Safety index of short concrete columns

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Indice de sécurité pour des colonnes courtes en béton

Sicherheitskennwert für kurze Stahlbetonstützen

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The goal of consistent safety is hampered by the fact that it is difficult to measure safety and to define what we mean by consistency. An ideal situation might be to have explicit probabilities and explicit costs of consequences, and to design for minimum expected cost. Probabilities are not really attainable, however, and other measures of safety are needed as substitutes.

The use of up to second moment measures of variability (means and variances), and a determination of safety index β to represent degree of safety (1, 2, 3, 4), has proved to be an appropriate bridge between traditional design procedures and explicit probabilistic design. A recent second moment method proposed by Hasofer and Lind (4) permits computation of the safety index in terms of any number of basic variables, thus overcoming several shortcomings of earlier approaches.

The safety index is a measure of safety that can be related to probability of failure when distribution assumptions are made. In the absence of such assumptions, it is in itself a measure of safety. Consistent safety may then involve design to a chosen safety index. Choice of new design rules can be based in part on approaching constant safety index for particular kinds of structures or elements. When the safety index varies, as in current design rules, economy may be achieved by bringing it to a consistent level (for example, constant).

The Hasofer-Lind criterion is formulated herein for the column interaction problem (bending and axial force) and the computation of safety index is described. Cases of normal and lognormal basic variables are considered, and correlation of axial and flexural loading is included. A few selected results are presented to indicate the variation of β under current ACI design rules.

A. The Hasofer-Lind Criterion for Safety Index

The safety index proposed by Hasofer and Lind (4) is computed as follows:

Let $\underline{X} = (X_1, X_2, \ldots, X_n)$ be the vector of basic random variables relevant to the design. Let the failure criterion for the design be $F(\underline{X}) < 0$. The criterion devides the space of $\underline{X}$ into a safe region $G(\underline{X})$ and a failure region $\bar{G}^*(\underline{X})$.

Now make an orthogonal transformation of the variables $\underline{X}$ to a new set of variables $\underline{Y} = (Y_1, Y_2, \ldots, Y_n)$, such that the new variables Y_i are uncorrelated. In addition, make suitable transformations such that the Y_i are approximately normally distributed.

Next introduce reduced variables $y_i = (Y_i - \bar{Y}_i)/\sigma_{Y_i}$. Let $y = (y_1, y_2, \ldots, y_n)$. To the failure criterion $F(\underline{X}) < 0$ there will correspond in the space of y a safe region G(y) and a failure region $\bar{G}^*(y)$.

The safety index β is the minimum radius from the origin of y to the failure region $G^*(y)$.

The short column interaction problem is an ideal one to demonstrate the power of this criterion, for the following reason: Previous attempts to formulate a second moment code format have required identification of the variables as load or resistance variables. The Cornell and Rosenblueth - Esteva formats (1 and 2) require the failure event to be expressed in terms of resistance R and load U. The Ditlevsen proposal (3), which handles more than two variables, requires identification of variables that increase safety and those that decrease safety. In the short column interaction problem the axial load P can play either role, depending on its value, because of the non-monotonic interaction curve.

B. Behavior of Short Concrete Columns

The model of physical behavior used in this study is the one commonly used in strength design of reinforced concrete, based on the Whitney rectangular stress block and linear strain variation across the section. Attention is restricted to symmetric tied columns with reinforcement only in the faces parallel to the axis of bending. Capacities and loads are treated in non-dimensional form by dividing axial quantities by btf_c' and flexural quantities by bt^2f_c'. A single interaction diagram in non-dimensional form is defined by the variables listed in Table I.

TABLE I

Symbol	Definition	Typical value
ϵ_c/ϵ_s	Ratio of concrete strain at failure to steel yield strain	2.0
d/t	Depth from face of concrete to face of reinforcement/total depth of section	0.15
A_s/bt	Ratio of steel to concrete area	0.04
f_y/f_c'	Ratio of steel yield to concrete compressive strength	14
k_1 and k_2	Whitney stress block parameters	0.85

C. Application of the Criterion to Short Concrete Columns

The random variables relevant to the design are P, M, and V, where P is the non-dimensional axial load (actual force divided by btf_c', M the non-dimensional flexural load (actual moment divided by $bt^2 f_c'$), and V a random variable, representing variability of flexural capacity on the interaction curve as a function of load P.

When P, M, and V are approximately normally distributed, the failure region is defined by

$$f(P) + V - M < 0 \tag{1}$$

where f(P) is the flexural capacity on the interaction curve as a function of load P, E(V) = 0, $\sigma(V) = g(P)$, the standard deviation of flexural capacity as a function of P.

When P, M, and V are approximately lognormally distributed, the failure region is defined by

$$f(p)\, V - M < 0 \tag{2}$$

where E(log V) = 0, $\sigma(\log V) = h(P) = g(P)/f(P)$, the coefficient of variation of flexural capacity as a function of P.

P and M may be correlated, while V is uncorrelated with both P and M.

1. Procedure for Normal Random Variables

Let R be the correlation coefficient of P and M, and write

$$P = \bar{P} + \sigma_p\, p \tag{3}$$

$$M = \bar{M} + \sigma_M\, (m \sqrt{1 - R^2} + p\, R) \tag{4}$$

$$V = g(P) \tag{5}$$

where p and m and v are uncorrelated variables with mean zero and unit standard deviation. It may be verified that $E(P) = \bar{P}$, $\sigma^2(P) = \sigma_p^2$, $E(M) = \bar{M}$, $\sigma^2(M) = \sigma_M^2$, Cov (P, M) = $= \sigma_p\, \sigma_M\, R$, as required. Moreover, the safety index obtained with the basic variables P, M, V, is the same as that obtained with the reduced variables, p, m, v.

The failure surface is

$$F(p, m, v) = F(P) + V - M \tag{6}$$

and the safety index β is the distance from the origin to the surface F(P, m, v) = 0.

To calculate β we use the iteration technique described in Hasofer and Lind (4). The iteration formulae are:

$$p^{(n+1)} = \lambda F_1$$

$$v^{(n+1)} = \lambda F_2$$

$$m^{(n+1)} = \lambda F_3$$

$$\beta^{(n+1)} = \left[(p^{(n+1)})^2 + (v^{(n+1)})^2 + (m^{(n+1)})^2 \right]^{1/2}$$

where

$$F_1 = \frac{\partial F}{\partial p}(n)$$

$$F_2 = \frac{\partial F}{\partial v}(n)$$

$$F_3 = \frac{\partial F}{\partial m}(n)$$

$$\lambda = \frac{p^{(n)} F_1 + v^{(n)} F_2 + m^{(n)} F_3 - F}{F_1^2 + F_2^2 + F_3^2}$$

$$F = F(p^{(n)}, v^{(n)}, m^{(n)})$$

and the partial derivatives of F can be shown to be

$$\frac{\partial F}{\partial p} = f'(P)\, \sigma_{\bar{p}} + g'(P)\, v\, \sigma_{\bar{p}} - \sigma_m R$$

$$\frac{\partial F}{\partial v} = g(P)$$

$$\frac{\partial F}{\partial m} = -\sigma_m \sqrt{1 - R^2}$$

A starting point $p^{(1)}$, $v^{(1)}$, $m^{(1)}$ must be chosen. There are a number of perpendiculars from the origin to the failure surface, and the iteration procedure may converge to any one of them, depending on the starting point of the iteration. It is necessary to choose several starting points and take the smallest of the β's obtained.

2. Procedure for Lognormal Random Variables

In this case we write

$$P = \bar{P} \exp(V_p p)$$

$$M = \bar{M} \exp\left[V_m (m \sqrt{1 - R^2} + p R) \right]$$

$$V = \exp\left[h(P)\, v \right]$$

where p, m, v, and R are defined as before, and E (log P) = log $\bar{P}$, σ^2 (log P) = V_p, E (log M) = log $\bar{M}$, σ^2 (log M) = V_m, Cov (log P, log M) = $V_p V_M R$.

The failure surface is

$$F(p, m, v) = f(p)\, V - M$$

and the safety index β is the distance from the origin to the surface F(p, m, v) = 0.

The iteration formulae for computing the distance β is the same as for normal random variables, except that the partial derivitaves are

$$\frac{\partial F}{\partial p} = VPV_p \left[f'(P) + f(P)\, h'\,(P)\, v\right] - MV_M R$$

$$\frac{\partial F}{\partial v} = Vf\,(P)\, h\,(P)$$

$$\frac{\partial F}{\partial m} = -\, MV_M \sqrt{1 - R^2}$$

The choice of starting points again must be made with care to avoid missing convergence to the smallest β . The authors encountered this problem in some instances, especially in the lognormal case.

D. The Information Base

Current ACI design criteria is generally assumed to relate to load information by ANSI Committee A58. Recent studies have resulted in prediction of mean lifetime loadings for offices that differ substantially from ANSI. Because of this bias in design load, β varies considerably. In order to focus on strength consistency, we assume that the mean peak lifetime load from Mc Guire and Cornell (5) is used as the live load in the ACI design procedure. This means that the β values discussed subsequently are for the single design area at which ANSI and Ref 5 give the same loading (about 1000 ft^2). The coefficient of variation of live load V_L is taken as 0.15. Mean dead load is taken as nominal dead load, and coefficient of variation of dead load is taken as 0.10. Current ACI load factors are used to relate design load to ultimate strength.

Strength of concrete columns is subject to prediction error due to model discrepancies, material and geometric variability, and professional uncertainty. Studies of these factors has led the authors to choose a bias on strength b = 0.95 as representative of ACI criteria, where b $U = \bar{U}$, U is ACI predicted strength and $\bar{U}$ is mean strength. The coefficient of variation of strength V_u was chosen as 0.20 on the strength as represented by radius to the interaction curve.

For a given "case" in this study, a dimensionless interaction curve is generated. For a particular ratio of nominal dead/live load intensity, a series of permissible ACI load cases (P, M) are generated and denoted by letters A to I to identify eccentricty. For each of these cases, β is computed as indicated in Section C. Dead/live ratio did not significantly affect β, and was taken as 0.5 in the calculations discussed here.

For any axial load P, the flexural capacity f(P) is computed from the parameters and principles discussed in Section B. The standard deviation of flexural capacity, g(P), is computed from the coefficient of variation of radius to the interaction curve (evaluated from studies of available data) and corrected to account for the fact that it must represent a "horizontal slice" through the interaction curve, rather than a "radial slice".

E. Selected Results

A large number of cases representing varying loading, geometry material strength, and correlation coefficient of axial and flexural load were evaluated, and sensitivity studies

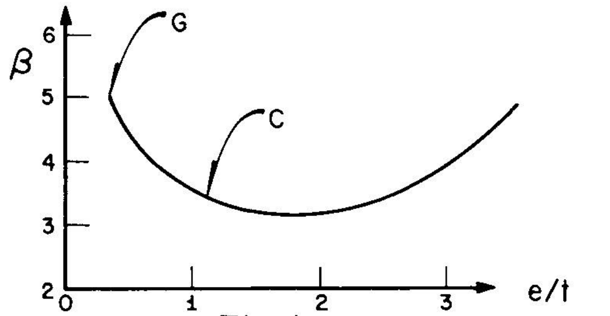
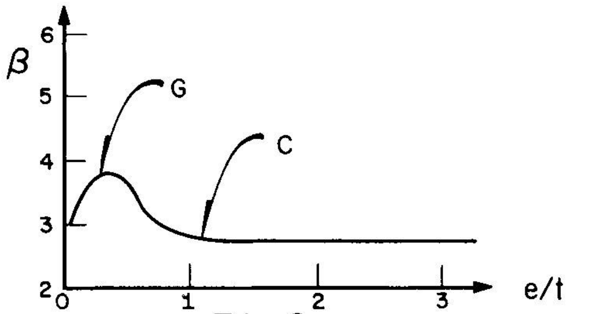


Fig 1



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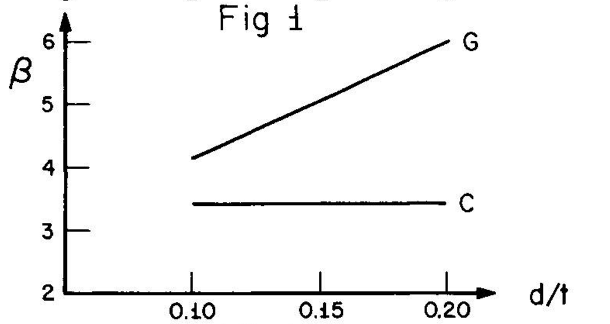

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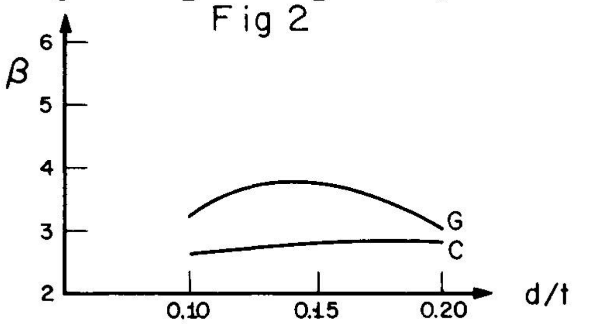

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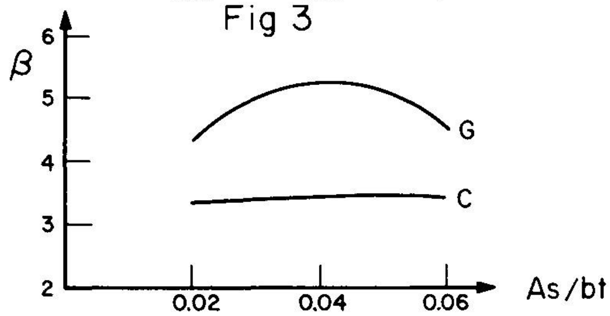

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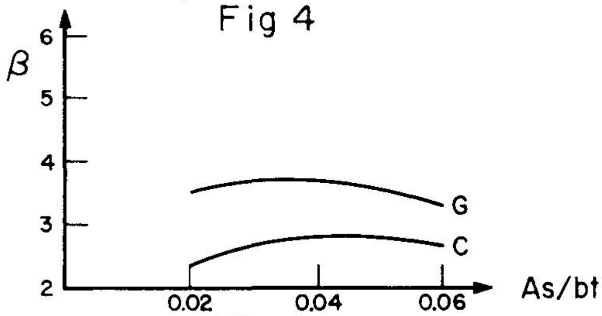

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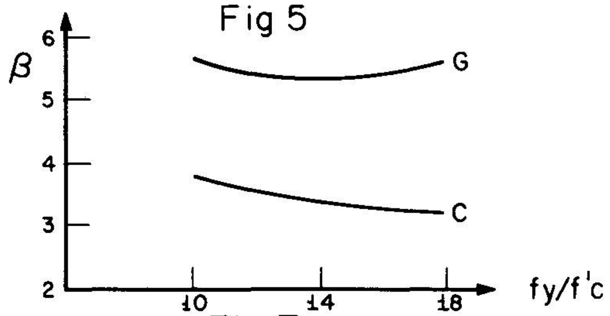

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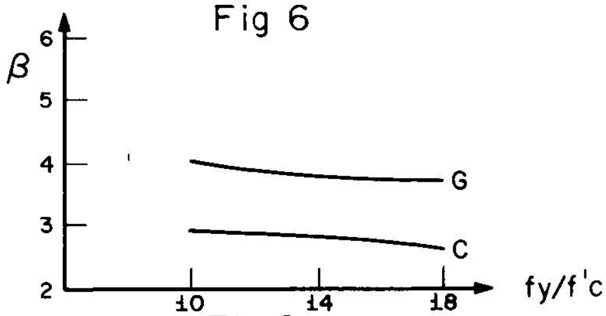

Fig 8

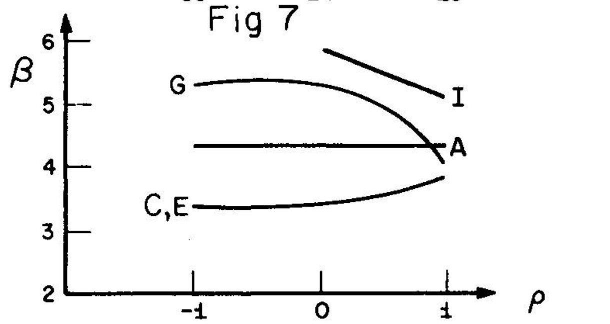

Fig 9

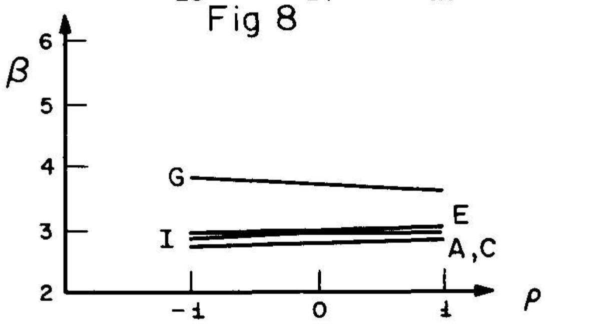

Fig 10

were carried out by varying parameters in these cases. The results presented herein are selected to illustrate the main conclusions of the study.

A basic column design was chosen using ACI design rules with the parameter values as given in Table I. For this design, point C on the interaction curve represents an e/t ratio of 1.1, in the tension zone close to the balanced design. Point G has e/t = 0.30, in the compression zone. Other designs are A (e/t = 3.3), E(e/t = 0.8), F(e/t = 0.52), and I (e/t = 0.05). Parameter variations discussed below represent reasonable designs with the named parameters varying from the basic case and the others held constant at the basic values of Table I.

The safety index appears very sensitive to the e/t ratio, particularly for the lognormal case, where (fig 1) β is high in the compression failure zone (G), decreasing to a minimum near the balanced point C. The normal case of fig 2 shows a peak at point G in the compression zone, and a constant value in the tension zone.

The results show that β is not very sensitive to d/t. Fig 3, lognormal case, shows an extreme situation in which β became larger as d/t increased, but for the normal case, fig 4, β remains constant. Thus geometry is well accounted for in present design practice. The variation of β with steel percentage A_s/bt is pronounced in the lognormal case (fig 5) and insignificant in the normal case of fig 6.

The effect of steel strength appears in figs 7 and 8 for the lognormal and normal cases respectively. The strength ratio f_y/f_c' does not appear to alter β in an important way.

The effect of correlation between axial load and moment is indicated for the lognormal case in fig 9. Designs A through E show increasing β with increasing correlation coefficient. This occurs because negative correlation results in increased chance of tension failure. Designs G and I in the compression zone show the converse, as expected. Positive correlation for compressive zone designs decreases the safety. These trends occur in the normal case (fig 10) but the variation is so slight that it is almost indiscernible.

The safety index appears to be sensitive to most variables in the lognormal case, less so in the normal case. It is higher in the lognormal case, as should be expected due to the improved strength situation when strength is lognormally distributed.

β appears to approach 3.2 in a number of situations in the lognormal case, and this may be a reasonable calibration point. One possible consistency rule for column designs would be to alter design rules to give β = 3.2. This would result in slightly more economical columns in many cases, but would not decrease strength in the cases where β is already at this calibration point.

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ACKNOWLEDGMENT

The authors wish to acknowledge the support of the Institute of Engineering, National University of Mexico, for the conduct of this research.

SUMMARY

The Hasofer-Lind method of computing a second moment measure of safety called the safety index is applied to the case of short reinforced concrete columns. The variation of safety index with the usual parameters is discussed for current ACI (1974) design rules.

RESUME

On applique au cas de colonnes courtes en béton armé la méthode de Hasofer-Lind consistant à calculer un indice de sécurité d'après la méthode des moments des variables statistiques. La variation de l'indice de sécurité selon les valeurs des paramètres de dimensionnement usuels est étudiée pour les règles de dimensionnement ACI (1974).

ZUSAMMENFASSUNG

Die Hasofer/Lind-Methode für die Berechnung eines Sicherheits-Kennwertes mittels den Methode der zweiten Momente, wird auf den Fall kurzer Stahlbetonstützen angewendet. Die Variation des Sicherheits-Kennwertes mit den üblichen Parametern wird für die gültigen ACI (1974) Normen diskutiert.