to be used to treat the sequence of loadings and load reversals. Extensive iterative calculations at various cross sections are avoided by considering the columns to be rigid except in a short segment in the region of maximum moment. This very much simplified approach shows good agreement with experimental results presented in condensed form.

### 3. Physical-Mathematical Models

The methods of analysis referred to above are generally directed towards an accurate and realistic treatment of column behaviour, and rely on the computer for detailed numerical calculations. Such methods provide the most reliable predictions of column performance, but frequently do not give a correspondingly clear picture of physical behaviour or of overall trends. Physical-mathematical models can sometimes provide a good qualitative (but less accurate quantitative) representation which is of heuristic value both to the design engineer and to the research worker attempting to develop a more precise analysis.

A physical-mathematical model usually contains one or more simplifications - often these are over-simplifications - which can be interpreted in purely physical terms. The bar-spring model of a slender column is a classic example of a physical-mathematical model and was used in the Introductory Report to develop a linear viscoelastic analysis to parallel Dischinger's classic analysis of a slender concrete compression member.

In their contributions to the Preliminary Report, CERVERA and CREUS use several bar-spring models to investigate non-linear viscoelastic behaviour in a slender column. A refined bar-spring assembly is proposed to account both for the effect of axial thrust on bending stiffness and the non-coincidence of the loading and unloading paths for concrete subjected to compressive stress.

A bar-spring assemblage with two springs (two degrees of freedom of deformation) is used by GRENACHER to investigate the deformation capacity of reinforced concrete columns with special end conditions. By varying the moment-rotation characteristics of the springs, Grenacher draws conclusions of practical importance concerning the ductility of fixed-ended columns. This work is particularly interesting, in that a much more refined and detailed computer analysis confirms the conclusions obtained from the simple model.

The basic idea of a physical-mathematical model is of course not restricted to bar-spring assemblages. A lumped, two-fibre idealization can for example be used in a simplified analysis of the load-deformation characteristics and even load-carrying capacity of column sections. The two-fibre model is particularly useful in the analysis of creep effects and is, in effect, used by CERVERA and CREUS. A three-fibre model is introduced by YAMADA and KAWAMURA in their contribution to Theme II.

Physical-mathematical models can also be used in the analysis of small frames and assemblages. The analysis of WAKABAYASHI et alia, already referred to, uses rigid bar models to obtain quantitative results for portal frames under complex loadings. By increasing the number of springs (ie degrees of freedom) in a bar-spring model, one can obtain an accurate numerical analysis which corresponds closely to the non-linear methods of analysis normally used for slender columns and frames

#### 4. CONCLUDING REMARKS

Attention in Theme I tended to concentrate on accurate methods of analysis of overload behaviour and load-carrying capacity. As a consequence several other topic areas of importance were to some extent neglected. A range of serviceability problems, such as the axial shortening of columns and the differential displacements of members in frames under sustained loading were thus ignored. In both The I and Theme II only minimal attention was given to intermediate methods of analysis such as those based on tri-linear and multi-linear moment-curvature relations.

Nevertheless, the range of questions studied in the Preliminary Report and in the subsequent Session I discussions was fairly wide, and the contributions, taken together, give an indication of the theoretical methods at present available for the analysis of reinforced concrete columns.