

**Zeitschrift:** IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

**Band:** 16 (1974)

**Artikel:** Effect of confining reinforcement on the flexural ductility of rectangular reinforced concrete column sections with high strength steel

**Autor:** Park, R. / Norton, J.A.

**DOI:** <https://doi.org/10.5169/seals-15741>

#### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

#### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

#### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 26.01.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

**Effect of Confining Reinforcement on the Flexural Ductility of Rectangular Reinforced Concrete Column Sections with High Strength Steel**

Influence des armatures en acier à haute résistance situées au bord de la section sur la ductilité à la flexion des colonnes rectangulaires en béton armé

Der Einfluss hochfester Bewehrung in rechteckigen Stahlbetonstützen auf das Verformungsvermögen infolge Biegung

<b>R. PARK</b>	<b>J.A. NORTON</b>
University of Canterbury	Ministry of Works
Christchurch	Wellington
New Zealand	

**INTRODUCTION**

The need to design structures capable of undergoing large post-elastic deformations is of considerable importance if very severe earthquakes are to be survived. Current seismic design codes rely on this energy dissipating characteristic as a major factor in justifying the present levels of seismic design load.

The ductility of compressed concrete can be significantly increased by the presence of closely spaced transverse steel in the form of steel spirals or hoops. Such steel confines the concrete and significantly improves the stress-strain characteristics of the concrete at high strains.

This paper is based on a theoretical study (1) into the effect of transverse confining steel on the flexural ductility of short reinforced concrete columns with rectangular cross sections. Moment-curvature characteristics of column sections under eccentric monotonic loading are determined from the stress-strain curves of concrete confined with rectangular steel hoops and of longitudinal steel including the effects of strain hardening. The degree of flexural ductility available from sections is indicated by the moment-curvature curves and the effect of transverse steel is assessed. This leads to a method for detailing columns for flexural ductility. A range of square column sections containing longitudinal steel with a yield strength of 60,000 psi ( $414 \text{ N/mm}^2$ ) is examined. This paper extends a previous study in which columns containing steel with a yield strength of 40,000 psi ( $276 \text{ N/mm}^2$ ) were examined (2).

**2. DETAILING FOR DUCTILITY**

**2.1 Ductility Demands During Seismic Loading**

A measure of the post-elastic performance of structures is given by the displacement ductility factor defined as  $\Delta_u/\Delta_y$  where  $\Delta_u$  is the lateral deflection at the end of the post-elastic range and  $\Delta_y$  is the lateral deflection at first yield. Several dynamic analyses of structures using recorded ground motion response from severe earthquakes have indicated that displacement ductility factors of the order 3 to 5 should provide

structures designed to the SEAOC (3) and ICBO (4) code loading with sufficient energy dissipation capacity. Those codes, and the ACI code (5), make detailing recommendations for design which are aimed at achieving adequate ductility. The post-elastic deformation of reinforced concrete frames is concentrated in regions of plastic hinging and to achieve a given displacement factor much larger local rotational ductilities are required within the hinging region. The rotation capacity of sections can be expressed by the curvature ductility factor  $\phi_u/\phi_y$ , where  $\phi_u$  is the section curvature at the end of the post-elastic range and  $\phi_y$  is the curvature at first yield. This assumes that flexural deformations predominate. An approximate procedure for assessing the likely demands, based on a static collapse mechanism of frames under lateral loading (6), has indicated that for a collapse mechanism involving plastic hinges in the beams and at the column bases, the  $\phi_u/\phi_y$  demands at the column bases could be in the order of three times the  $\Delta_u/\Delta_y$  value. For a collapse mechanism involving plastic hinges only in the columns, inducing an inter-storey sidesway collapse mechanism, the  $\phi_u/\phi_y$  demands may be much higher.

## 2.2 Capacity of Reinforced Concrete Column Sections for Ductile Behaviour

The moment-curvature relationship provides a measure of the plastic rotation capacity of sections. One problem in assessing this in relation to the  $\phi_u/\phi_y$  demand is the definition of the ultimate curvature  $\phi_u$ . It has been pointed out previously (2) that many sections maintain considerable capacity for plastic rotation beyond the peak of the moment-curvature curve and that it would be reasonable to recognise this and define  $\phi_u$  as the curvature when the moment capacity of the section has reduced to 80-90% of the maximum moment. A criterion for satisfactory ductile performance of columns can thus be established on the basis of reaching a curvature ductility  $\phi_u/\phi_y$  of at least 15, with  $\phi_y$  defined as above. This should ensure structures achieving a displacement ductility factor  $\Delta_u/\Delta_y$  of at least 4 with a limited reduction in strength provided sidesway mechanisms with plastic hinging concentrating in the columns of one storey, and brittle shear and anchorage failures, are avoided. The derivation of moment-curvature relationships from the stress-strain curves of confined concrete and longitudinal steel allows this criterion to be applied and enables the required amount of transverse steel to be determined.

Such an approach is more rational than existing seismic code procedures for determining transverse steel requirements in columns. The shortcomings of the existing code (3), (4), (5) requirements have been discussed previously (2), (7). These existing code requirements are based on the increase in concrete strength observed in tests on axially loaded columns with confined concrete. The code approach involves a philosophy of maintaining the axial load carrying capacity of the column. Sufficient transverse steel is provided to enhance the strength of the core concrete so that when the cover concrete spalls the axial load strength is not reduced. In deriving the requirements to meet this philosophy for columns with rectangular hoops, the hoop steel is assumed to be 50% as effective as circular spirals as confining reinforcement. The philosophy itself is assumed to produce structures with sufficient ductility. This code approach may maintain the axial load strength of columns but fails to relate the detailing requirements to the required rotational capacities of eccentrically loaded sections. The approach based on the moment-curvature relationships obtained from stress-strain curves outlined previously could form a rational basis of detailing columns for ductility.

## 3. DERIVATION OF MOMENT-CURVATURE CHARACTERISTICS

### 3.1 Factors Taken Into Account

In deriving the moment-curvature characteristics of eccentrically loaded rectangular reinforced concrete column sections the following factors are taken into account: the level of axial load on the column, the proportion

of column section confined, the longitudinal steel content, and the material strength characteristics. Ideally the effects of cyclic loading should also be considered but the complexity of a cyclic analysis makes it difficult to study a large range of cases. In this study monotonic loading will be analysed and it is felt that this should give a reasonable assessment in the first instance.

### 3.2 Assumptions

The analysis of the moment-curvature characteristics is derived on the basis of the following assumptions:

- (i) Plane sections remain plane after flexure.
- (ii) The stress-strain curve of steel reinforcement in tension or compression follows the curve of Fig. 1a. There are three regions as follows:

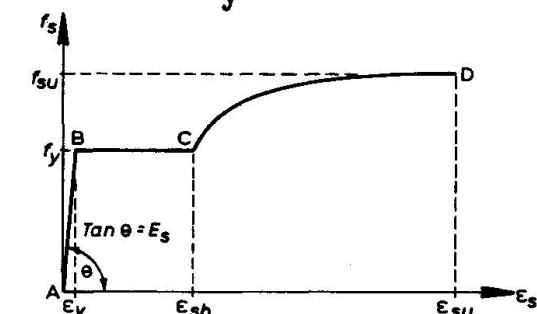
Region AB  $\epsilon_s \leq \epsilon_y$ ,  $f_s = \epsilon_s E_s$  ... (1)

Region BC  $\epsilon_y \leq \epsilon_s \leq \epsilon_{sh}$ ,  $f_s = f_y$  ... (2)

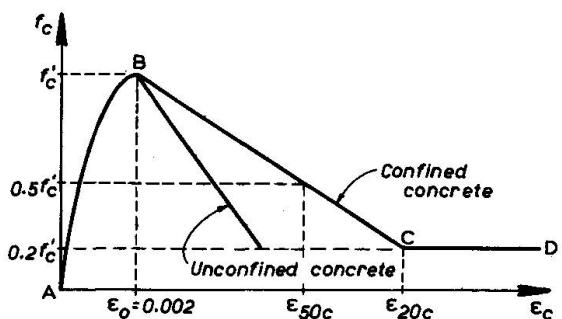
Region CD  $\epsilon_{sh} \leq \epsilon_s \leq \epsilon_{su}$ ,  

$$f_s = f_y \left\{ \frac{m(\epsilon_s - \epsilon_{sh}) + 2}{60(\epsilon_s - \epsilon_{sh}) + 2} + \frac{(\epsilon_s - \epsilon_{sh})(60 - m)}{2(30r + 1)^2} \right\} \quad \dots (3)$$

where  $m = \left\{ \frac{f_{su}}{f_y} (30r + 1)^2 - 60r - 1 \right\} / 15r^2$  and  $r = \epsilon_{su} - \epsilon_{sh}$



(a) Steel in tension or compression



(b) Concrete in compression confined by rectangular hoops

This curve follows that reported previously (2).

For columns with high yield ( $f_y = 60,000$  psi =  $414$  N/mm $^2$ ) longitudinal steel studied in this paper, the following steel parameters are used:  $\epsilon_{sh} = 4 \epsilon_y$ ,  $\epsilon_{su} = 0.12$ ,  $f_{su} = 1.58 f_y$ . These values were obtained from a series of tests conducted by Norton (1) on New Zealand produced reinforcing steel of this grade.

- (iii) The tensile strength of concrete is ignored.
- (iv) The stress-strain curve for confined concrete in compression follows the curve of Fig. 1b. There are three regions as follows:

Region AB  $\epsilon_c \leq 0.002$

$$f_c = f'_c \left\{ \frac{2\epsilon_c}{0.002} - \left( \frac{\epsilon_c}{0.002} \right)^2 \right\} \quad \dots (4)$$

Region BC  $0.002 \leq \epsilon_c \leq \epsilon_{20c}$

$$f_c = f'_c \left\{ 1 - z \left( \epsilon_c - 0.002 \right) \right\} \quad \dots (5)$$

Fig. 1 Assumed Stress-Strain Curves.

where  $Z = 0.5 / \left( \frac{3}{4} \rho_s \sqrt{\frac{b''}{s_h}} + \frac{3 + 0.002 f'_c}{f'_c - 1000} - 0.002 \right)$  with  $f'_c$  in psi ... (6)

Region CD  $\epsilon_{20c} \leq \epsilon_c$ ,  $f_c = 0.2 f'_c$  ... (7)

This curve follows that proposed previously (8). From Eq. (6) and Fig. 1b it can be seen that the parameter  $Z$  defines the slope of the falling of the stress-strain curve in non-dimensional terms. It describes the effect of the transverse rectangular confining steel on the stress carried by the concrete at high strains.

- (v) The stress-strain curve for the cover concrete (outside the hoops) in compression is identical to that of the confined concrete core up to a strain of 0.004. The cover concrete at strains greater than 0.004 is considered to have spalled and to have zero strength.

### 3.3 Calculation of the Parameter Z for Arrangements of Transverse Steel

In practice, various arrangements of transverse steel involving overlapping hoops, or hoops with supplementary cross ties, may be required in columns to provide lateral support to the longitudinal bars in the section. The column section of Fig. 2 shows one possible arrangement. The stress-strain relationship for confined concrete given by Eqs. (5) and (6) was obtained from test results for small concrete specimens with hoops only

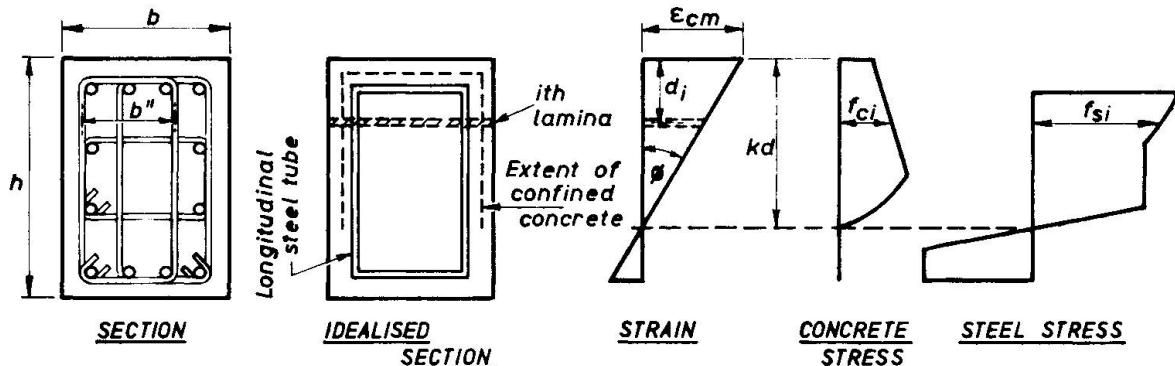


Fig. 2 Section With Stress and Strain Distribution.

around the perimeter of the specimen. Additional transverse bars across the section will help confine the concrete and need to be taken into account. To include the effect of such additional transverse bars the parameter  $Z$  in the stress-strain relationship may be calculated for the partitioned concrete section. For example, for the section with overlapping hoops shown in Fig. 2,  $Z$  may be calculated using  $b'' =$  width of side of one hoop,  $s_h =$  spacing of sets of overlapping hoops and  $\rho_s =$  ratio of volume of one hoop to volume of concrete core within that hoop. This definition of  $\rho_s$  is more conservative than the alternative of taking  $\rho_s$  as the ratio of the total volume of hoops to total volume of concrete core, but in view of the lack of test data on the efficiency of overlapping hoops it is probably wise to use the more conservative definition. It is also to be noted that Eqs. (5) and (6) were derived from test results in which the whole of the concrete core was in compression, whereas in a column part of the concrete section will be in tension. However the lowly stressed concrete near the neutral axis will help confine the highly stressed concrete and hence the fact that the hoop terminates in the tension zone is of no great significance. It is suggested that  $\rho_s$  be defined as above by ratio of volume of hoop to volume of concrete enclosed, rather than by any new definition which considers an effective hoop volume and the compressed concrete volume.

### 3.4 Moment-Curvature Analysis

The procedure for evaluating the moment-curvature relationship of a section has been described in detail elsewhere (2), (1). Briefly, the bending moments and curvatures associated with an axial load and a range of deformations are determined from the requirements of strain compatibility and equilibrium of stress resultants. The section is divided into a number of discrete laminae each having the orientation of the neutral axis. The longitudinal steel is replaced with an equivalent thin tube so that each lamina contains a particular amount of cover concrete, core concrete and longitudinal steel at a strain consistent with the strain profile. Fig. 2 shows a cross section with strain and stress distribution. For a value of extreme compression fibre strain  $\epsilon_{cm}$  the neutral axis depth  $kd$ , for which the sum of the internal forces found from the laminae stress resultants equals the axial load  $P$ , is determined by an iterative process. The internal moment  $M$  corresponding to that value of  $\epsilon_{cm}$  and  $P$  is then

calculated and the corresponding curvature  $\phi$  is determined from  $\epsilon_{cm}^c / kd$ . The theoretical moment-curvature relationship is obtained by successively incrementing the value of  $\epsilon_{cm}^c$ .

### 3.5 Variables Studied

Using the analysis procedure outlined above, a series of square column sections with high strength longitudinal reinforcement were studied (1) with the following range of variables:

- (i) Axial load intensity,  $\frac{P}{f'A} = 0.1, 0.2, 0.3$  and  $0.5$ .
- (ii) Cover ratio,  $c_v/b = 0.1, 0.075$  and  $0.05$  corresponding to square column sections of  $15$  in ( $381$  mm),  $20$  in ( $508$  mm) and  $30$  in ( $762$  mm) dimensions with concrete cover thickness to the outside of the transverse steel of  $1.5$  in ( $38$  mm).
- (iii) Concrete strength,  $f'_c = 4000$  psi ( $27.6$  N/mm $^2$ ) and  $6000$  psi ( $41.4$  N/mm $^2$ ).
- (iv) Longitudinal steel strength,  $f_y = 60,000$  psi ( $414$  N/mm $^2$ ), with  $E_s = 29 \times 10^6$  psi ( $200,000$  N/mm $^2$ ).
- (v) Longitudinal steel content uniformly distributed around the four sides of the section,  $\rho_t = 0.02, 0.04$  and  $0.06$ , where  $\rho_t = A_{st}/A$ .
- (vi) Transverse steel content, various arrangements of transverse rectangular hoops as defined by a range of  $Z$  values. For a given arrangement of transverse hoops the corresponding  $Z$  value may be calculated using Eq. (6) as discussed in Section 3.3. For example, for the arrangement of three overlapping hoops shown in the column section of Fig. 2, if the hoop diameter is  $\frac{1}{2}$  in ( $12.7$  mm), the spacing of each set of hoops is  $s_h = 4$  in ( $102$  mm), column dimensions are  $b = h = 20$  in ( $508$  mm), cover thickness  $c_v = 1.5$  in ( $38$  mm), concrete strength  $f'_c = 4000$  psi ( $27.6$  N/mm $^2$ ), and assuming that  $b'' = \frac{2}{3}(b-2c_v)$ , Eq. (6) gives

$$Z = \frac{0.5}{0.75 \times \frac{2(17+11.33)0.2}{17 \times 11.33 \times 4} \sqrt{\frac{11.33}{4}} + \frac{3+8}{3000} - 0.002} = 24.8$$

Table 1 gives some values of  $Z$  calculated as above for the arrangement of three overlapping hoops shown in Fig. 2 for various hoop diameters and spacing of hoop sets. The  $Z$  values for other spacings can also be calculated. The  $Z$  values corresponding to the ACI code (5) requirements for special transverse steel using the above arrangement when  $P > P_b$  are also shown in the note under the table.

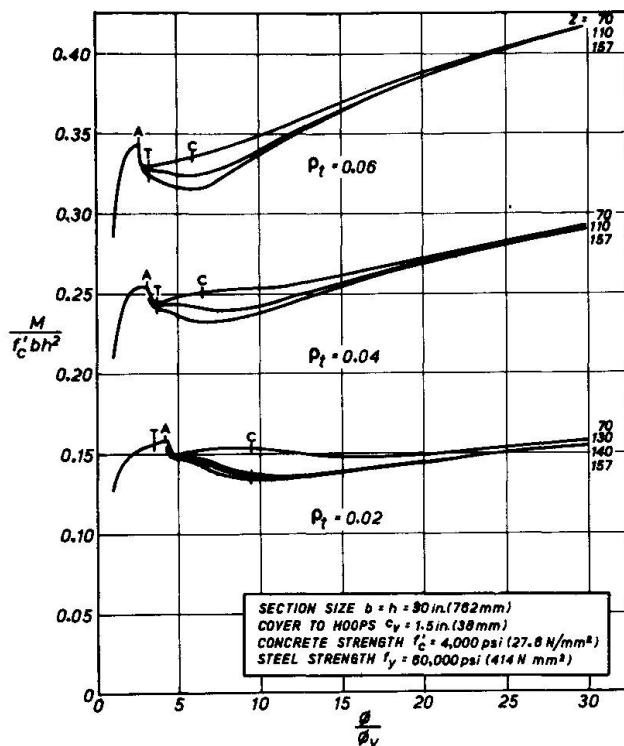
Table 1 Z Values for Square Sections With Three Overlapping Rectangular Hoops with Unsupported Length of Hoop Side =  $\frac{2}{3} \times$  dimension of confined core,  $f'_c = 4000$  psi ( $27.6$  N/mm $^2$ ) and  $c_v = 1.5$  in ( $38$  mm).

Transverse Steel	Section size		
	15 in (381 mm)	20 in (508 mm)	30 in (762 mm)
$\frac{3}{8}$ in ( $9.5$ mm) dia. at 12 in ( $305$ mm) centres	125	138	155
$\frac{3}{8}$ in ( $9.5$ mm) dia. at 4 in ( $102$ mm) centres	36	42	51
$\frac{1}{2}$ in ( $12.7$ mm) dia. at 4 in ( $102$ mm) centres	21	25	31
$\frac{3}{8}$ in ( $15.9$ mm) dia. at 4 in ( $102$ mm) centres	14	16	20
$\frac{1}{2}$ in ( $19.1$ mm) dia. at 4 in ( $102$ mm) centres	9.9	12	15

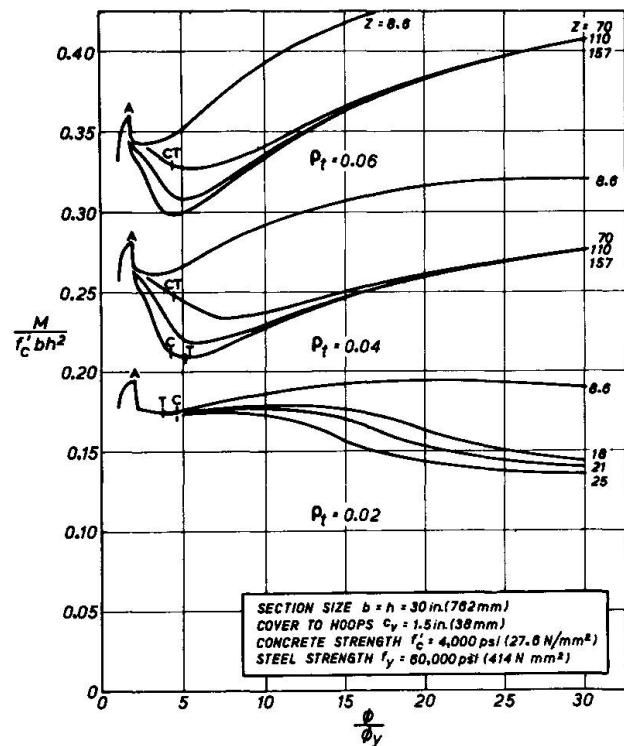
Note: ACI 318-71 (5) requires for  $P > 0.4P_b$ ,  $\frac{2}{3}$  in ( $15.9$  mm) dia. hoops with  $f_y = 40,000$  psi ( $276$  N/mm $^2$ ) spaced as follows:  
 For 15 in ( $381$  mm) section, spacing = 3.1 in ( $79$  mm), giving  $Z = 9.6$   
 For 20 in ( $508$  mm) section, spacing = 3.2 in ( $81$  mm), giving  $Z = 12$   
 For 30 in ( $762$  mm) section, spacing = 2.9 in ( $74$  mm), giving  $Z = 13$

### 3.6 Moment-Curvature Results

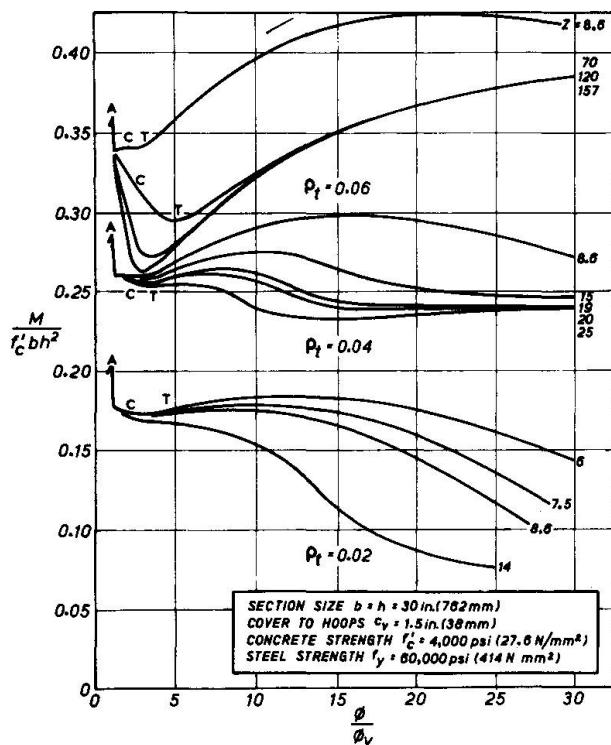
Typical moment-curvature curves from the analysis for bending about a principal axis of the section are shown in Figs. 3, 4 and 5. On each curve,  $A$  denotes the onset of spalling of the cover concrete ( $\epsilon_{cm}^c = 0.004$ ),



**Fig. 3** Moment-Ductility Curves  
for  $P = 0.1f' A_c g$



**Fig. 4** Moment-Ductility Curves  
for  $P = 0.3f' A_c g$



**Fig. 5** Moment-Ductility Curves  
for  $P = 0.5f' A_c g$

**Steel Strength:** High strength steel with early strain hardening maintains the moment capacity better than mild steel in which strain hardening does

and C and T denote the onset of strain hardening of the compression and tension steel, respectively.

The curves are drawn from first yielding of the tension steel. Curves for other column sections are available elsewhere (1). The influence of the variables on the moment capacity at high strains may be summarized as follows:

**Transverse Steel Content:** A large transverse steel content leads to better concrete stress-strain characteristics and therefore the moment capacity is better maintained at large strains.

**Longitudinal Steel Content:** Sections with large longitudinal steel content rely less on the concrete capacity and therefore the moment capacity is better maintained at large strains.

**Axial Load Intensity:** A large axial load means a large proportion of the load is carried by the concrete and therefore the moment capacity is reduced at higher strains.

not commence until higher strains.

Concrete Strength: High strength concrete results in a greater reduction in moment capacity at spalling of the cover concrete.

Cover Ratio: The larger proportion of cover concrete in smaller sections results in a greater reduction in moment capacity at spalling of the cover concrete.

#### 4. APPROACH FOR DETAILING COLUMNS FOR DUCTILITY

The transverse steel requirements for columns can be assessed from the moment-curvature curves by establishing the Z value required to achieve adequate ductility. For seismic design, as discussed in Section 2, the requirement could be a curvature ductility factor  $\phi_u/\phi_y$  of 16 at a moment capacity of not less than 0.85 of the maximum moment capacity. Table 2 shows the Z values meeting this criterion for the sections considered. Where this criterion could not be met with reasonable amounts of transverse steel no Z value is indicated in the table. The Z values corresponding to the ACI code (5) requirements for an axial load level greater than  $0.4P_b$  are in the note under Table 1. An axial load level  $0.4P_b$  corresponds to approximately 0.12 to 0.18 f' A<sub>g</sub> for the sections studied. In general the ACI requirements appear to be conservative for most cases in contrast with results obtained for sections containing mild steel reported previously (2) and by Norton(1). The early onset of strain hardening of high strength longitudinal steel allows the moment capacity to be maintained without such high transverse steel contents.

The Z values shown in Table 2 and Eq. (6) can be used to design the transverse steel of ductile columns. It must be noted however, that the analysis assumes the compression steel does not buckle. The calculated strain in the compression steel at  $\phi_u/\phi_y = 16$  for each case is included in brackets in Table 2. This is an area of considerable concern since little experimental evidence on the buckling of compression steel is available. Some tests have indicated that in sections with closely spaced hoops strain hardening of mild compression steel can take place without

**Table 2** Approximate Z Values Required for a Moment Capacity of 85% of the Maximum Moment Capacity at  $\phi/\phi_y = 16$  for Concrete  $f'_y = 4000$  psi (27.6 N/mm<sup>2</sup>), Longitudinal Steel  $f_y = 60,000$  psi (414 N/mm<sup>2</sup>),  $c_v = 1.5$  in (38 mm).

Section Size	Longitudinal Steel Content, $\rho_t$	Axial Load Level, $\frac{P}{f'_y A_g}$			
		0.1	0.2	0.3	0.5
15 in square (381 mm)	0.02	55(0.019)	16(0.023)	5(0.030)	-
	0.04	127(0.025)	127(0.033)	25(0.037)	9(0.074)
	0.06	127(0.027)	127(0.034)	127(0.040)	127(0.073)
20 in square (508 mm)	0.02	90(0.021)	28(0.025)	14(0.032)	-
	0.04	140(0.026)	140(0.033)	70(0.041)	13(0.067)
	0.06	140(0.027)	140(0.033)	140(0.040)	140(0.068)
30 in square (762 mm)	0.02	157(0.023)	39(0.028)	21(0.034)	7.5(0.057)
	0.04	157(0.025)	157(0.033)	157(0.042)	19(0.059)
	0.06	157(0.027)	157(0.033)	157(0.041)	157(0.061)

buckling occurring. The high Z values in Table 2 should be modified to limit the spacing of hoops to a maximum of 4 in (102 mm) but not greater than 6 longitudinal bar diameters. Serious doubts may exist as to whether some of the large compression steel strains associated with the high axial load levels indicated in Table 2 can be achieved without steel buckling.

It should also be noted that transverse steel is also required in columns to resist shear forces. The transverse steel can play a simultaneous role of confining the concrete, restraining buckling of bars and

resisting shear force, but it should always be checked that the transverse steel content is sufficient to resist the shear force induced at the flexural capacity of the column.

### 5. CONCLUSIONS

The results of this study lead to the following conclusions:

1. The capacity of column sections for post-elastic behaviour is dependent on the level of axial load, the concrete cover to steel ratio, the longitudinal and transverse steel contents and the material strengths. Procedures for detailing transverse steel for ductility should take these factors into account.

2. Columns containing high strength longitudinal reinforcement and sufficient transverse steel can exhibit good ductile characteristics. The required quantity of transverse steel may be obtained from the moment-curvature relationships.

3. The transverse steel required to prevent buckling of the compression steel and shear failure may override the requirements for concrete confinement in some cases.

### 6. NOTATION

$A_g$  = gross area of column section,  $A_{st}$  = total area of longitudinal steel,  $b$  = width of section,  $b''$  = width of confined core measured to the outside of hoops,  $c$  = thickness of cover concrete,  $E$  = modulus of elasticity of steel,  $f$  = stress in concrete,  $f'_c$  = compressive cylinder strength of concrete,  $f$  = stress in steel,  $f_{su}^s$  = ultimate strength of steel,  $f_y$  = yield strength of steel,  $h$  = overall depth of member,  $k_d$  = neutral axis depth,  $M$  = moment of resistance,  $P$  = axial load on section,  $P_b$  = axial load capacity at simultaneous attainment of  $\epsilon_{cm}^s = 0.003$  and yielding of the tension steel,  $s_h$  = spacing of transverse hoops,  $Z$  = constant defined by Eq. (6),  $\rho_s$  = ratio of volume of transverse reinforcement to volume of concrete core,  $\rho_t$  = ratio of total area of longitudinal steel to gross area of column section,  $\epsilon$  = strain in concrete,  $\epsilon_{cm}$  = strain in concrete at extreme compression fibre,  $\epsilon_s$  = strain in steel,  $\epsilon_{sh}$  = steel strain at commencement of strain hardening,  $\epsilon_{su}$  = steel strain at ultimate steel stress,  $\epsilon_y$  = strain in steel at onset of yielding,  $\phi$  = section curvature,  $\phi_u$  = curvature at end of post-elastic range,  $\phi_y$  = curvature at commencement of yielding of tension steel,  $\Delta$  = lateral deflection at first yield,  $\Delta_u$  = lateral deflection at end of post-elastic range.

### 7. REFERENCES

1. Norton, J.A., "Ductility of Rectangular Reinforced Concrete Columns", Master of Engineering Report, University of Canterbury, Christchurch, New Zealand, 1972.
2. Park, Robert., and Sampson, Richard A., "Ductility of Reinforced Concrete Column Sections in Seismic Design", ACI Journal, Proceedings V.69, No. 9, Sept. 1972.
3. "Recommended Lateral Force Requirements and Commentary", Seismology Committee, Structural Engineers' Association of California, San Francisco, 1968 (with 1970 amendment).
4. Uniform Building Code, International Conference of Building Officials, Pasadena, V.1, 1970.
5. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-71)", American Concrete Institute, Detroit, 1971.
6. Park, R., "Ductility of Reinforced Concrete Frames Under Seismic Loading", New Zealand Engineering (Wellington), V.23, No.11, Nov. 1968.
7. Park, Robert., Discussion of "Proposed Revision of ACI 318-63, Building Code Requirements for Reinforced Concrete", by ACI Committee 318, ACI Journal, Proceedings V.67, No. 9, Sept. 1970.
8. Kent, D.C. and Park, R., "Flexural Members with Confined Concrete", Proceedings, ASCE, V.97, ST7, July 1971.

## SUMMARY

The ductility required of eccentrically loaded reinforced concrete column sections in seismic design is discussed. A theoretical method for the determination of the amount of transverse steel required for flexural ductility is described. The method is based on assumed stress-strain curves for confined concrete and steel, and takes into account the required ultimate curvature, the level of axial load, the longitudinal steel content, and material strengths. Moment-curvature curves for columns reinforced longitudinally by high strength steel are derived and transverse steel contents for such columns suggested.

## RESUME

On traite de la ductilité des colonnes en béton armé à charge excentrée qui doivent résister aux séismes. On décrit une méthode théorique pour déterminer la quantité d'armature transversale nécessaire pour obtenir la ductilité requise. Cette méthode se base sur l'hypothèse de courbes tension-déformation pour le béton et les armatures, et tient compte également de la courbure ultime exigée, de la grandeur de la charge axiale, du pourcentage d'armature longitudinale et de la résistance des matériaux. On obtient des diagrammes moment - courbure pour les colonnes avec armature longitudinale en acier à haute résistance, et on propose des pourcentages pour les armatures transversales.

## ZUSAMMENFASSUNG

Im vorliegenden Beitrag wird das erforderliche Verformungsvermögen exzentrisch belasteter Stahlbetonstützen-Querschnitte bei Erdbebeneinflüssen diskutiert. In der Folge wird eine theoretische Methode zur Bestimmung des Bewehrungsgehalts beschrieben, welcher ausreichendes Verformungsvermögen gewährleistet. Die Methode beruht auf angenommenen Spannungs-Dehnungs-Diagrammen für Beton und Stahl und berücksichtigt die erforderliche maximale Krümmung, die Grösse der Normalkraft, den Längsbewehrungsgehalt und die Materialfestigkeiten. Es werden Moment/Krümmungs-Diagramme für in der Längsrichtung mit hochfestem Stahl bewehrte Stützen abgeleitet und der Anteil an Querbewehrung für solche Stützen vorgeschlagen.

Leere Seite  
Blank page  
Page vide