Slender Reinforced Concrete Columns Subjected to Biaxially Eccentric Loads

Colonnes en béton armé sous une charge d'excentricité biaxiale

Schlanke bewehrte Betonsäulen unter zweiaxig aussermittiger Belastung

N.J. GARDNER
Associate Professor

S.I. ABDEL-SAYED
Research Associate

Department of Civil Engineering
University of Ottawa
Ottawa, Canada

Over the past few years increasingly refined and accurate methods of analysis for complex structures have become available to the structural engineer. However to use such analyses more realistic imput information is needed concerning the moment-thrust-curvature characteristic of the various members comprising the structure. In general the frame analysis methods available, and their associated computer programs, have not taken account of the reduction in flexural stiffness of the columns due to axial load even though the theoretical treatment has long been available (5).

For detailing purposes the behaviour of slender reinforced concrete columns has been treated as the behaviour of a short column modified by a factor to account for slenderness effects. Currently the Building Code Requirements of the American Concrete Institute ACI 318-71 (3) require that for a slender column the moment be increased to account for slenderness and the column be designed as a short column with this increased moment. The short column axial load moment interaction is easily obtained from equilibrium considerations. Neither the moment magnifer nor the short column interaction consider strain compatibility.

The minimum characteristics needed to completely define the behaviour of a reinforced concrete column are the interrelationships between axial load-moment-curvature-strain. By integrating the moment curvature relation along the length of a given column the internal strain-lateral displacement behaviour of the column can be determined.

This paper presents a new form of interaction diagram relating the axial load, moment, maximum strain and curvature for square section corner reinforced concrete columns under uniaxial and biaxial eccentricities.

Design Method

To completely describe the behaviour of a reinforced concrete section the load, eccentricity of load about both axes, maximum strain and curvature (or position of neutral axis to determine curvature) need to be known.

(a) Section capacity calculations

Design charts have been presented by Abdel-Sayed and Gardner (1,2) for square section columns with corner reinforcement. These charts were prepared assuming a planar distribution of strain across the section, a bilinear stress strain characteristic for the steel reinforcing and Hagnestad's (4) stress strain curve for the concrete. The cross section of the column was divided into a number of elements and for a given planar strain distribution the stress on each element, either concrete or steel, determined. Hence the equilibrating force and moment due to the assumed strain distribution are determined by summing the elemental forces. The curvature implicit in the assumed strain distribution is obtained from the magnitude of the maximum strain and the perpendicular distance z between the position of maximum strain and the neutral axis.

Figure 1 is a typical interaction curve, prepared by the above described method, which relates the non-dimensional load P_u'/btf_c', eccentricity e, maximum strain ε and location of neutral axis z (curvature) for a column with a steel ratio $pf_y/f_c' = 0.60$ and a cover ratio $g = 0.75$ for angles of eccentricity of 0^o and 45^o. Linear interpolation is used to obtain results for angles of eccentricity between 0^o and 45^o. To use the interaction curves to design columns with cover ratio's other than $g = 0.75$ a single value function F is given in Figure 2 to modify the actual eccentricity of a column with $g \neq 0.75$ to its analogous column with $g = 0.75$.

(b) Slenderness or PΔ Effects

As the axial load is constant along the length of any given column there are only a number of maximum strains and curvatures (curvature = maximum strain ε/distance to the neutral axis z) which satisfy equilibrium for a given cross section. The curvature and maximum strain are known at the ends of a column of given cross section where the load, eccentricity of the load and the angle of the eccentricity are defined. By assuming a strain and curvature of the critical section of the column the lateral deflection of that section can be calculated if the deflected shape is assumed. If the lateral deflection plus the eccentricity is greater than that possible for the chosen section then a larger section must be chosen.

Experimental Investigation

An experimental investigation into the actual behaviour of slender reinforced concrete columns under biaxial load was undertaken to verify the theoretical results.

A total of 44 columns were cast and loaded to failure. All columns were cast and tested in a horizontal position. All columns had a nominal concrete section of 6 ins. x 6 ins. (15.2 cm. x 15.2 cm.) with enlarged ends to allow loads to be applied outside the section. Two sizes of steel reinforcement #3 (0.95 cm. diam.) and #7 (2.25 cm. diam.) bars were used giving steel ratio's of 1.22% and 6.67% respectively. All reinforcement had a specified minimum yield strength of 60,000 psi (4150 bars). Two concrete cylinder strengths were used; nominally 3,000 psi (207 bars) and 5,500 psi (380 bars). To investigate length effects, two groups of specimens were made with effective lengths of 90 ins. (230 cm.) and 134 ins. (340 cm.) respectively. Columns were tested horizontally in a pinned-pinned configuration. Deflections were measured in two orthogonal directions, normal to the faces of the section at mid length of the column. The load deflection behaviour of the column was taken to failure. Unfortunately, it was not possible to obtain reliable deflections at loads near the failure load.

A summary of the experimental results and comparison with the results predicted by the method presented are given in Table 1.

Comparison shows the agreement between the measured and predicted ultimate loads to be very good with a mean of 1.065 and a coefficient of variation of 0.083. However, agreement between the measured and predicted final displacements is less good with a mean value of 0.92 and a coefficient of variation 0.129.

Example of Design Method

To illustrate the method a typical column will be designed.

Determine the section and reinforcement required for a reinforced concrete column 168 ins. (425 cm.) long monolithically cast with slabs at both ends to carry a design ultimate axial load of 150 kips (68,000 kg) and design ultimate moments of 665 kip ins. (770,000 kg. cm.) and 1000 kip ins. (1,115,000 kg. cm.) respectively about the two axes. The column is assumed bent in single curvature and a design understrength factor of 0.7 is to be used.

Assume a concrete strength of 4,000 psi (271 bars), a steel yield stress of 60,000 psi (4,100 bars), 4% reinforcement and a cover ratio of g = 0.70.

$$\frac{pf_y}{f'_c} = 0.60$$

Assume column size 13 ins. (33 cm.) x 13 ins. (33 cm.) and cover ratio 0.70.

$$P'_u = \frac{P_u}{\phi} = 214 \text{ kips } (97{,}000 \text{ kg.})$$

$$\frac{P'_u}{f'_c bt} = \frac{214{,}000}{4{,}000 \times 13 \times 13} = 0.317$$

$$e = \sqrt{(4.42)^2 + (6.67)^2} = 8 \text{ ins. } (20.4 \text{ cm.})$$

From Figure 2 with $g = 0.70$ and $\frac{pf_y}{f_c'} = 0.6$

$$F = 1.08$$

$$e \text{ modified} = Fe = 1.08 \times 8 = 8.64 \text{ ins. } (21.9 \text{ cm})$$

$$\theta = \tan^{-1} \frac{665}{1000} = 33.6^o$$

$$\frac{e}{t} = \frac{8.64}{13.00} = 0.665$$

At the ends of the column, using Figure 1, and estimating θ between $\theta = 0$ and $\theta = 45^o$

Maximum strain $\varepsilon = 0.0018$

$\frac{z}{t} = 0.72$ $\quad \therefore z = 9.36$ ins. (23.8 cm)

(z is the perpendicular distance between the point of maximum strain and the neutral axis).

$$\text{curvature} = \frac{0.0018}{9.36} = 0.000193 \text{ ins.}^{-1} \ [0.000075 \text{ cm}^{-1}]$$

At mid column height the load is the same but the eccentricity and its angle are unknown.

Assume maximum strain = 0.0030. Again from Figure 1

$\frac{e}{t} = 0.765$ $\quad \therefore e = 9.95$ ins. (25.2 cm)

$\frac{z}{t} = 0.710$ $\quad \therefore z = 9.22$ ins. (23.4 cm)

$$\text{curvature} = \frac{0.0030}{9.22} = 0.000326 \text{ ins.}^{-1} \ (0.000128 \text{ cm}^{-1})$$

From moment area the lateral deflection about the neutral axis

$$\Delta = 1.08 \text{ ins. } (2.74 \text{ cm})$$

Hence e actual $= e \text{ top} + \Delta = 8.64 + 1.08 = 9.72$ ins. (24.6 cm)

e possible = 9.95 ins. (25.2 cm)

e possible > e actual $\quad \therefore$ section OK.

Conclusion

An improved form of the short column axial load-biaxial moment interaction diagram is presented which also gives information concerning the maximum concrete strain and curvature necessary to develop the load capability of the section. Long column behaviour can be determined by integrating the curvature(s) along the column.

References

1. Abdel-Sayed, I. "Behaviour of slender reinforced concrete columns under biaxially eccentric loading." Ph.D. Thesis 1974, University of Ottawa.

2. Abdel-Sayed, I. and Gardner, N.J. 'Design of Symmetric Square Slender Reinforced Concrete Columns under Biaxially Eccentric Loads.' Column Symposium, ACI Fall Convention, October 1973, Ottawa.

3. Building Code Requirements, ACI 318-71, American Concrete Institute, Detroit, U.S.A., 1971.

4. Hagnestad, A. 'A study of combined and axial load in reinforced concrete members'. Bulletin #399, University of Illinois Experiment Station.

5. Livesley, R.K. and Chandler, D.B. 'Stability Functions for Structural Frameworks', Manchester University Press, 1955.

Notation

b	smaller dimension of column cross section
e	eccentricity of load, e_x about y axis, e_y about x axis
F	factor to correct for cover ratio - defined in text
f'_c	specified cylinder strength of concrete
f_y	specified yield strength of reinforcement
g	cover ratio
P_u	design ultimate load
P'_u	ultimate load
P_{exp}	experimental ultimate load
P_{ca}	calculated ultimate load
p	steel ratio
t	larger dimension of column cross-section
z	perpendicular distance from neutral axis to point of maximum strain
Δ	lateral displacement of column
ε	maximum concrete strain
ϕ	understrength factor

SUMMARY

An improved axial load biaxial moment interaction diagram including information on strain and curvature is presented. Results predicted by the proposed method are shown to be in good agreement with those measured in a companion experimental investigation. Finally, a design example using the interaction diagram is presented.

RESUME

Cette étude présente un diagramme amélioré d'interaction, donnant des informations sur la charge, les flexions biaxiales, la déformation spécifique et la courbure. Les résultats prédits par cette méthode sont en accord avec ceux de l'étude expérimentale. Un exemple de calcul employant le diagramme d'interaction est présenté.

ZUSAMMENFASSUNG

Es wird ein verbessertes Interaktions-Diagramm vorgestellt, welches neben der Beziehung zwischen Last und Exzentrizität zusätzlich Information über Dehnung und Krümmung enthält. Die nach der vorgeschlagenen Methode erhaltenen Ergebnisse sind in guter Uebereinstimmung mit Versuche. Zum Schluss wird ein Bemessungsbeispiel unter Anwendung des Interaktionsdiagramms durchgerechnet.

Set Number	Specimen Number	e_x in	e_y in	P_{exp} kip	P_{ca} kip	$\frac{P_{exp}}{P_{ca}}$	δ_{exp}	δ_{ca}	$\frac{\delta_{exp}}{\delta_{ca}}$
Set A	A1	0	2.5	62.5	64.5	0.97	0.90	0.90	1.00
90" long	A2	1.25	2.5	56.9	56.9	1.00	0.81	0.76	1.06
#7 bars	A3	2.5	2.5	46.2	44.6	1.03	0.83	0.71	1.17
	A4	0	5.0	39.2	38.8	1.01	1.1	1.28	0.86
	A5	1.67	5.0	33.8	33.7	1.00	1.0	0.95	1.05
	A6	3.33	5.0	26.7	26.5	1.01	0.96	0.87	1.11
	A7	5.0	5.0	22.0	20.9	1.05	0.94	0.79	1.19
	A8	0	7.5	29.7	29.3	1.01	1.26	1.45	0.87
	A9	2.5	7.5	23.4	22.0	1.06	1.00	1.02	0.98
	A10	5.0	7.5	22.6	20.3	1.11	0.87	0.89	0.98
	A11	7.5	7.5	15.8	15.0	1.05	0.87	0.88	0.99
Average						1.03			1.02
Set C	C1	0	2.5	48.5	56	0.87	2.41	2.42	1.00
134" long	C2	1.25	2.5	39	47.5	0.82	1.84	2.0	0.92
#7 bars	C3	2.5	2.5	36	36.6	0.98	1.82	1.72	1.06
	C4	0	5.0	31.9	31.7	1.01	3.00	3.05	0.98
	C5	1.67	5.0	28.8	28.7	1.01	2.05	2.10	0.98
	C6	3.33	5.0	20.6	20.5	1.01	1.83	1.87	0.98
	C7	5.0	5.0	17.9	17.9	1.00	1.84	1.64	1.12
	C8	0	7.5	22.6	22.4	1.01	2.41	3.14	0.77
	C9	2.5	7.5	19	17.4	1.09	1.74	1.82	0.96
	C10	5.0	7.5	14.9	14.8	1.01	1.76	1.95	0.90
	C11	7.5	7.5	13.8	13.2	1.04	1.81	1.82	0.99
Average						0.99			0.97
Set B	B1	0	2.5	44	38.4	1.14	0.8	1.06	0.76
90" long	B2	1.25	2.5	31.2	27.9	1.12	0.61	0.82	0.75
#3 bars	B3	2.5	2.5	27.2	22.5	1.21	0.65	0.78	0.83
	B4	0	5.0	18.1	14.7	1.23	0.97	1.34	0.73
	B5	1.67	5.0	16.5	14.0	1.18	0.86	1.07	0.80
	B6	3.33	5.0	12.4	11.4	1.09	0.76	1.04	0.73
	B7	5.00	5.0	10.4	9	1.15	0.75	0.97	0.78
	B8	0	7.5	10.1	8.7	1.15	1.18	1.48	0.80
	B9	2.5	7.5	9.7	8.1	1.20	1.05	1.11	0.95
	B10	5.0	7.5	8	6.95	1.15	1.02	1.09	0.94
	B11	7.5	7.5	6.4	5.2	1.23	1.03	1.04	0.99
Average						1.17			0.82
Set D	D1	0	2.5	28.5	27.7	1.03	1.4	1.67	0.84
134" long	D2	1.25	2.5	24	22.4	1.07	1.3	1.64	0.79
#3 bars	D3	2.5	2.5	16	16	1.00	1.2	1.32	0.91
	D4	0.0	5.0	14	13.4	1.04	1.99	2.34	0.85
	D5	1.67	5.0	11.9	11.5	1.04	1.51	1.95	0.78
	D6	3.33	5.0	10.7	10.4	1.03	1.41	1.85	0.76
	D7	5.0	5.0	8.3	7.9	1.05	1.46	1.59	0.92
	D8	0.0	7.5	9.8	8.20	1.19	2.19	2.33	0.94
	D9	2.5	7.5	8.7	7.38	1.18	2.23	2.29	0.93
	D10	5.0	7.5	7.7	6.84	1.12	1.83	2.11	0.87
	D11	7.5	7.5	7	6.3	1.11	1.84	1.96	0.94
Average						1.08			0.87
Overall Average						1.07			0.92

Table 1 Comparison of predicted and experimental results

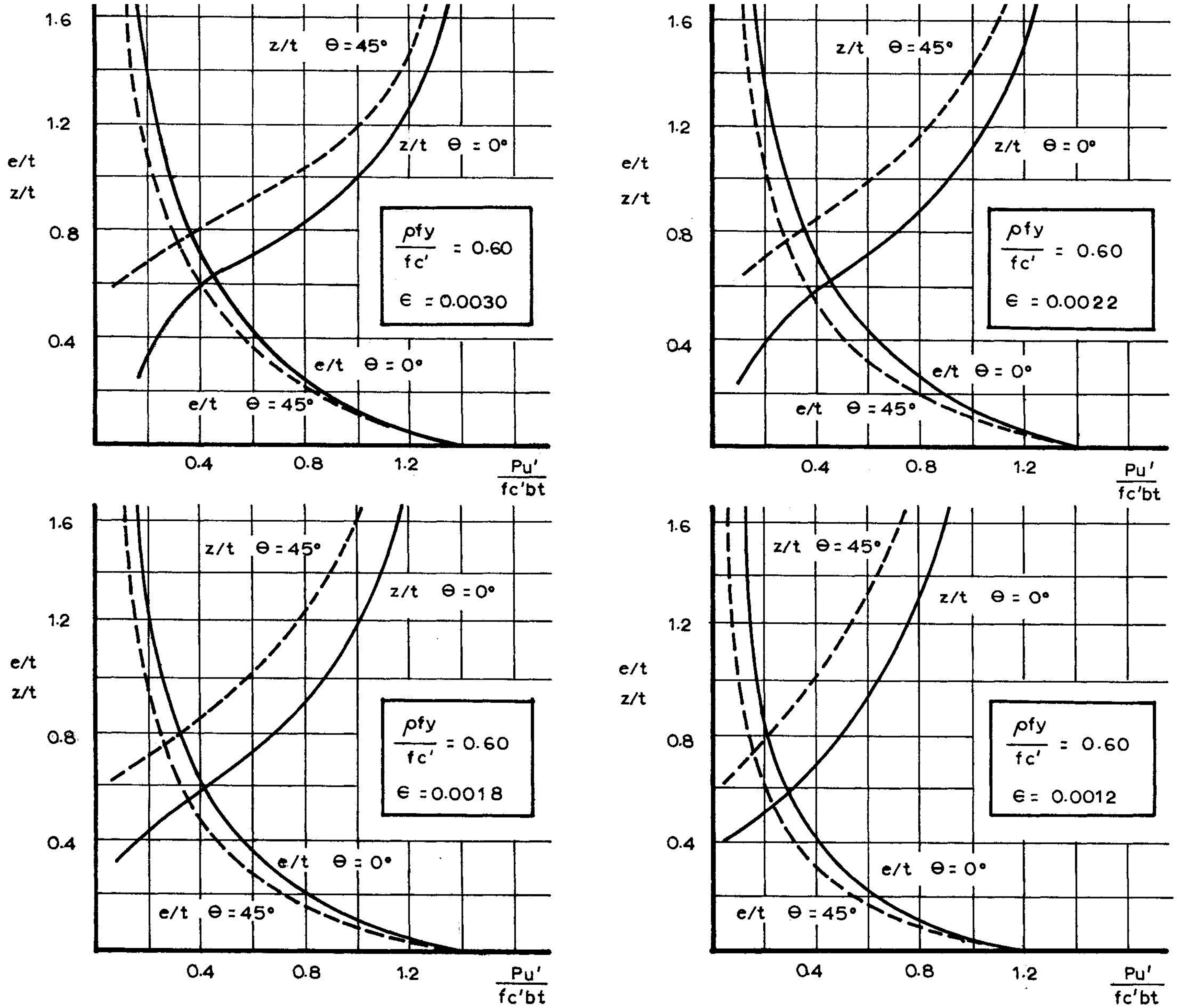

Figure 1 Interaction curves for $\frac{\rho f_y}{f_c'} = 0.60$

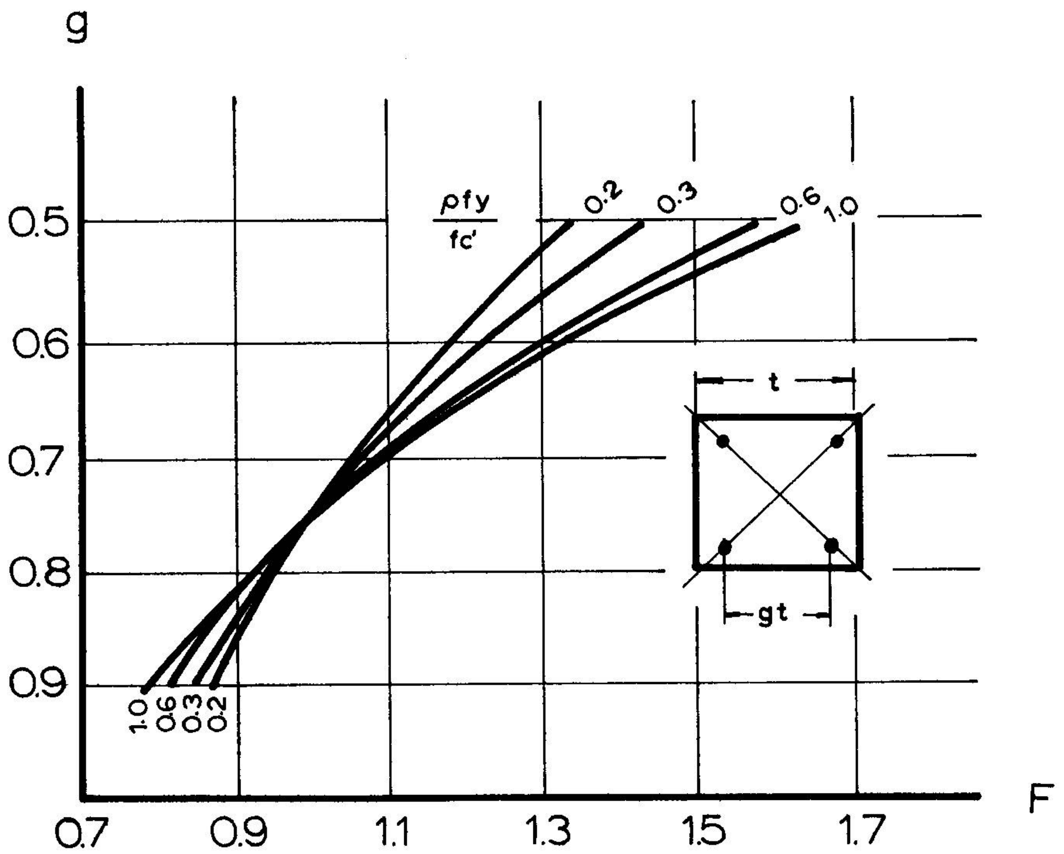

Figure 2 Factor F to modify results with a cover ratio g=0.75 to any other cover ratio