Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 16 (1974)

Artikel: Strength of reinforced or prestressed concrete slender columns under biaxial load

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DOI: https://doi.org/10.5169/seals-15730

Strength of Reinforced or Prestressed Concrete Slender Columns under Biaxial Load

Résistance des poteaux élancés en béton armé ou précontraint en flexion déviée

Tragfähigkeit von schlanken Säulen aus Stahlbeton oder Spannbeton bei schiefer Biegung

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INTRODUCTION. Theoretical research aiming to "exact" solutions for reinforced and prestressed concrete structures in non-linear range has been very active in recent times, improving the accuracy of results for basic problems and engaging gradually with more complex cases of structural behavior.

A problem which has received up to now only limited solutions (see ref.|1|,|7|,|8|,|9|) is that of slender structures acted by normal force and biaxial bending. For problems like those of columns of buildings having typical cross sections and restraint, solutions are available in the form of tables, graphs or approximate formulae (|3|,|4|), and this is certainly sufficient for design needs. More variable conditions of shape and loading occur frequently: an important example is offered by tall bridge piers. A realistic analysis of these structures requires both geometrical and mechanical nonlinearities to be considered, and neither the hollow shaped sections, variable together with reinforcement along the height, nor the complex loading patterns allow for the use of any simplified formula.

The method presented here, and the corresponding computer program, are based on a quite comprehensive analytical model, as described in the following; the result is a convenient tool for a complete analysis of that kind of structural problems.

DESCRIPTION OF THE MODEL.

Geometry. The model (Fig. 1) consists of a cantilever column, discretized by a sufficiently large number of cross-sections. These are contained in parallel planes, and their geometry is defined in a fixed coordinate system x, y, z. The shape of the sections is polygonal -eventually with poligonal voids- being described as a sum of positive and negative triangular areas (Fig. 2). The shape may vary along the vertical axis z.

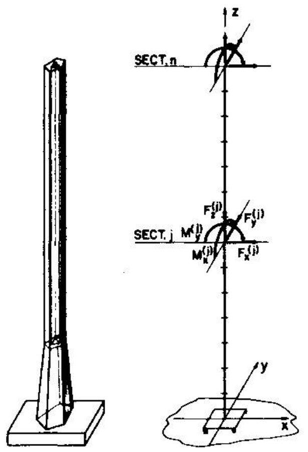


Fig.1 Idealization of the structure

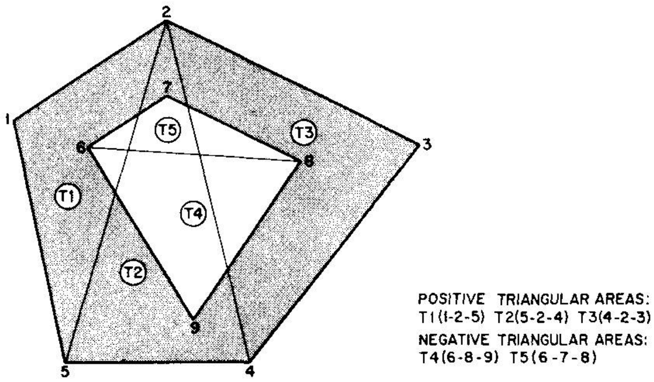


Fig.2 Description of a section by triangles

External loads. External loads are applied to every cross-section by two vectors of five components: $\{F_x, F_y, F_z, M_x, M_y\}^j$ for the dead load and the live load respectively (Fig. 1). Prestressing is introduced giving tendons areas and pretensioning strains along the sections.

Materials. Three separate constitutive laws can be assigned to concrete, reinforcing and prestressing steels. Figure 3 illustrates the "non-linear elastic" types presently adopted. The law for steels can describe, by varying its coefficients, the behavior of ordinary and high strength prestressing reinforcement.

The laws are assumed independent of time. Perfect bond is assumed between concrete and ordinary reinforcement; prestressing steel is considered free during the application of dead load, and bonded under the effect of live load. The deformations in the model are entirely attributed to axial and bending forces, shear and torsional effects being neglected.

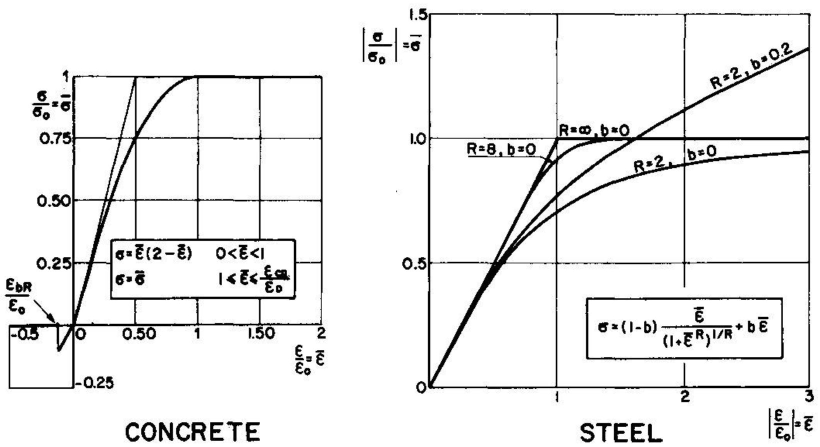


Fig.3 Constitutive laws adopted for the three materials

DESCRIPTION OF THE METHOD. The procedure calculates the stress and strain state of the structure for a given set of loads.

The second-order solution is pursuited through the following iterative scheme ("general" iteration):

- for every section, normal force and bending moments are calculated according to the deformed shape of the axis previously obtained.
- each section is then analized, in order to calculate the axial deformation ε_c, referred to the point C (appertaining to the conventional axis of the structure), and the curvatures K_x and K_y.
- by numerical integration of the curvatures a new deflected shape is obtained, which is used for starting a new "general" cycle.

Analysis of the section. The analysis is performed by means of an iterative procedure ("internal" iteration), which arrives at the determination of the neutral axis n and of the curvature K about it, for a given set of applied forces N, M_x, M_y.

Numerical analysis requires a discretization of the section. For each cycle, the concrete part is automatically discretized into strips parallel to the last neutral axis n, triangle by triangle.(The m axis is defined as orthogonal to n and passing through point C). The technique of subdivision is illustrated in Fig. 4. Each triangle is first divided into two sub-triangles by a line parallel to n. Then each one is discretized into strips of equal thickness (an upper bound is assigned for the structure). If the tensile strength axis ($\varepsilon = \varepsilon_{br}$)cuts a triangle, the cracked zone is cut off the discretization. The areas of ordinary and pretensioned steel are treated individually.

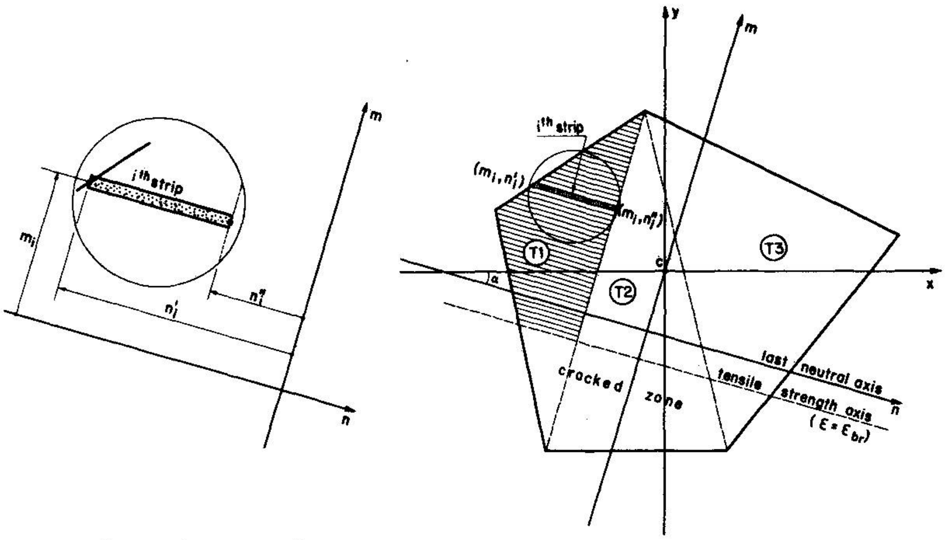


Fig.4 Automatic discretization of the concrete area in each step

If the actual values of n and K were known, the strain ε_i of the i^{th} concrete or ordinary steel area would be given by the expression:

$$\varepsilon_i = K \cdot m_i \tag{1}$$

m_i being the distance from n.

The actual secant modulus of the i^{th} area would be defined thus:

$$E_i = \frac{\sigma_i(\varepsilon_i)}{\varepsilon_i} \tag{2}$$

being σ_i the stress corresponding to ε_i according to the constitutive law of the material. By the use of E_i the section can be treated as if composed of linear elastic parts with varied moduli (Ref. |10|). To the homogenized section can then be applied the elastic relationships between external forces and internal stresses. The following quantities must be first calculated:

$$\begin{aligned} A &= \Sigma_i (B_i E_i) + \Sigma_\ell (A_\ell \cdot E_\ell) \\ S_n &= \Sigma_i (B_i E_i m_i) + \Sigma_\ell (A_\ell E_\ell \cdot m_\ell) \qquad (B_i = (n_i'' - n_i') \cdot s) \\ S_m &= \Sigma_i \left(B_i E_i \frac{n_i'' + n_i'}{2}\right) + \Sigma_\ell (A_\ell E_\ell n_\ell) \qquad (A_\ell = \text{individual steel area}) \end{aligned} \tag{3}$$

The coordinates of the homogenized centroid 0 of the section, referred to the system n, m, are:

$$n_o = S_m/A \quad ; \quad m_o = S_n/A \tag{4}$$

The homogenized moments of inertia, referred to the central coordinate system n_o, m_o, parallel to n, m (Fig. 5)are:

$$\begin{aligned} J_{n_o} &= \Sigma_i \left| B_i E_i (m_i^2 + s^2/12) \right| + \Sigma_\ell (A_\ell E_\ell m_\ell^2) - A \cdot m_o^2 \\ J_{m_o} &= \Sigma_i \left| E_i \left| (n_i'')^3 - (n_i')^3 \right| \cdot s/3 \right| + \Sigma_\ell (A_\ell E_\ell n_\ell^2) - A \cdot n_o^2 \\ J_{n_o m_o} &= \Sigma_i \left| A_i E_i m_i (n_i'' + n_i')/2 \right| + \Sigma_\ell (A_\ell E_\ell m_\ell n_\ell) - A \cdot m_o \cdot n_o \end{aligned} \tag{5}$$

The Culmann ellipse, modified with the introduction of the varied E_i, has center in 0 , and is defined by its axes and by the angle α_o between its major axis ξ and the neutral axis n (Fig. 5).

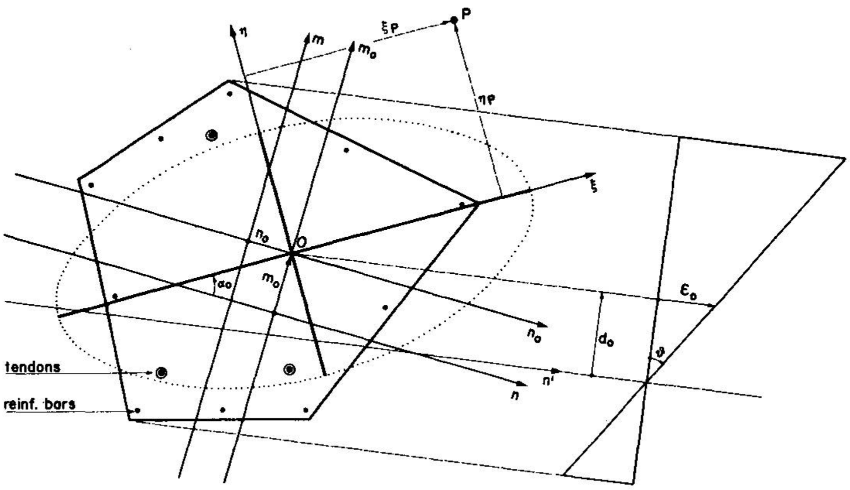


Fig.5 Homogenized cross-section

The homogenized principal moments of inertia of the section are:

$$J_{\xi,\eta} = \frac{J n_o + J m_o}{2} \pm \sqrt{\left(\frac{J n_o - J m_o}{2}\right)^2 + J_{n_o m_o}^2} \tag{6}$$

and the angle α_o :

$$\alpha_o = -\frac{1}{2}\ arctg\ \frac{2\ J_{n_o m_o}}{J_{n_o} - J_{m_o}}\ (+\frac{\pi}{2}) \qquad (7)$$

The neutral axis obeys the antipolarity relationship with the center of application of the external forces, P.

As the neutral axis is being researched through an iterative procedure, the antipolarity is employed to find a new approximate position n' of it (Fig. 5). The equation of the n' axis, referred to the coordinate system ξ, η, is:

$$(\frac{\xi_p}{J_\eta})\ \xi + (\frac{\eta_p}{J_\xi})\ \eta + \frac{1}{A} = 0 \qquad (8)$$

in which ξ_p and η_p are the coordinates of the point P.

Repeated discretization of the section and use of the expressions (1) to (8) end up with coincidental n and n'.

Finally, being $\varepsilon_o = \frac{N}{A}$ the strain on the homogenized centroid 0, the curvature K of the section is given by (see Fig. 5):

$$K = \frac{\varepsilon_o}{d_o} \qquad (9)$$

Cases of sections subjected to pure compression or pure bending are dealt with analogous homogenization and expressions derived from the elastic theory.

Analysis of the structure. Given the different positions of the neutral axes in each section and the associated curvatures K^j, the curvatures K^j_x K^j_y about the fixed axes x, y and the vertical deformations ε^j_c for all the sections of the column are:

$$K^j_x = K^j \cdot cos\ \alpha^j \qquad K^j_y = K^j \cdot sin\ \alpha^j \qquad \varepsilon^j_c = K^j \cdot d^j_c \qquad (10)$$

Numerical integration of these quantities gives the deflected shape of the column axis, with five components of displacement for each section j :

$$\{S_x,\ S_y,\ S_z,\ \phi_x,\ \phi_y\}^j \qquad (11)$$

The "general" iteration terminates when convergence on the deflected shape is reached.

Test for convergence. In the procedure outlined, the iterative solutions are in a monotonic progression from below (constitutive laws are monotonic, and only "small" displacements are considered). This requires checking both increments size and rate of convergence. A test wich has some theoretical bases, and proved to be effective is:

$$\frac{\Delta S_x{}^j}{\overline{S}} < \gamma \cdot \left(1 - \left|\frac{\Delta S_x{}^j}{\Delta S'^j_x}\right|\right) \qquad (j = 1,2,\ldots,n) \qquad (12)$$

and analogous for direction y. In the (12) $\Delta S_x{}^j$ and $\Delta S'^j_x$ are the last and the penultimate increments of $S_x{}^j$, $\overline{S}$ is an averaged displacement and γ is the assigned tolerance (the value adopted is 0,01).

When the second-order effects become important, the iteration converges very slowly. Therefore, an extrapolation formula is applied to the solution (after a certain number of cycles). This formula derives from the hypothesis that the successive solutions form a geometrical series, and makes use of the last three calculated solutions to extrapolate a tentative one, as the limit of the series. The factor to be applied at the last solution S_x results:

$$r = \frac{1}{S_x} \cdot \frac{S''_x \dfrac{\Delta S_x}{\Delta S'_x} - S'_x \dfrac{\Delta S'_x}{\Delta S''_x}}{\dfrac{\Delta S_x}{\Delta S'_x} - \dfrac{\Delta S'_x}{\Delta S''_x}} \tag{13}$$

Prestressing. With the steel strains initially assigned, the forces in the tendons are calculated, for each section, and hence their contribution to the external normal force and bending moments.

The stress and strain variations in the prestressing steel after the bonding, due to the application of live loads, are calculated in this way: a preliminary analysis is made, in which only the dead load and prestressing (as external action) are applied. This analysis yields the concrete strains at all the points where tendons are located. In the subsequent iterative analysis, in which both dead and live loads are applied, the strain variations in the steel due to live loads are obtained as:

$$\Delta\varepsilon_i = (\varepsilon_i - \hat{\varepsilon}_i)$$

($\hat{\varepsilon}_i$ ≡ concrete strain from the preliminary analysis)

The corresponding stress variations produce $\overline{\Delta N}$, $\overline{\Delta M_x}$ and $\overline{\Delta M_y}$: these are added to external forces, and the cycle proceeds to the analysis of the section as for the ordinary case.

Ultimate conditions. After the solution for the given loading has been found, the program can automatically increase the set of live loads by a multiplier λ . It is thus possible to draw load-displacement curves, from the service load conditions up to collapse. For better approximation, the increments of λ are automatically reduced when the collapse value is reached.

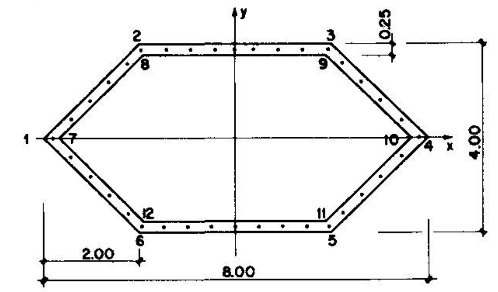
APPLICATION.

The results obtainable by the method are illustrated with reference to a simplified scheme of a bridge pier.

Geometry of cross-sections details of reinforcement and material characteristics are shown in Fig. 6a.

The pier is 100 m high: eleven equidistant sections have been considered in the analysis.

The Table 1 contains the loading conditions examined. The dead load consists of the weight of the pier and of the bridge deck. A first live load ("X") consists of horizontal forces parallel to x



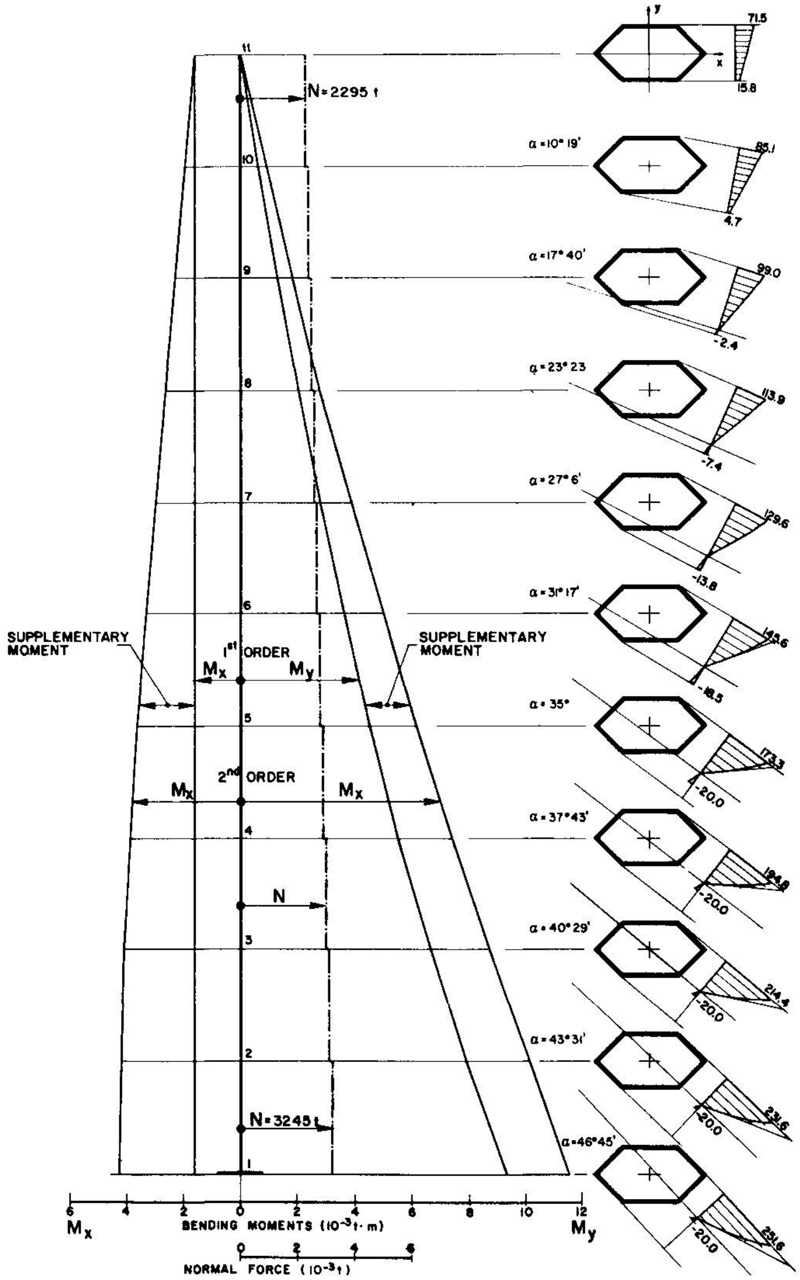
CONCRETE f_c = 300 Kg/cm² ; E_{in} = 300,000 Kg/cm²

STEEL f_y = 4,000 Kg/cm² ; E_s = 2,100,000 Kg/cm²

THICKN. OF CONC. STRIPS : 5 cm

NUMBER OF STEEL AREAS : 42 x 20 cm²

SUBDIVISION IN TRIANGLES :
4 positive,, 4 negative



LOADING CONDITIONS OF THE PIER

SECTION	Z(m)	DEAD LOAD F_z (t)	LIVE LOADS "X" F_x (t)	LIVE LOADS "Y" F_z (t)	LIVE LOADS "Y" M_x (t·m)
1	0.	– 50.	-	-	-
2	10.	– 100.	8.	-	-
3	20.	– 100.	8.	-	-
4	30.	– 100.	8.	-	-
5	40.	– 100.	8.	-	-
6	50.	– 100.	3.	-	-
7	60.	– 100.	3.	-	-
8	70.	– 100.	3.	-	-
9	80.	– 100.	3.	-	-
10	90.	– 100.	3.	-	-
11	100.	– 1500.	40.	–500.	1000.

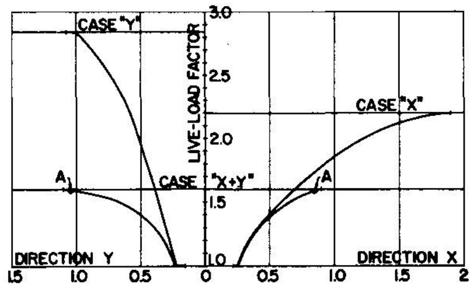


Fig.6 : a) cross section and reinforcement
b) stress-state of the pier in near-collapse stage
c) load-deflections curves of the top section

axis distributed along the column; the second live load ("Y") comprises a vertical load and a couple M_x applied to the top section.

Both "X" and "Y" loading conditions were increased up to collapse; in a third case, "X" and "Y" were applied together.

The load-displacement curves appear in Fig.6c. The "X + Y" condition produces biaxial bending; as can be seen, the live load factor at collapse is noticeably less than for the separate cases. The onset of collapse is due to instability in the y direction. The Δy displacement under service loads (load factor = 1) is less than for x direction: under the proportional increase of live loads Δy overpasses Δx and tends to horizontal tangent. The stress state on the pier at a near-collapse condition (point A in Fig.6c, $\lambda = 1.59$; collapse value: $1.59 < \lambda < 1.60$) is illustrated in Fig. 6b.

Diagrams show the shifting and rotating of neutral axis from the base to the top. It can be noted that limit stresses have not been attained yet in any point of the structure (only concrete stresses are plotted beside the sections). In fact, this case of collapse is due to instability appearing almost in the "elastic" range.

The slenderness of the pier is confirmed by the bending moment M_x which is at the base more than 2.5 times the corresponding first order value (Fig.6b).

B I B L I O G R A P H Y

|1| AAS-JAKOBSEN, A. - "C.E.B. - BULLETIN D'INFORMATION - N.76 & N.77".

|2| ARONI, S. - "Slender Prestressed Concrete Columns" Jnl. of Str. Div., ASCE Vol.94, Apr. 1968.

|3| C.E.B. - "BULLETIN D'INFORMATION N. 93" - Manuel de Calcul "Flambement - Instabilite", Juillet 1973.

|4| DIN 1045 German Code for the Design of Reinforced Concrete Structures.

|5| DRYSDALE, R.G., HUGGINS, M.W. - "Sustained Biaxial Load on Slender Prestressed Columns". Jnl. of Str. Div., ASCE Vol. 97, May 1971.

|6| GESUND, H., VANDEVERLDE, C.E. - "Stifness of Reinforced Concrete Columns in Biaxial Bending". - Proc.Int.Conf. on Planning and Design of Tall Buildings - Lehigh Univ., 1972 - Tech. Comm. N.22.

|7| KORDINA, K., RAFLA, K., HJORTH, O. - "Traglast von Stahlbetondruckgliedern unter schiefer Biegung mit Achsdruck" - Techn. Univ. Braunschweig, Juli 1973.

|8| Mac GREGOR, J.C. - "Simple Design Procedures for Concrete Columns" Introd.Rep., Theme II, IABSE Symp., Quebec 1974.

|9| MARTIN, I. - State of Art Report, N.5 - "Reinforced Concrete Columns" Proc.Int.Conf. on Planning and Design of Tall Buildings-Lehigh Univ., 1972 - Tech.Comm.N.22.

|10| MENEGOTTO, M., PINTO, P.E. - "Method of Analysis for Cyclically Loaded R.C. Plane Frames Including Changes in Geometry and

Non-Elastic Behavior of Elements under Combined Normal Force and Bending" Prel.Rep. IABSE Symposium, Lisbon, 1973. IABSE Vol. 13.

|11| PARME, A.L., NIEVES, J.M., GOUWENS, A. - "Capacity of Reinforced Rectangular Columns Subjected to Biaxial Bending". Jnl. A.C.I. Vol.63, Sept. 1966.

|12| WARNER, R.F. - "Biaxial Moment Thrust Curvature Relations" Jnl. of Str. Div., ASCE Vol. 95, May 1969. Introd.Rep., Theme I, IABSE Symposium, Quebec 1974.

|13| ZIA, P., MOREADITH, F.L. - "Ultimate Load Capacity of Prestressed Concrete Columns" Jnl. A.C.I., Vol. 63, July 1966.

SUMMARY

A procedure is presented for the analysis of slender columns in biaxial bending, for general loading conditions and cross sections variable in shape and reinforcement. The behavior of the three materials is described by different non-linear laws. Discretization of the sections and numerical integration of the actual strains yield the deformed shape of the column for any given set of loads. Ultimate conditions can be found by automatic incrementing of part of the loads.

RESUME

On décrit une méthode pour l'analyse des poteaux élancés en béton armé ou précontraint soumis à flexion biaxiale. Les charges dans les deux plans ont des distributions quelconques: la forme et les armatures des sections sont variables. Trois lois constitutives non-linéaires décrivent le comportement des trois matériaux. L'étude des sections est réalisée au moyen d'une discrétisation, et la ligne déformée du poteau s'obtient en intégrant les déformations de chaque section. La condition d'équilibre limite est atteinte en accroissant automatiquement l'intensité d'une partie ou de la totalité des charges.

ZUSAMMENFASSUNG

Es wird eine Methode vorgestellt für die Berechnung von schlanken Säulen unter schiefer Biegung, für allgemeine Belastungsbedingungen und veränderliche Form des Querschnitts. Das Verhalten der drei Baustoffe wird mittels verschiedener nicht-linearer Gesetze beschrieben. Eine Diskretisierung der Querschnitte und numerische Integration der effektiven Dehnungen ergibt die verformte Axe der Säule für jede gegebene Belastungsbedingung. Die maximale Tragfähigkeit kann durch automatische Steigerung der Belastung ermittelt werden.

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