

**Zeitschrift:** IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

**Band:** 16 (1974)

**Artikel:** Elastic-postelastic analysis of the cyclic behaviour of reinforced concrete columns taking account of the effort of bond

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**DOI:** <https://doi.org/10.5169/seals-15743>

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## IV

### **Elastic-Postelastic Analysis of the Cyclic Behaviour of Reinforced Concrete Columns Taking Account of the Effort of Bond**

Calcul élastique et post-élastique du comportement sous charge répétée de colonnes en béton armé, en tenant compte de la cohésion béton—acier

Elastisch-überelastische Berechnung des Verhaltens von Stahlbetonstützen unter wiederholter Belastung unter Berücksichtigung der Haftung

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#### 1. Introduction

In our countries subjected to earthquakes it is important that structures are capable of deforming in a ductile manner under the action of severe seismic motions. Recently, in the design of reinforced concrete frames it is necessary to consider the behavior of frames in the postelastic range, when researching the accurate response of the frames to the seismic motions. The accurate behavior of frames at the ultimate load for these aspects is dependent on grasping the accurate behavior of members in the elastic-postelastic range. The elastic-postelastic behavior of reinforced concrete columns under cyclic loading is discussed herein for the purpose of researching the accurate behavior of reinforced concrete frames under cyclic loading.

The elastic-postelastic analysis of the cyclic behavior of reinforced concrete columns is discussed herein in the process as;

1. Properties of concrete under cyclic stress in a member.
2. Properties of steel under cyclic stress.
3. Moment-curvature relation in the cross section of a member.
4. Longitudinal distribution of curvature.
5. The effect of bond.
6. Shear deformation.
7. Method of analysis.

And the experiment is done to investigate the general cyclic behavior of columns and to make sure the method of analysis.

The author propose that considering the effect of beam-column connection behavior under cyclic load, the analysis is capable to utilize for elastic-postelastic analysis of the cyclic behavior of the reinforced concrete frames.

#### 2. Properties of concrete

Many equations for the stress-strain curve of concrete under uniaxial compression have been proposed in past years. Proba-

bly the most widely accepted curve in our countries is the proposed by Umemura(1) which consists of a e-function up to the maximum stress  $\sigma_B$  at a strain  $\epsilon_B$  and down of the same function as shown in Fig1.

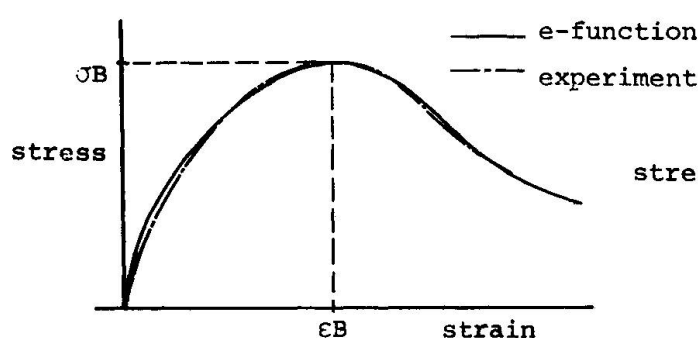


FIG1. e-FUNCTION CURVE

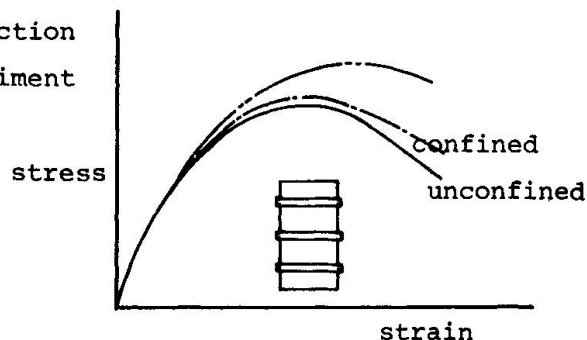


FIG2. CONFINED CONCRETE CURVE

Umemura's curve showed a little stress over the experimental value in initial range but is become widely used. Concrete which is restrained in the directions at right angles to the applied stress will be referred to as confined concrete. The test conducted by Hanajima(2) showed that both the strength and ductility of the concrete are greatly increased by such stress, as shown in Fig2. However, the tests of reinforced concrete members by many investigators in the past, showed that rectangular hoops do not confine the concrete as effectively as circular spirals.

On the basis of the experimental evidence it is proposed that the curve as shown in Fig3 is able to cover the cyclic stress-strain curve for confine concrete by rectangular hoops.

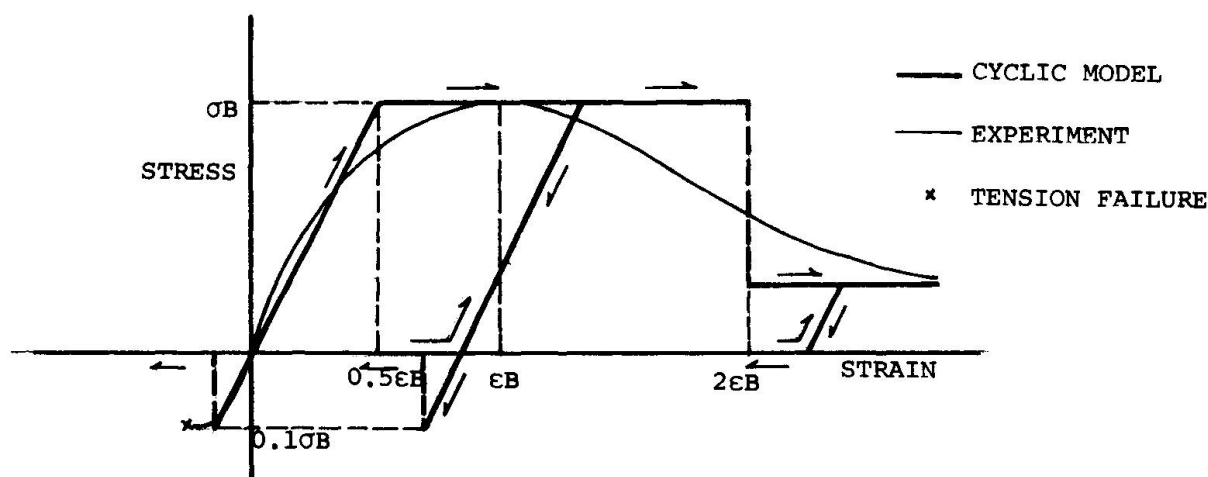


FIG3. PROPOSED STRESS-STRAIN CURVE OF CONCRETE

### 3. Properties of steel

The behavior of structural reinforcing steel under tensile loading has been extensively explored within both its elastic and inelastic range, but little information is available regarding its inelastic behavior under reversed loading. By reversing the graph paper between tensile and compressive testing, it was possible to obtain continuous graphs as shown in Fig4. In assessing the factors that are responsible for the difference between the virgin

stress-strain curves and those obtained after previous cycles of inelastic loading, the following variables are relevant; (1) the virgin properties of the material; (2) the entire previous history of loading; and (3) the rate of straining.

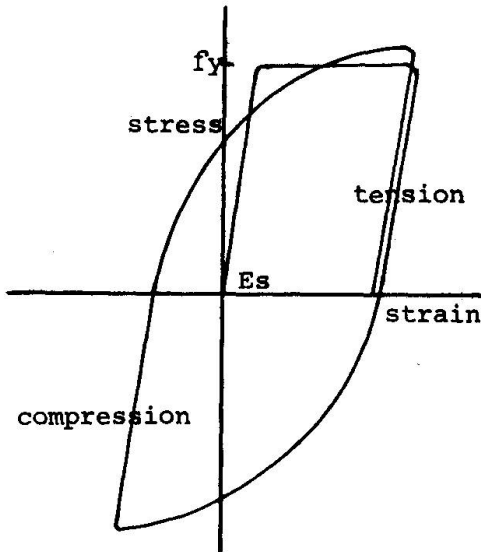


FIG4. EXPERIMENTAL CURVE

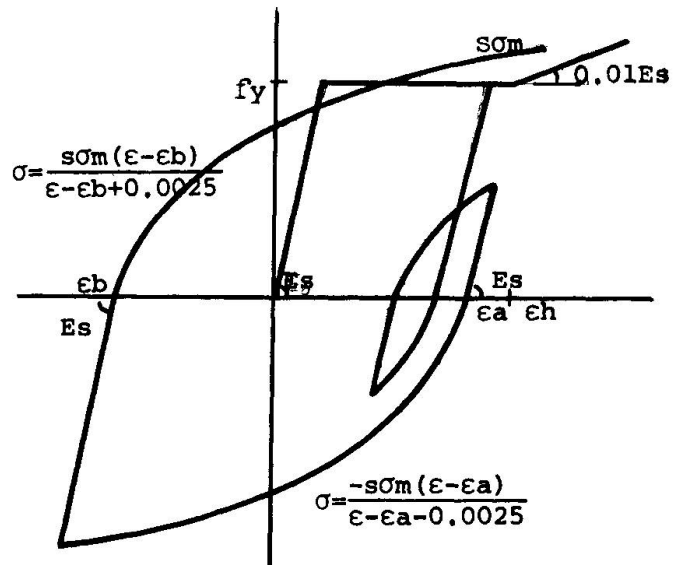


FIG5. PROPOSED CURVE OF STEEL

However, considering the state of existing columns, the influence of the plastic strain at previous reversal is being considered quantitatively, and proposed cyclic stress-strain curve for reinforcing steel is shown in Fig5.

#### 4. Flexural strength and curvature analysis

In the analysis of sections for flexure the following assumptions will be made.

1. The longitudinal strain in the steel and concrete at the various section levels is directly proportional to the distance from the neutral axis.
2. The cyclic stress-strain curve for the steel and concrete have general shapes shown in Fig3, Fig5.
3. The 50 concrete blocks are taken in the section, and each block has the properties of concrete, as shown in Fig6.

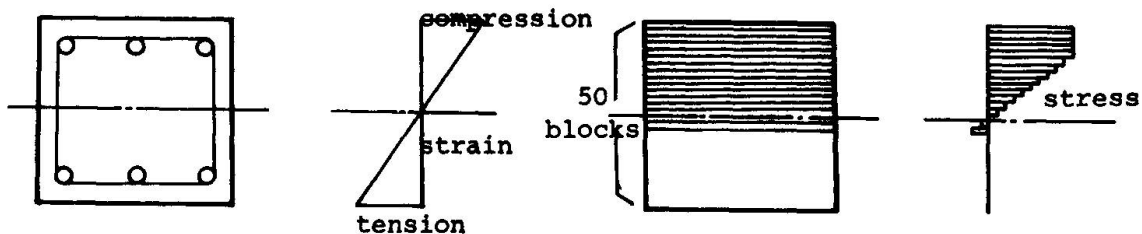


FIG6 STRESS DISTRIBUTION OF CONCRETE BLOCKS

4. Cyclic behavior of the concrete blocks is shown in Fig7.

5. The equilibrium of forces in the section.

Fig8 shows some theoretical moment-curvature curves obtained for rectangular concrete columns sections with different amount of longitudinal steel and axial stress, compared to the e-function theory.

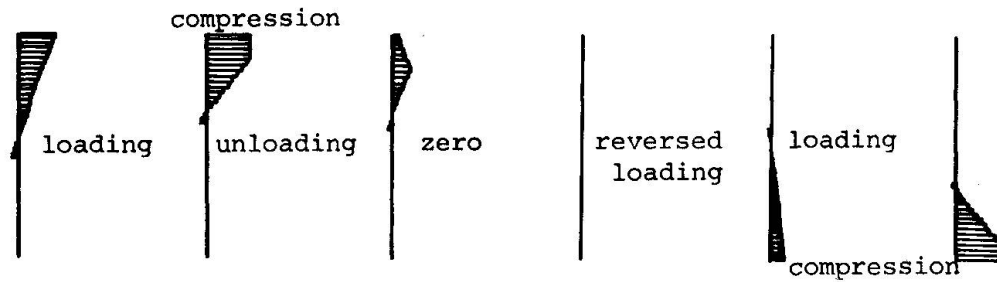


FIG.7 CYCLIC STRESS OF CONCRETE BLOCKS

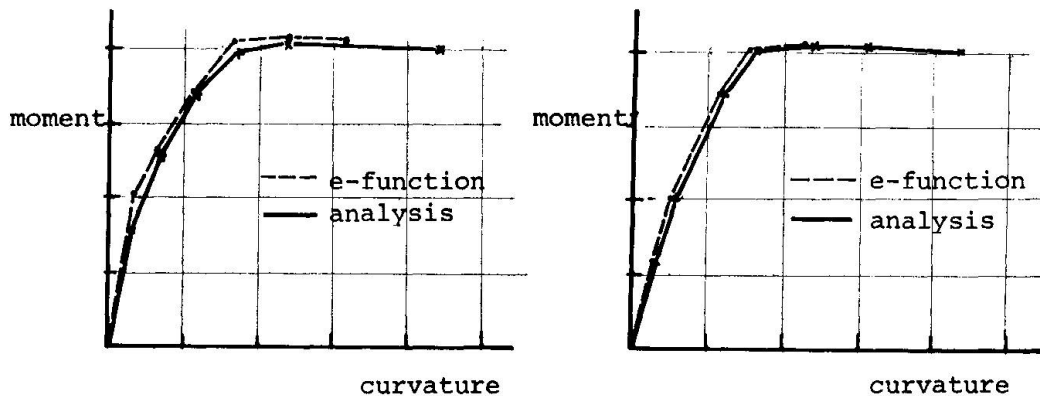


FIG.8 THEORETICAL MOMENT-CURVATURE RELATION

### 5. Distribution of curvature

By longitudinal distribution of curvature, the deformation of member due to bending moment can be obtained. It is not good for obtaining the longitudinal distribution of curvature to use the moment-curvature relation of cross section because many sections have a considerable capacity for plastic rotation beyond the peak of the moment-curvature curve. Tests by many investigators in the past, have shown that such longitudinal distribution of curvature in reinforced concrete column can be considerably obtained by simple model having a flexible zone at the end of member.

The simple model and the experimental distribution of curvature in reinforced concrete column testing by Kokusho(3) are shown in Fig9.

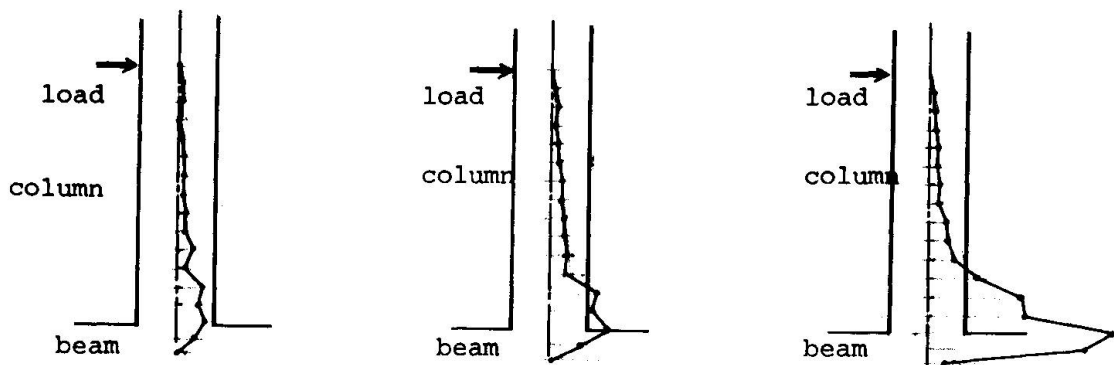


FIG.9 EXPERIMENTAL DISTRIBUTION OF CURVATURE

Proposed distribution considering the flexible zone of curvature is shown in Fig10.

#### 6. The effect of bond

Maximum bond stress for concrete and steel due to form of reinforcing bar, concrete strength, concrete cover for reinforcement. While in the beam-column connection, a large shear force occurs during an earthquake and as a result large shear and bond stresses are produced in concrete and reinforcements. Test by Okita(4) showed the bond stress-slip curve under cyclic loading in Fig11.

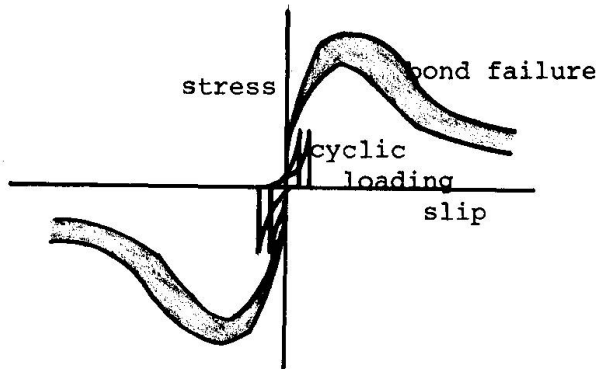


FIG.11 BOND STRESS-SLIP CURVE

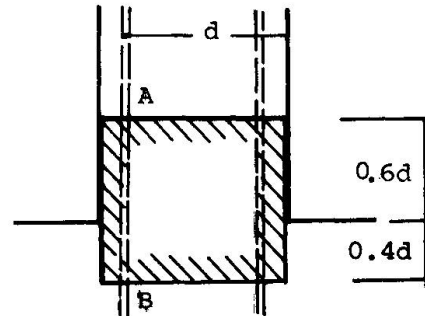


FIG.10 FLEXIBLE ZONE

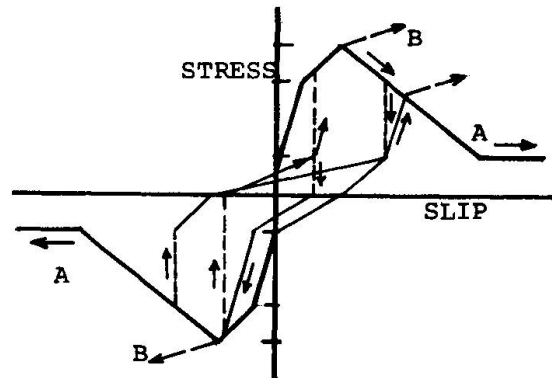


FIG.12 PROPOSED CYCLIC CURVE

The permissible bond stress along the reinforcement can be increased by making the surface of a bar rough or irregular.

In this research, considering of such deformed bars are generally used for longitudinal reinforcement. On the basis of the experimental and analytical evidence it is proposed that the curve bond stress-slip relation shown in Fig12, separating the cases of in column(A) and in pannel(B).

The average curvature of flexible zone due to bond slip is shown as follows.

$$1/\phi_a = \frac{s_1 + s_2 - s_3 - s_4}{f \times D}$$

Where  $f$ ; length of the flexible zone,  $D$ ; spacing of tension and compression reinforcement,  $s_1$   $s_4$ ; bond slip to flexible zone.

The curvature due to bond effect is added to bending curvature.

#### 7. Deformation due to shearing force

In reinforced concrete members, it is difficult to obtain the accuracy deformation due to shearing force. The author propose the experimental method to obtain the deformation as follows.

In the experimental study of reinforced concrete columns, the deformation due to shearing force is regarded as the difference between the total measured deformation and the deformation calculated by the longitudinal distribution of the measured curvature including crack's width. The experimental results are shown in Fig13.

In considering shear strength, the study by Arakawa(5) showed the average shear strength of reinforced concrete beams as follows. On the basis of the existing experimental evidence it is proposed that the curve shown in Fig14 gives a good representation of the shear stress-deformation relationship for general

reinforced concrete column.

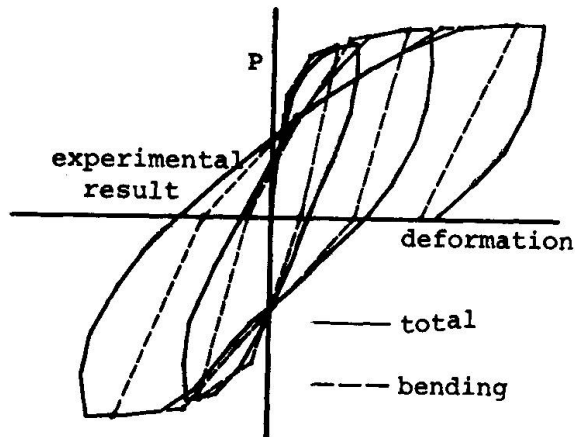


FIG.13 BENDING DEFORMATION

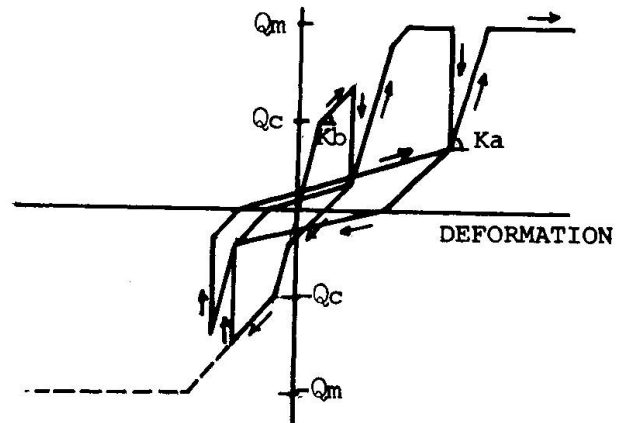


FIG.14 PROPOSED CYCLIC CURVE

### 8. Method of analysis

In the analysis developed in this study assumed that previous steps, a computer program was used embodying the step shown in Fig15.

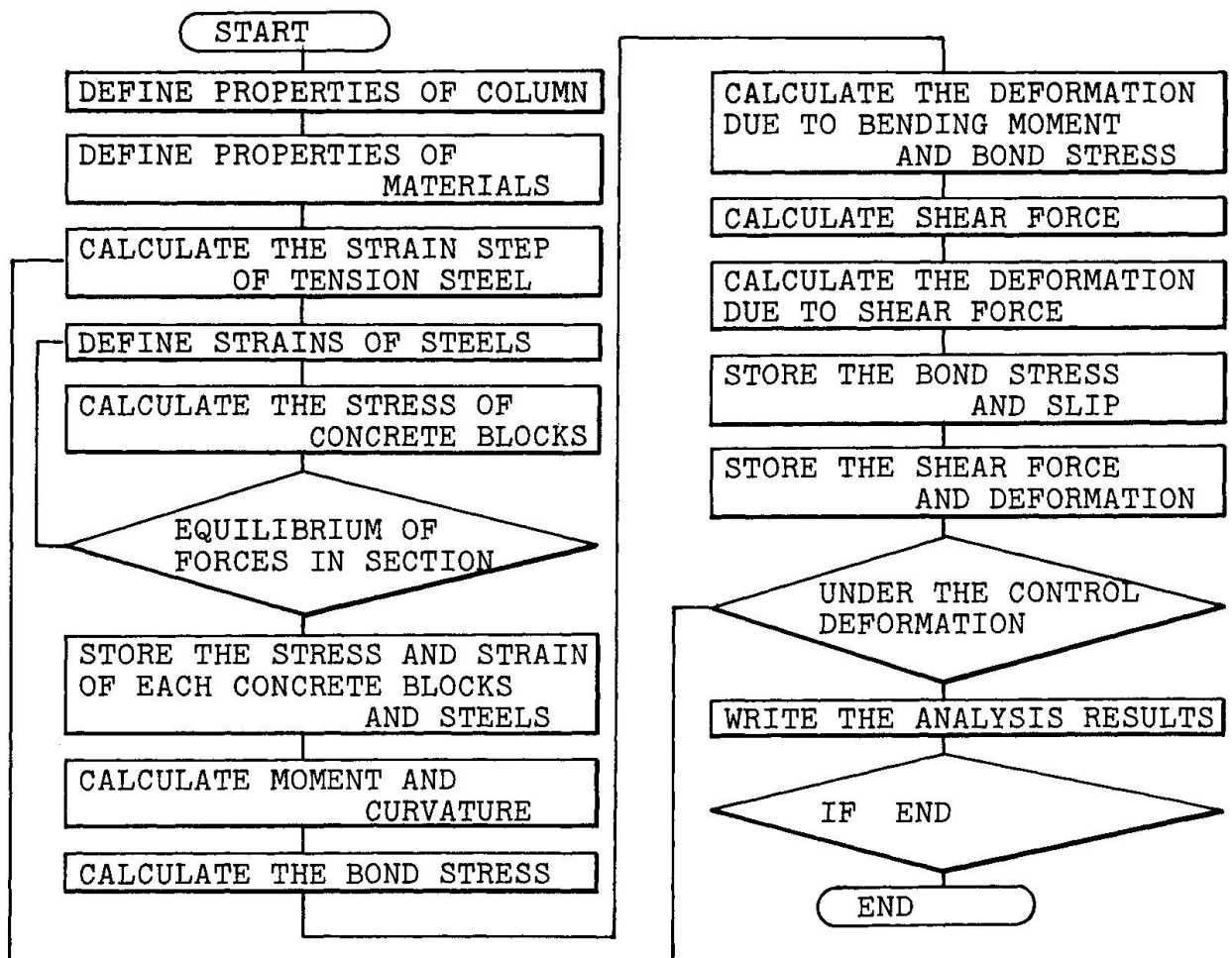


FIG. 15 ANALYSIS PROGRAM FLOW

### 9. Results of analysis

The analysis described in this paper was compared to the results of test on 16 columns.

Experiment of columns ; In the test, the principal variables are the ratio of span to depth ( $a/D$ ), the axial stress ( $\sigma_0$ , in  $\text{kg/cm}^2$ ), the amount of longitudinal tension steel ( $P_t$ , in %), and the shear steel ( $P_w$ , in %). The test setup shown in Fig.16 consists of steel reaction frames which is anchored to the test floor, hydraulic jacks for lateral forces and reaction steel bars and a oil pressure jack for constant axial forces. The test specimens are anti-symmetrically and reversally loaded by two pairs of jacks to simulate the moment distribution in actual column subjected to lateral forces.

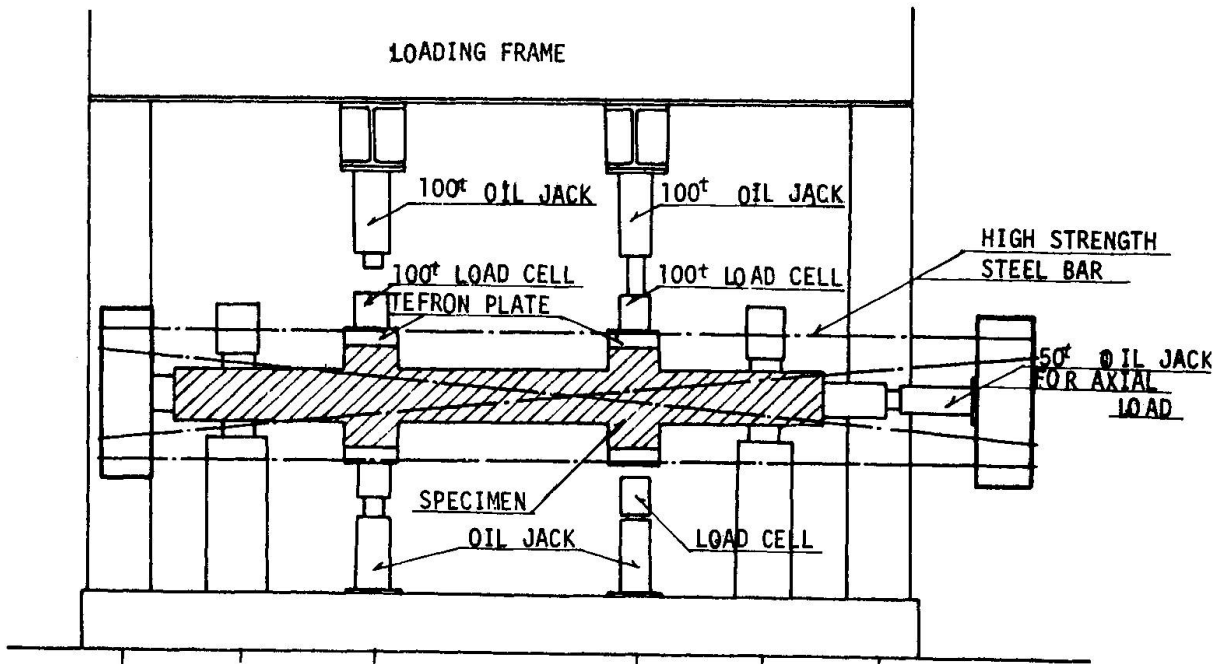


FIG.16 TEST SETUP

The typical results of load deformation curves compared with the results of the analysis are shown in Fig.17, Fig.18. The analytical results show a good agreement with the experimental results.

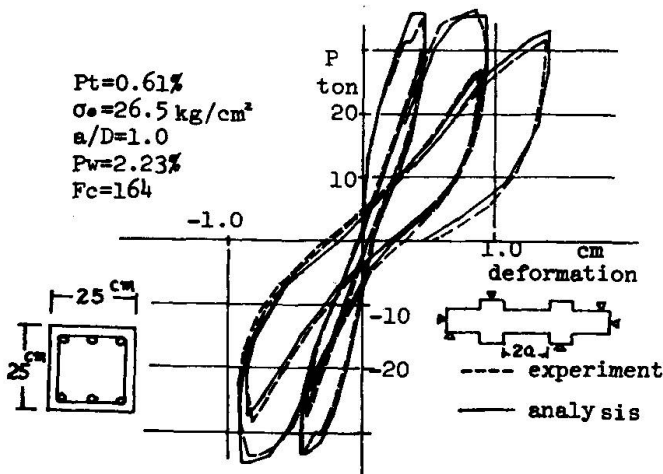


FIG.17 LOAD-DEFORMATION CURVE

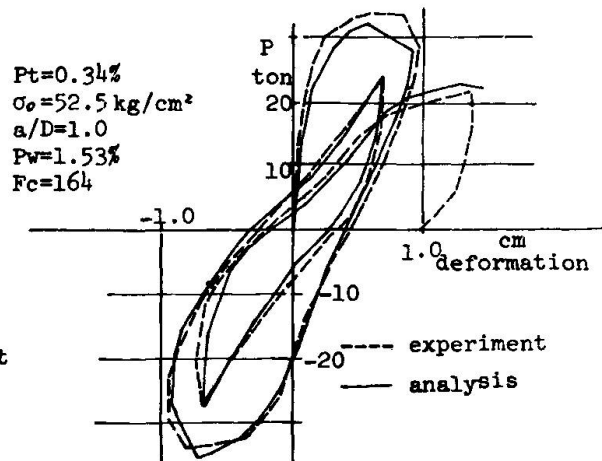


FIG.18 LOAD-DEFORMATION CURVE



## 10. Conclusions

The calculated load deformation characteristics of columns show a good agreement with the test results of reinforced concrete columns. The analysis is proposed to use the analysis of reinforced concrete frame directly. The author hope to propose the analysis of reinforced concrete frame in the other paper.

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## SUMMARY

The elastic-postelastic behaviour of reinforced concrete columns under cyclic loading is discussed herein for the purpose of the analysis of reinforced concrete frames. In this paper, the elastic-postelastic analysis of the cyclic behaviour of reinforced concrete columns taking account of the effect of bond and the effect of shear force is proposed. The analytical results show a good agreement with the experimental results.

## RESUME

On traite du comportement élastique et post-élastique de colonnes en béton armé soumises à une charge répétée, pour en déduire un calcul des cadres en béton armé. Dans ce rapport, on propose un calcul élastique et post-élastique du comportement sous charge répétée de colonnes en béton armé, en tenant compte de la cohésion entre l'armature et le béton, et de l'influence de l'effort tranchant. Les résultats analytiques concordent bien avec les résultats expérimentaux.

## ZUSAMMENFASSUNG

Das elastisch-überelastische Verhalten von Stahlbetonstützen unter wiederholter Belastung wird - als Grundlage für die Berechnung von Stahlbetonrahmen - diskutiert. Im Beitrag wird eine elastisch-überelastische Berechnung des Verhaltens von Stahlbetonstützen unter wiederholter Belastung und unter Berücksichtigung der Haftung zwischen Beton und Bewehrung und der Querkraft vorgeschlagen. Die Berechnungsergebnisse zeigen gute Übereinstimmung mit den Versuchsergebnissen.