Additional moments in slender prestressed concrete columns

Additional Moments in Slender Prestressed Concrete Columns

Moments additionnels dans des colonnes élancées précontraintes

Zusatzmomente in schlanken Spannbetonstützen

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Introduction

The derivation and application of the additional moment method for the design of slender R.C. columns is dealt with in a recent report by Cranston[1]. The report includes an extensive comparison between the design method and test data, and is shown to be suitable for a wide range of columns covering the principle variables. The present study was carried out in order to examine the suitability of applying the method directly to prestressed concrete columns. Similar comparisons between calculated and test results were made and differences in the trends relative to R.C. columns led to a closer study of the problem being carried out.

The additional moment expressions for the design of slender R.C. columns are given in Section 3.5 of the British Standards Code of Practice CP 110[2].

For long columns bent about the minor axis, the cross-section should be designed to resist the ultimate axial load N and a total moment given by

$$M_t = M_i + \frac{Nh}{1750}\left(\frac{l_e}{h}\right)^2 \times \left(1 - 0.0035\,\frac{l_e}{h}\right).\ K \qquad (1)$$

Where M_i is the maximum moment in the column due to ultimate loads, h is the overall depth of the cross-section in the plane of bending, l_e is the effective length, and K is an adjustment factor depending on N/N_{uz} where N_{uz} is the axial load capacity.

All tests reported and analyses carried out in this study were of pin-ended columns subject to loads acting at equal eccentricities with respect to both ends of the column. In such cases, $l_e = l_c$ where l_c is the clear height, and $M_i = N \times e_i$ where e_i is the initial eccentricity of load.

The test series used in the comparisons are reported in references 3, 4 and 5, and the results listed in Table 1. Details of the method used for calculating the failure loads are given in the appendix to this report.

A histogram of $(N_u)_{Test}/(N_u)_{Calc}$ is shown in Figure 1(b), and includes all the test cases analysed. In Figure 1(a) the corresponding histogram for pinned R.C. columns abstracted from reference (1) is shown. There is insufficient data available for the significance of individual parameters to be observed clearly, but the results did indicate somewhat different trends to those shown by the R.C.

column study. In general, the reported test work was not sufficiently detailed for study, and a series of computer analyses was therefore carried out. All combinations of the parameters listed in Table 2 were considered, but only one grade of concrete and one grade and percentage of steel used in the analyses.

Slenderness ratio	$- l/h$	20	30	40
Initial eccentricity	e_i/h	0.5	0.2	0.5
Level of prestress	f_{cp}/f_{cu}	0	0.15	0.3

Table 2

$$f_{cu} = 40 \text{ N/mm}^2$$

$$f_{pu} = 1700 \text{ N/mm}^2$$

$$\frac{Aps}{bh} = 0.01$$

Where

f_{cu} is the characteristic cube strength

f_{pu} is the characteristic strength of the prestressing tendons

f_{cp} is the compressive stress in the concrete due to prestress

Aps is the area of the prestressing tendons

The stress strain curves assumed in the analyses were those given in Figures 1 and 3 of reference (2) with γ taken as 1.0 in each case, and the modulus of elasticity of the tendons assumed to be 200 KN/mm^2. The computer programme used was essentially the same as that used in the R.C. column study, and full details are given in reference (6).

The computer analyses have merely been used to augment the information available on tests reported, for the effects of the various parameters may be observed without the inherent variability of laboratory testing obscuring the trends. The histogram of the results from the analyses is given in Figure 1(c) and it may be seen that the distribution is similar to that of Figure 1(b) but with a lower mean value.

The main differences between the histograms shown in Figure 1 are the lower mean value for the prestressed columns, and the percentage of the population values below 0.95. For the R.C. columns, there are 7% below, and for the P.S.C. columns, 17%. There are basically two reasons for these differences, both related, and being functions of the level of prestress and the additional moment philosophy.

In order to understand these effects, it is necessary to consider the basic additional moment expression, the derivation of which is covered in detail in reference (1).

Essentially, the method depends upon forecasting the deflection of the column at ultimate conditions, and this is assumed to be given by:

$$a_u = l_e^2/10r_u \qquad (2)$$

Where

a_u is the maximum deflection corresponding to material failure conditions.

$1/r_u$ is the corresponding curvature.

CP 110 assumes $1/r_u = K \times (1/r)_{bal}$ Where $(1/r)_{bal}$ is the curvature at 'balanced' conditions on the cross-section, and K is defined as $(N_{uz} - N)/(N_{uz} - N_{bal}) \leqslant 1.0$.

$(1/r)_{bal}$ is taken $(\varepsilon_{uc} + f_y{*}/E_s - l_e/50000h)/h$ and fixed values of ε_{uc}, the ultimate strain in the concrete in compression, and $f_y{*}/E_s$, the yield strain of the steel, of 0.00375 and 0.002 respectively give

$$(1/r)_{bal} = \frac{1 - 0.0035\ le/h}{175h}$$

Substituting the above in equation (2) yields the additional deflection or eccentricity implied in equation (1).

If the above criterion is applied to a P.S.C. section, then the value of 0.002 adopted for the steel yield strain will obviously be inappropriate. The corresponding condition in a P.S.C. section should, therefore, take into account the actual yield strain of the tendons, and also the prestrain applied. The corresponding strain diagram, due to loading at the balance point is shown in Figure 2.

The effect of ε_{pc} may be neglected as being insignificant relative to 0.00375 and, in analogy with R.C. formula, a value of 0.008 for ε_y may be assumed to cover the likely range of prestressing steels used. Hence, a modified expression for $(1/r)_{bal}$ is given by

$$(1/r)_{bal} = (0.00375 + 0.008 - \varepsilon_{ps} - l_e/50000h)/h \qquad (4)$$

Figure 3 shows the variation in the ratio of a_u for a prestressed section to a_u for reinforced section against level of prestress. It is apparent that for the low levels of prestress the R.C. expression considerably underestimates the deflection at ultimate.

Using the modified expression above, the test columns and the computer 'tests' were re-analysed, and the new histograms are shown in Figure 4. There are now less than 2% of the values below 0.95, and the mean value is close to that obtained from the R.C. column study.

Although the modified formula improves the results it was noticed that there was still a significant decrease in the value of $(N_u)_{Test}/(N_u)_{Calc}$ as the level of prestress was increased. For example, Figure 5 (a) shows some of the values obtained from the tests reported in reference 3 and Figure 5 (b) the results obtained from the computer analyses. It was also observed that the $(N_u)_{Test}/(N_u)_{Calc}$ values decreased with increasing eccentricity and increased with increasing slenderness. The reason for these trends is essentially a function of the strength based failure criterion that is assumed in the additional moment method. In the case of a slender column loaded axially or with small eccentricity, the failure mode is that of instability and, in general, the design method results in a conservative estimate of the failure load. This behaviour is illustrated in Figure 6 in which the effects of prestress, slenderness and eccentricity on the load - maximum moment curves are shown.

Also plotted on the figures are the load versus total moment lines as calculated from the additional moment expression.

The effects of initial eccentricity and slenderness produce the marked skewness of the histogram shown in Figure 1 (a). The additional effect of prestress which tends to inhibit instability failures may be observed in the reduced skewness of Figure 4 relative to Figure 1 (a). It may be concluded therefore, that provided the modified expression is used, the additional moment method may be applied to P.S.C. columns, and will in general result in a rather better forecast of the failure load than in the case R.C. columns.

The skewness of the histograms in Figures 1 and 4 could be eliminated by introducing an alternative failure criterion for slender columns with low levels of prestress and loaded at small eccentricities. Mikhailov(7) tackles this aspect by proposing a failure criterion based on first cracking. Analysis on this basis gives $(N_u)_{Test}/(N_u)_{Calc}$ results close to unity for the conditions outlined above, but is not applicable over the whole range of parameters.

There is a further consideration in that the additional moment method makes an implicit allowance for long term effects in the calculation of the deflection at ultimate. If an alternative failure criterion such as outlined above were adopted, the possibility of creep buckling failures at reduced load levels would have to be explicitly taken into account. Since the additional moment method is generally conservative - albeit unnecessarily so in some cases - it seems unwarranted to introduce a dual failure criterion. In particular, the interaction of the three main parameters makes it very difficult to set limits on the values of these parameters in order to define the transition from one failure criterion to the other.

Appendix - Calculation of failure loads

The various test series considered are given in Table 1, all results having been reduced to a common basis in S.I. Units. The interaction diagrams giving the section strengths for each individual cross-section were produced by a computer program using the stress strain curves given in Figures 1 and 3 of reference 2. Where cylinder strengths were given in the reported test work, the cube strength has been assumed to be 1.25 times the quoted values. The failure loads are obtained by the intersection of the load versus total moment line with the interaction diagram, taking account of the reduction factor K.

References

(1) CRANSTON, W.B. Analysis and Design of Reinforced Concrete Columns - Research Report 20, Cement and Concrete Association, London 1972.

(2) British Standards Code of Practice CP 110 (Part 1). The Structural Use of Concrete. 1972.

(3) ARONI, S. Slender Prestressed Concrete Columns. Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Volume 94, No. ST4. April 1968.

(4) BROWN, H.R. and HALL, A.S. Tests on Slender Prestressed Concrete Columns - Paper No. 8, Symposium on Reinforced Concrete Columns. American Concrete Institute Special Publication SP-13. 1966.

(5) LIPSON, S.L. Experimental Investigation of Buckling of Slender Prestressed Columns. Rilem International Symposium. Buenos Aires, Argentina, September 1971.

(6) CRANSTON, W.B. A computer Method for the Analysis of Restrained Columns. Technical Report TRA 402, Cement and Concrete Association. London 1967.

(7) MIKHAILOV, V.V. Design of Slender Prestressed Concrete Columns Based on Stability Criteria, P.C.I. Journal, Sept/Oct 1972.

TABLE 1

ARONI[3]

Test column	100 ρ	f_{cu} N/mm²	f_{pu} N/mm²	l/h	e_i/h	$\frac{f_{cp}}{f_{cu}}$	$(N_u)_{Test}$ kN	$(N_{uz})_{Calc}$ kN	$(N_{bal})_{Calc}$ kN	$(N_u)_{Calc}$ kN	$\frac{(N_u)_{Test}}{(N_u)_{Calc}}$
A1 20c 3	2.03	49	1730	20	1.98	0.208	11.8	137	8.3	9.8	1.21
A1 30a 3				30	0.124	0.208	46.4	137	8.3	37.8	1.24
A1 40b 3				40	0.743	0.208	13.8	137	8.3	12.9	1.07
A2 20b 5				20	0.743	0.32	22.8	112	-10.8	21.1	1.08
A2 30c 5				30	1.98	0.32	8.8	112	-10.8	8.2	1.07
A2 40a 5				40	0.124	0.32	26.4	112	-10.8	25.6	1.03
B1 20a 1				20	0.124	0.072	83.7	166	35	61.2	1.37
B1 30b 1				30	0.743	0.072	17.3	166	35	15.8	1.10
B1 40c 1				40	1.98	0.072	7.3	166	35	7.1	1.03
B2 20b 3				20	0.743	0.2	21.7	139	9.7	21.5	1.01
B2 30c 3				30	1.98	0.2	8.7	139	9.7	8.4	1.04
B2 40c 3				40	0.124	0.2	26.6	139	9.7	21.5	1.23
C1 20c 1				20	1.98	0.08	12.0	164	33	9.4	1.27
C1 30a 1				30	0.124	0.08	51.9	164	33	30.0	1.73
C1 40b 1				40	0.743	0.072	11.9	166	35	11.5	1.03
C2 20a 3				20	0.124	0.2	56.9	139	9.7	64.0	0.89
C2 30b 3				30	0.743	0.192	17.6	140	11	16.7	1.05
C2 40c 3				40	1.98	0.192	7.2	140	11	6.9	1.03
D1 20c 5				20	1.98	0.336	12.5	109	-17	9.3	1.35
D1 30b 5				30	0.743	0.336	20.1	109	-17	17.3	1.16
D1 40a 5				40	0.124	0.336	26.3	109	-17	25.8	1.02
D2 20b 3				20	0.743	0.2	24.8	139	9.7	21.5	1.15
D2 30b 3				30	0.743	0.2	18.9	139	9.7	17.1	1.10
D2 40b 3				40	0.743	0.2	13.0	139	9.7	12.8	1.02
E1 20a 5				20	0.124	0.336	58.3	112	-10.8	55.1	1.06
E1 30b 5				30	0.743	0.336	19.3	109	-17.0	17.3	1.14
E1 40c 5				40	1.98	0.336	8.4	109	-17.0	7.5	1.12
E2 20b 1				20	0.743	0.072	22.6	166	35	20.7	1.10
E2 30c 1				30	1.98	0.08	8.9	164	33	8.1	1.10
E2 40a 1				40	0.124	0.072	29.8	166	35	17.4	1.71
F1 20b 1				20	0.743	0.08	24.8	164	33	21.1	1.17
F1 30b 1				30	0.743	0.072	16.5	166	35	16.1	1.02
F1 40b 1				40	0.743	0.072	11.1	166	35	11.5	0.97
F2 20b 5				20	0.743	0.32	23.6	112	-10.8	21.1	1.12
F2 30a 5				30	0.124	0.32	42.8	112	-10.8	40.1	1.07
F2 40b 5				40	0.743	0.32	13.4	112	-10.8	13.7	0.98

BROWN AND HALL[4]

Test column	100 ρ	f_{cu} N/mm²	f_{pu} N/mm²	l/h	e_i/h	$\frac{f_{cp}}{f_{cu}}$	$(N_u)_{Test}$ kN	$(N_{uz})_{Calc}$ kN	$(N_{bal})_{Calc}$ kN	$(N_u)_{Calc}$ kN	$\frac{(N_u)_{Test}}{(N_u)_{Calc}}$
A1	2.09	35.9	1558	33	0.04	0.019	56	145	32.6	21.3	2.63
A2					0.045		53.2			21.1	2.53
A3					0.13		33.5			19.1	1.75
A4					0.25		22.1			17.0	1.30
A5					0.75		12.0			10.9	1.10
A6					1.965		7.9			6.5	1.21
B1		42.0			0.01	0.182	64.2	130	10.4	32.5	1.98
B2					0.11		43.0			27.3	1.58
B3					0.165		37.8			24.4	1.55
B4					0.305		25.8			20.4	1.27
B5					0.815		13.9			13.0	1.07
B6					1.960		8.0			7.3	1.10
C1		40.3			0.05	0.26	53.4	113	-0.8	31.0	1.72
C2					0.11		46.7			29.0	1.61
C3					0.15		40.0			26.4	1.51
C4					0.285		31.3			21.4	1.46
C5					0.875		15.1			12.8	1.18
C6					1.97		7.9			6.9	1.15

TABLE 1 continued

BROWN AND HALL[(4)] continued

Test column	100 ρ	f_{cu} N/mm^2	f_{pu} N/mm^2	l/h	e_i/h	$\frac{f_{cp}}{f_{cu}}$	$(N_u)_{Test}$ kN	$(N_{uz})_{Calc}$ kN	$(N_{bal})_{Calc}$ kN	$(N_u)_{Calc}$ kN	$\frac{(N_u)_{Test}}{(N_u)_{Calc}}$
D1	2.09	39.0	1558	33	0.025	0.382	57.8	89.5	-20.6	35.8	1.61
D2					0.13		6.7			27.8	1.68
D3					0.285		33.4			21.9	1.52
D4					0.42		26.4			18.4	1.43
D5					0.82		17.5			12.7	1.38
D6					1.995		8.9			6.6	1.34
E1					-0.015	0.471	67.4	73.3	-30.0	36.3	1.86
E2					0.055		57.7			30.1	1.92
E3					0.15		44.5			24.6	1.81
E4					0.35		32.1			18.3	1.75
E5					0.885		18.4			11.7	1.57
E6					2.18		9.3			5.9	1.59
LIPSON[(5)]											
A1	0.23	75.6	1724	40	0.102	0.03	165	773	115	48.7	3.39
A2	0.23	77.3			0.177	0.029	90	790	119	45.0	2.00
B1	0.45	63.8			0.041	0.069	210	638	43.4	64.4	3.27
C1	0.60	56.1			0.081	0.103	175	551	-5.0	74.4	2.35
C2	0.60	61.1			0.050	0.095	246	602	4.8	78.3	3.15
C3	0.60	66.0			0.121	0.088	153	653	14.4	78.3	1.95
C4	0.60	75.6			0.103	0.077	190	753	33.1	82.8	2.30
C5	0.60	77.3			0.037	0.075	246	770	36.2	84.7	2.91
D1	0.91	75.6			0.077	0.116	214	735	-34.5	108.7	1.96
D2	0.91	63.7			0.035	0.138	231	611	-58.0	102.0	2.27
D3	0.91	77.3			0.058	0.114	239	752	-31.6	110.5	2.16
E1	1.20	62.8			0.032	0.184	244	584	-123	126.7	1.92
E2	1.20	57.5			0.080	0.201	165	529	-133	119.6	1.38
COMPUTER ANALYSES											
1 a	1.00	40	1700	20	0.05	0	204	335	-10.3	114	1.42
1 b						0.15	167	272	-16.1	176	0.95
1 c						0.3	127	207	13.5	146	0.87
2 a					0.2	0	109	335	-10.3	85	1.29
2 b						0.15	110	272	-16.1	104	1.06
2 c						0.3	92	207	13.5	99	0.93
3 a					0.5	0	50	335	-10.3	52	0.97
3 b						0.15	58	272	-16.1	60	0.98
3 c						0.3	57	207	13.5	59	0.96
4 a				30	0.05	0	149	335	-10.3	51	2.91
4 b						0.15	128	272	-16.1	72	1.79
4 c						0.3	100	207	13.5	102	0.98
5 a					0.2	0	66	335	-10.3	43	1.52
5 b						0.15	84	272	-16.1	60	1.41
5 c						0.3	72	207	13.5	71	1.02
6 a					0.5	0	31	335	-10.3	33	0.93
6 b						0.15	45	272	-16.1	41	1.10
6 c						0.3	46	207	13.5	45	1.01
7 a				40	0.05	0	103	335	-10.3	28	3.70
7 b						0.15	96	272	-16.1	42	2.30
7 c						0.3	77	207	13.5	59	1.30
8 a					0.2	0	41	335	-10.3	26	1.59
8 b						0.15	63	272	-16.1	36	1.74
8 c						0.3	57	207	13.5	47	1.22
9 a					0.5	0	20	335	-10.3	22	0.93
9 b						0.15	36	272	-16.1	29	1.25
9 c						0.3	38	207	13.5	34	1.10

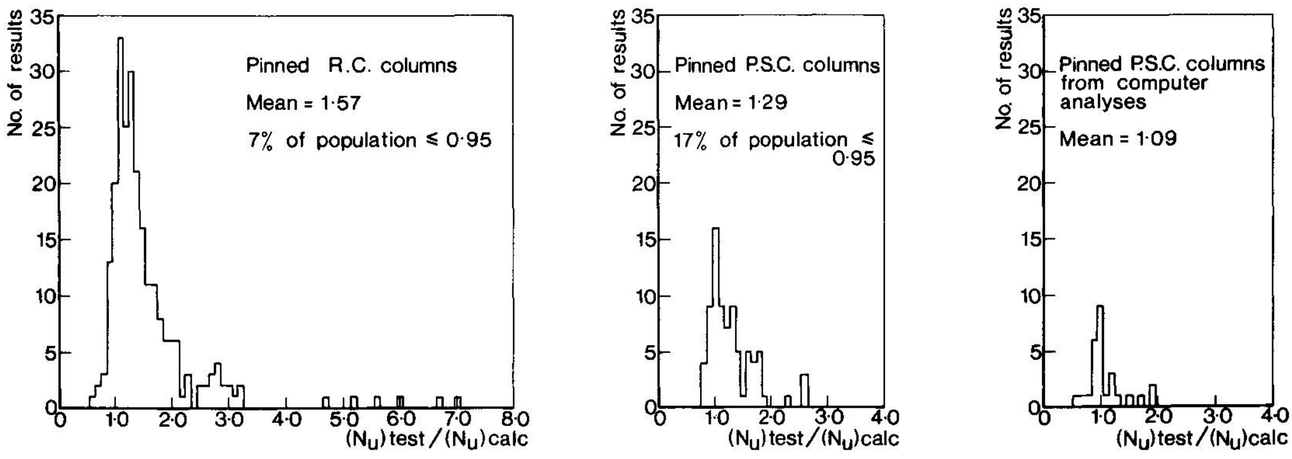

FIGURE 1 (N_u) TEST/ (N_u) CALC USING ADDITIONAL MOMENT EXPRESSION FOR R.C. SECTIONS

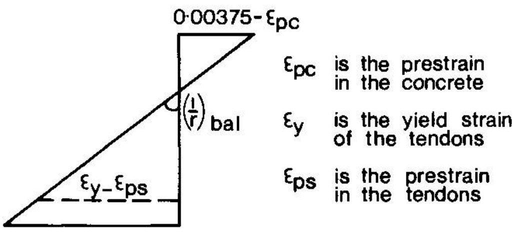

FIGURE 2 STRAIN PROFILE ACROSS SECTION AT THE "BALANCE POINT"

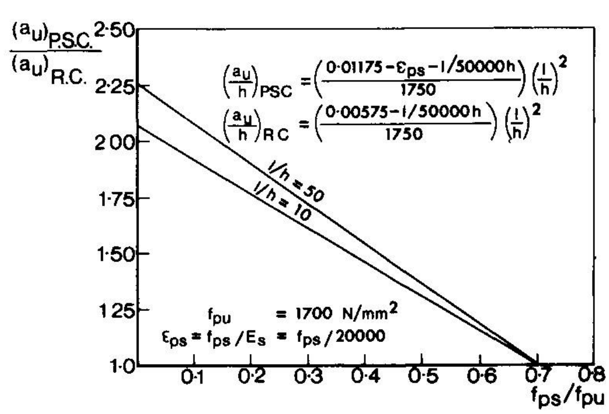

FIGURE 3 EFFECT OF PRESTRESS ON ULTIMATE DEFLECTION

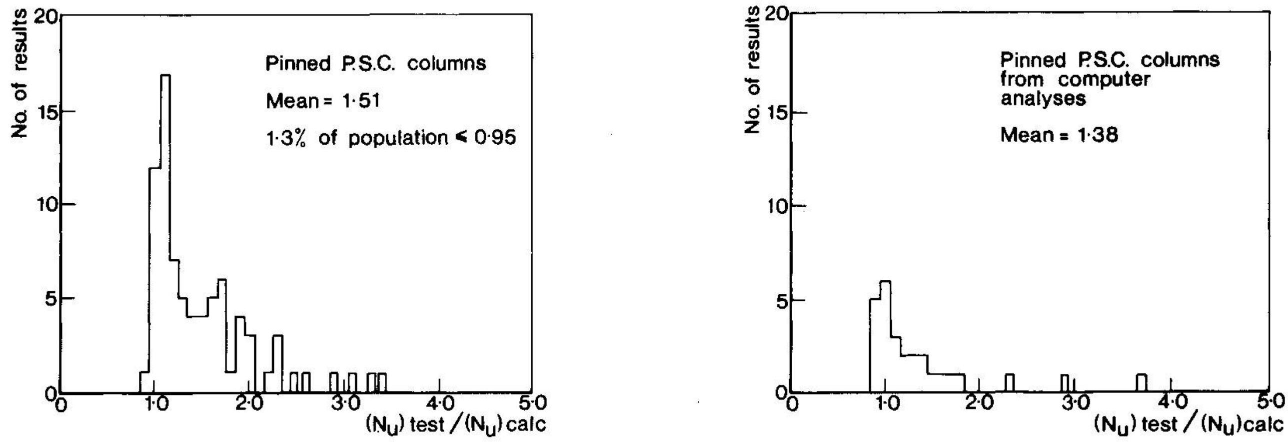

FIGURE 4 (N_u) TEST/ (N_u) CALC USING MODIFIED ADDITIONAL MOMENT EXPRESSION

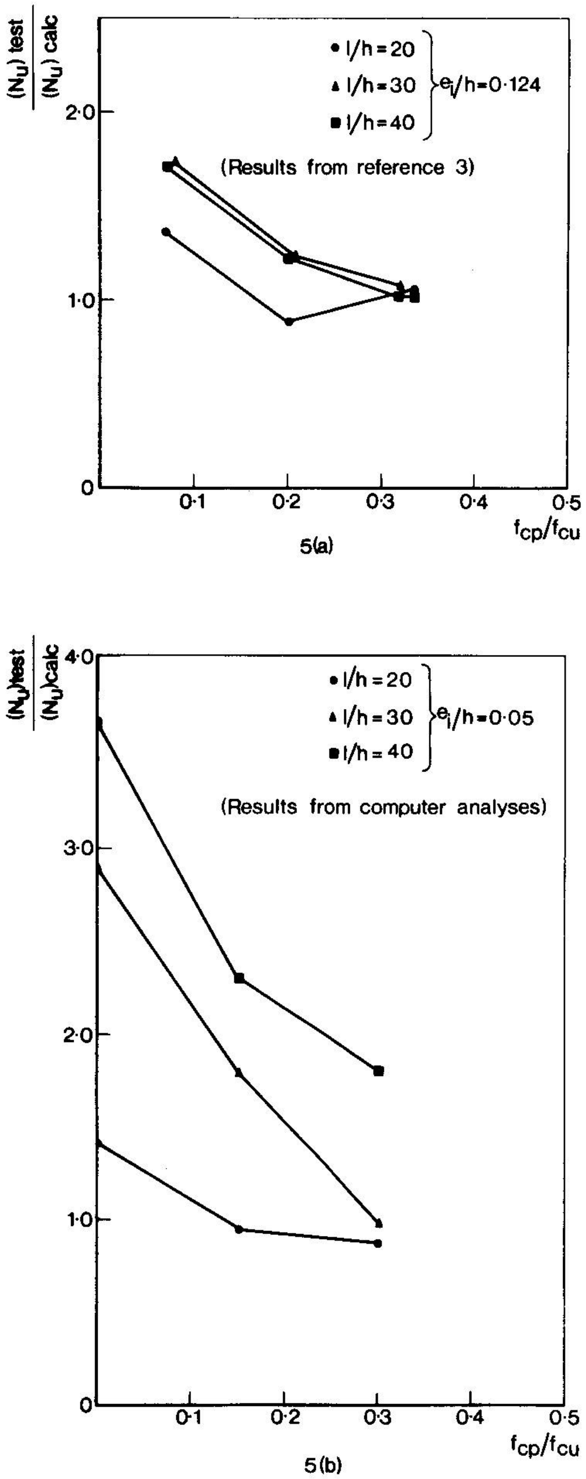

FIGURE 5 EFFECT OF PRESTRESS ON (N_U)TEST / (N_U)CALC

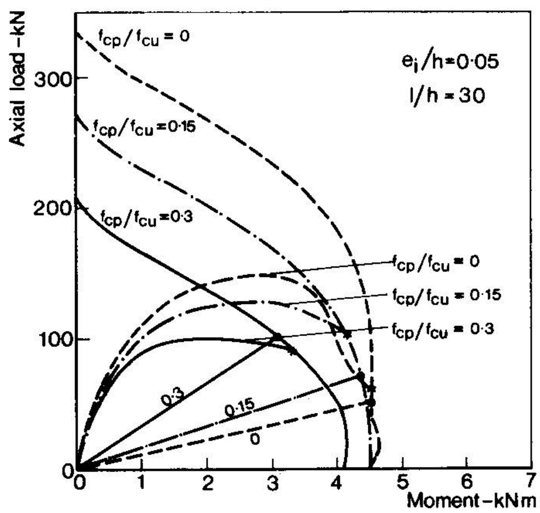

FIGURE 6A EFFECT OF PRESTRESS ON LOAD v MAXIMUM MOMENT

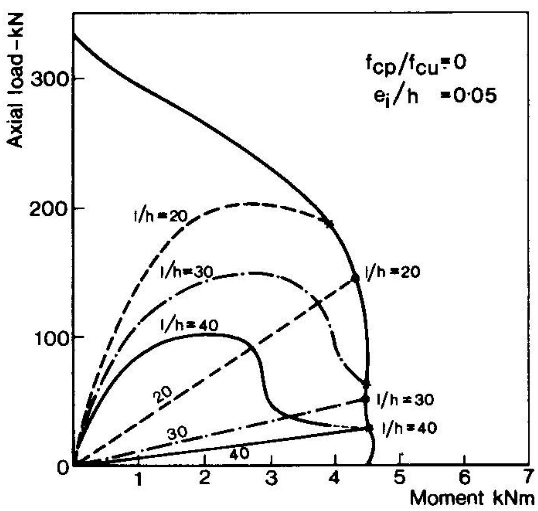

FIGURE 6B EFFECT OF SLENDERNESS ON LOAD v MAXIMUM MOMENT

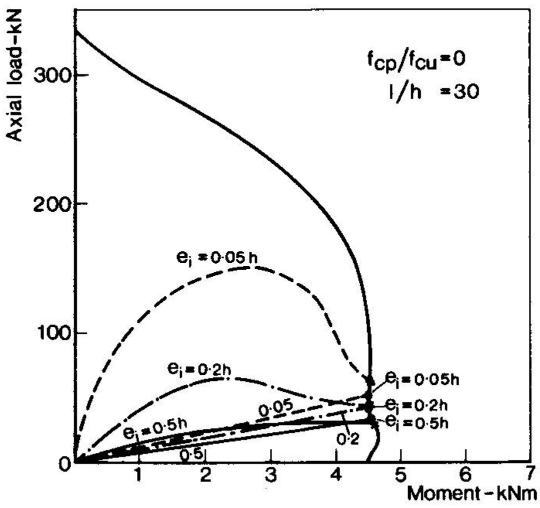

FIGURE 6C EFFECT OF INITIAL ECCENTRICITY ON LOAD v MAXIMUM MOMENT

SUMMARY

A comparison between calculated and test failure loads of prestressed concrete columns is presented. The calculated values were obtained by applying the additional moment method as derived for R.C. columns to the prestressed concrete columns in the test work reported. The comparisons showed somewhat different trends relative to R.C. columns and a series of computer analyses was carried out in order to study more closely the reasons for these differences. The behaviour of prestressed columns as influenced by slenderness, initial eccentricity of loading and level of prestress is discussed and recommendations for design proposed.

RESUME

La contribution présente une comparaison entre les charges ultimes calculées et celles constatées à l'essai, pour les colonnes précontraintes en béton armé. Les valeurs calculées ont été obtenues en appliquant aux colonnes précontraintes la méthode des moments additionnels des colonnes en béton armé. Les comparaisons ont montré des tendances assez différentes de celles des colonnes en béton armé; une série d'analyses par ordinateur ont été exécutées en vue d'étudier de plus près les raisons de ces différences. On a discuté le comportement de colonnes précontraintes sous l'influence de l'élancement, de l'excentricité initiale à la charge et du degré de précontrainte, et on propose des recommandations pour le dimensionnement.

ZUSAMMENFASSUNG

Der vorstehende Bericht behandelt einen Vergleich zwischen berechneten und Versuchsbruchlasten vorgespannter Stahlbetonstützen. Die berechneten Werte für die dem Versuch unterworfenen Spannbetonstützen ergeben sich nach der für Stahlbetonstützen hergeleiteten Methode der Zusatzmomente. Die Vergleiche zeigen ein im Vergleich zu Stahlbetonstützen unterschiedliches Verhalten; eine Reihe von Computerberechnungen wurde durchgeführt, um die Ursachen für diese Abweichungen genauer zu untersuchen. Das Verhalten vorgespannter Stützen unter dem Einfluss von Schlankheit, Anfangsexzentrizität und Grad der Vorspannung wird diskutiert und Empfehlungen für die Bemessung gegeben.