Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 16 (1974)

Artikel: Effects of slenderness and sustained load on the capacity of reinforced concrete columns: an analysis of the design parameters

Autor: Drysdale, R.G. / Sallam, S. / Tan, K.B.

DOI: https://doi.org/10.5169/seals-15725

Effects of Slenderness and Sustained Load on the Capacity of Reinforced Concrete Columns: An Analysis of the Design Parameters

Effets de l'élancement et de charges soutenues sur la capacité des colonnes en béton armé: Une Analyse des Paramètres de Dimensionnement

Der Einfluss von Schlankheit und Dauerlast auf die Tragfähigkeit von Stahlbetonstützen: Eine Untersuchung über die Bemessungsparameter

R.G. DRYSDALE
Associate Professor
McMaster University
Hamilton, Ontario, Canada

S. SALLAM
Ph.D. Student
McMaster University
Hamilton, Ontario, Canada

K.B. TAN
Assistant Structural Engineer
Housing and Development Board
Republic of Singapore

1. INTRODUCTION

The design of column cross sections for known axial loads and moments has reached the stage where practical methods give results which agree very closely with tests and with accurate analyses. However considerable uncertainty exists with regard to methods employed to take into account the effects of column length. It is generally agreed(2,6) that the rational way to compute the reduced capacity is to include directly the effects of the additional moments caused by deflection of columns (PΔ effects). Theoretical calculations(3) can be used to accurately predict the loads at which material failure or column instability will occur. However designers require simpler techniques which are sufficiently general in nature to be equally applicable to the large variety of design cases.

The effect of column slenderness which is further complicated by consideration of creep under sustained load is the main topic of this paper. It is suggested that a realistic appraisal of design methods must be based on the idea of consistent safety factors. Thus slender columns subjected to sustained load must retain sufficient reserve capacity so that failure loads when compared to design loads provide equal safety factors. The National Building Code of Canada is being revised to include the relevant provisions of ACI 318-71(1). Therefore the columns analysed in this paper were designed in accordance with ACI 318-71.

2. DESIGN PARAMETERS

The magnitude and effect of the additional moments due to deflection must be determined for the full range and combinations of design parameters. Such a comprehensive evaluation was not attempted here. The values of the design parameters chosen were selected to be representative of normal design practice. Those parameters which are included in this study are briefly discussed below.

2.1 Column Properties

For simplicity of interpretation square cross sections with symmetric reinforcing in exterior layers were analysed. The reinforcement was positioned so that the distance, g, between the exterior layers was 0.8h. Most of the results presented are for P_t =3.0% although some results for 1.5% of steel are provided for comparison. The concrete strength used is f'_c=27.5 MN/m^2 and the steel yield stress is σ_y=412 MN/m^2.

Individual columns with slenderness ratios, $^l/_r$, from 0 to 100 were analysed for various combinations of end eccentricities. Also an example of the behaviour of columns in frames is presented.

2.2 Loading Conditions

The columns to be analysed were designed according to ACI 318 -71[1,7] where, in addition to knowing the section properties, the values of the end moments, the effective length and the level of sustained load were required. For the analyses of individual columns (kl=1.0) the majority of the results are for the case of symmetric single curvature where the effect of PΔ should be largest. The eccentricities used in the investigation are 0.1h, 0.4h and balanced eccentricity, e_{bal}.

The short term capacity of each column was determined. This only has meaning if live load, L, is 100% of the total loading. Two cases which are more realistic (D=L and D=100% of the total load) were analysed to find the effect of sustaining the dead load, D. The remaining capacities after sustained loading were also determined. Finally analyses were performed for the case of D=100% of the total load and with the ultimate dead load of 1.4D being sustained.

3. DESCRIPTION OF THE METHOD OF ANALYSIS

A computer program has been developed to predict the behaviour and capacity of reinforced concrete frame structures[8]. Details of the major features of the method of analysis have been reported elsewhere[3,4]. The accuracy of this method has been verified by the comparison[3,4,9] of the analytical results with tests of columns and frames subjected to short term and sustained loading. Therefore only a brief description with emphasis on those aspects which are particularly important to this study will be provided.

3.1 General Description

The response of cross sections to axial load and moment is found by dividing the section into strips. For any plane distribution of strain the stress on each strip is calculated taking into account the amount of creep and shrinkage which has occurred at the centre of each strip. The sum of the forces and the moments of the forces from each cross section strip and from the reinforcing steel are compared to the applied axial load and moment. The magnitude and slope of the plane strain distribution are varied until the internal forces balance the applied forces. Failure of the cross-section is defined when the internal forces cannot be increased to balance the applied forces.

The strain distributions for equilibrium of internal and applied forces provide values of equivalent stiffnesses ($M/K=EI$ and $P/\varepsilon=EA$) which can be used in the elastic structural analysis of a column or frame. In this analysis the members are divided into short elements. These elements are assigned stiffnesses which are the average of those calculated for the cross sections at each end. Using a matrix analysis format the forces and displacements at the ends of each element are computed. For the axial load and moment (including the $P\Delta$ effect calculated using the displacement information) at the ends of each element the new stiffnesses are found and compared to the previous values. Using an iterative process for changing the stiffnesses of each element, convergence for equilibrium and compatible displacements is achieved when all calculated values of stiffness coincide with the values used in the previous structural analysis.

For sustained load the magnitudes of creep and shrinkage are calculated and accumulated at regular intervals. The externally applied loads may change according to any predetermined pattern. Usually a period of constant sustained load is followed by short term loading to failure. Material failure is defined as before and instability is identified when the stiffness values for a particular element do not converge.

3.2 Material Properties

The stress strain properties for the reinforcing steel are idealized as being elastic-plastic. Inclusion of the effect of strain hardening would increase the column cross section capacity.

The properties of the concrete were based on test results(3) for a particular concrete which was specifically designed to have a lower than average aggregate to cement ratio and therefore a higher than average creep and shrinkage. The compressive failure strain was taken as .0038. In this analysis no increases of concrete strength or modulus of elasticity were taken into account after 28 days. Also the tensile strength of the concrete was disregarded. These factors plus the fact that creep and shrinkage values were found for 50% relative humidity and a temperature of 24°C create the situation where the calculated strains in the concrete will be greater than would actually occur. Thus the analysis will underestimate the resistance of the concrete section.

For this study sustained load was maintained for only 2 years. Previous analyses(3) had shown that most of the effects of creep and shrinkage will have occurred during this time. This is because the rate of creep is nearly proportional to the logarithm of time and because the stresses in the concrete decrease as the reinforcement carries a larger share of the load. A modified superposition method for calculating creep strain on each cross section strip was used to account for stress history.

Load	L=100%, D=0% $P_\phi=0$			L=D=50% $P_\phi=D$			L=0%, D=100% $P_\phi=D$			L=0, D=100% $P_\phi=1.4D$		
$^l/_r$ \ e	0.1h	0.4h	e_{bal}	0.1h	0.4h	e_{bal}	0.1h	0.4h	e_{bal}	0.1h	0.4h	e_{bal}
0	2.73	2.68	2.66	2.50	2.37	2.36	2.20	2.15	2.13	2.20	2.06	1.97
20	2.70	2.65	2.63	2.42	2.33	2.33	2.18	2.11	2.09	2.17	2.03	1.96
40	2.83	2.72	2.66	2.45	2.50	2.52	2.38	2.38	2.35	2.31	2.20	2.10
60	2.95	2.82	2.76	2.90	2.60	2.60	3.00	2.54	2.50	2.94	2.31	2.37
80	3.45	2.95	2.81	3.62	3.00	2.75	4.20	3.12	2.80	4.14	3.01	2.65
100	4.20	3.42	3.09	5.00	3.50	3.05	5.80	3.70	3.20	5.74	3.66	3.15

Table 1 Comparison of Computed Safety Factors for Columns Designed by ACI 318-71(1). ($M_1=M_2$, $P_t=0.03$, $f'_c=27.5$ MN/m^2, $\sigma y=412$ MN/m^2)

4. ANALYTICAL RESULTS

4.1 Comparison of Safety Factors

Table 1 contains the safety factors for different combinations of P_ϕ, e, and $^l/r$. The safety factor was determined by dividing the computed capacity after sustained loading by the design load found from ACI 318-71(1). The nominal ACI safety factor is (1.4D + 1.7L)/0.7 which gives values of 2.43, 2.21, 2.0 and 2.0 for the 4 cases of loading. The safety factors for $^l/r=0$ were up to 13% higher than the ACI values. The difference is mainly due to the ACI's use of a rectangular approximation for the stress distribution. This difference (which is less for larger eccentricities) is considered to be acceptable for calculation of cross section capacities. Therefore the design provisions to account for the additional moment, PΔ, should be evaluated in terms of the change in safety factor compared to the computed value for $^l/r=0$.

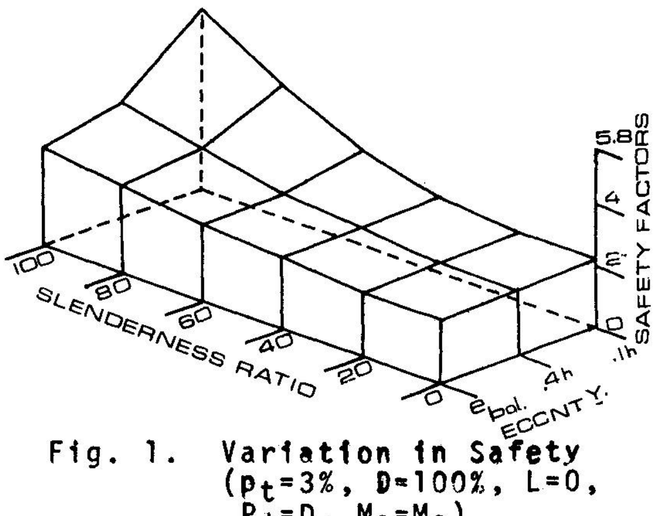

Fig. 1. Variation in Safety ($p_t=3\%$, D=100%, L=0, $P_\phi=D$, $M_1=M_2$)

The results show that the ACI design is most conservative for high $^l/r$ ratios, low $^e/h$ and high sustained loads. Even for the unrealistic case of sustaining 1.4D where L=0, the safety is not affected much. Reasons for these trends are suggested in the Conclusions.

The variation in the actual safety factor is shown graphically in Figure 1 for the case where L=0, D=100% and the design D is sustained. The safety factors for $P_t=1.5\%$ and e=0.4h are shown in Figure 2 for the same loading conditions. A similar trend is observed.

4.2 Evaluation of Moment Magnification

Figure 3 contains a graph of the computed moment magnifications, F, at failure versus l/r for e=0.1h and e=0.4h. The computed F values at the ACI failure load are substantially less than the ACI values. Because the computed failure loads are much greater the moment magnifications at failure are closer to the ACI values. The differences between computed and ACI values are even more pronounced for lower levels of sustained load.

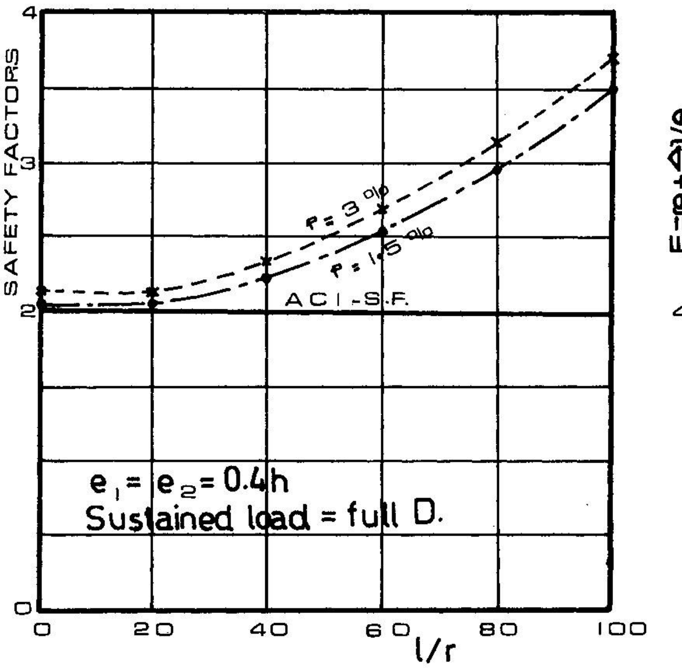

Fig. 2. Influence of p_t on Safety (D=100%, L=0)

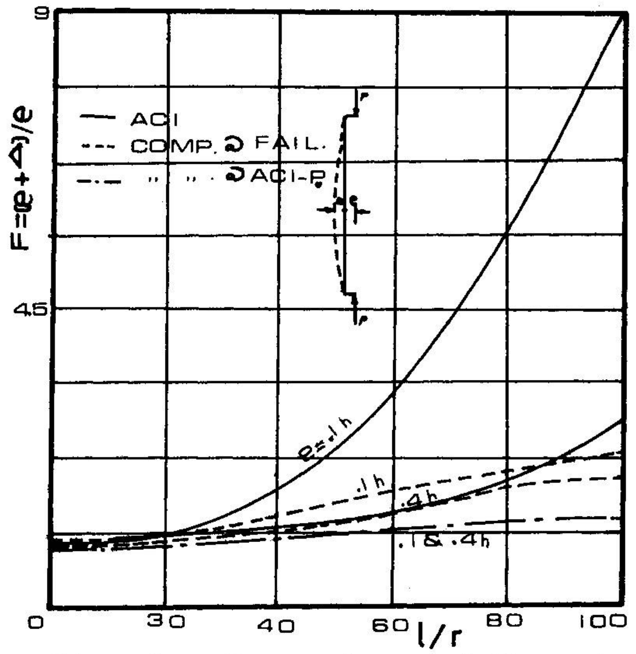

Fig. 3. Comparison of Moment Magnification (D=100%, L=0, P_ϕ=D, p_t=3%)

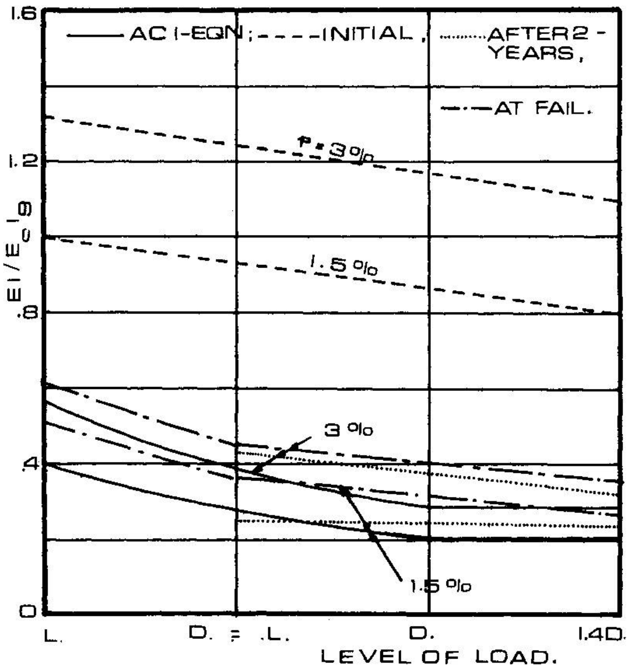

Fig. 4. Comparison of EI Values (e_1=e_2=0.4h, P_ϕ=D, l/r=60)

One of the main features in the calculation of the PΔ effect is the determination of the flexural stiffness, EI, of the cross section. For e=0.4h, the computed values of EI=Pe/K for different combinations of loading are shown in Figure 4 along with the ACI values. Computed values are shown for the following cases:

a) initial application of the design load
b) after two years of sustained dead load
c) at failure as the load was increased after the period of sustained loading.

The EI values after sustained load are slightly higher than the ACI values. As more load is applied to determine failure, the concrete again begins to carry a greater share of the load and the computed EI values show large increases. Even as the failure load is reached the EI values remain higher than they were at the sustained load stage. The influence of EI will be discussed in the Conclusions.

4.3 Effect of End Conditions

Analyses performed for different ratios of M_1/M_2 indicate that some variation in safety factor occur where the ACI provision for modifying the magnification factor, F, to account for this effect is used. The effect of multiplying F by the term $(0.6 + 0.4 M_1/M_2) > 0.4$ did not seem to create major changes in the spread of safety factor values.

One of the structures which have been analysed(8) is presented in Figure 5 to illustrate behaviour. A portion of the frame was analysed for sustained load followed by short term loading to failure. The dimensions, initial stiffnesses and loading are shown. Using ACI 318-71 the nominal safety factor for column 1 is 2.31. The computed safety factor is 3.15. Since this column had a low slenderness ratio the reserve strength is partially due to the

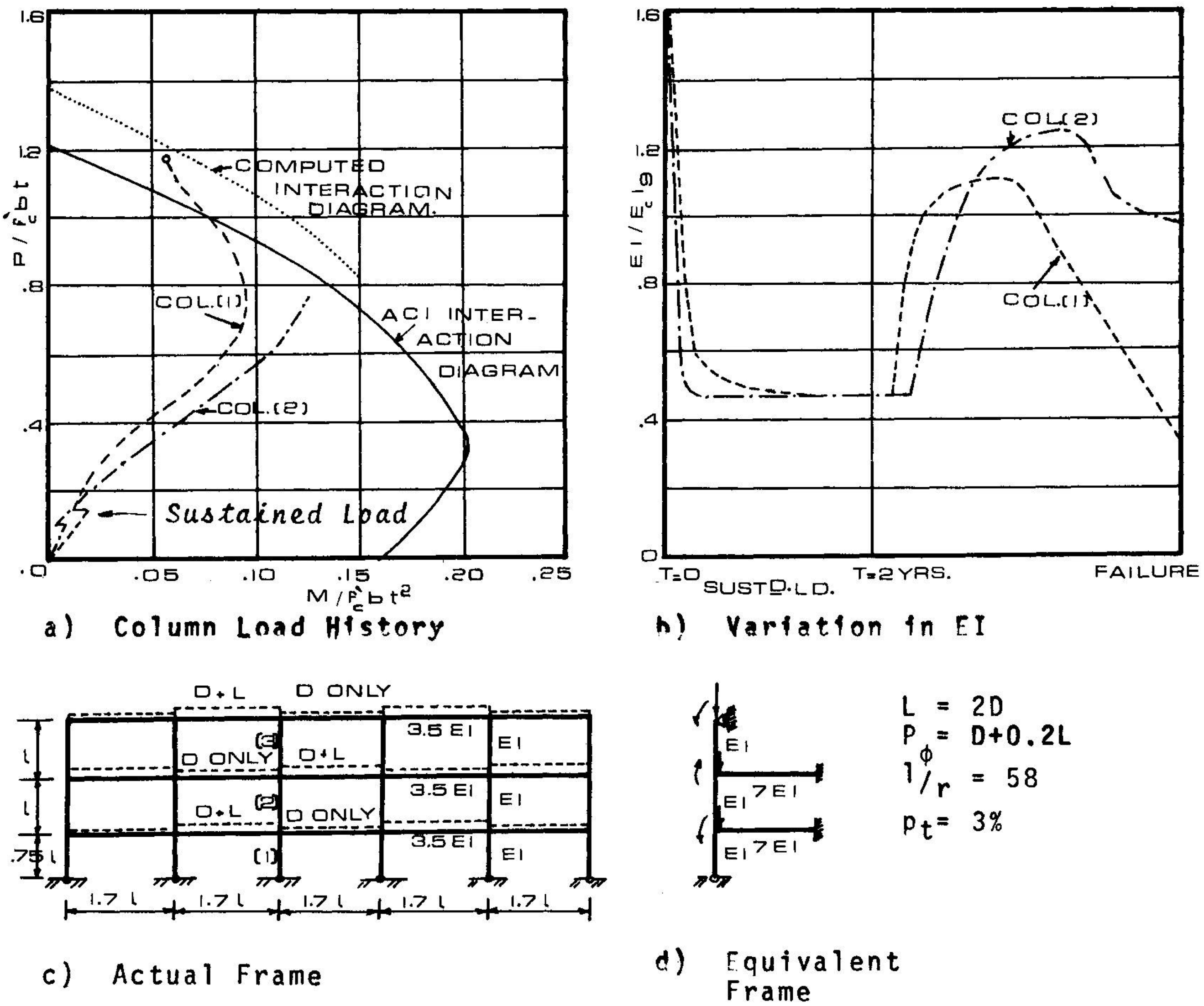

Fig. 5. Example of Behaviour of Columns in a Structure

reduction in moment which occurs as the column becomes more flexible.

The axial load and maximum moment on columns 1 and 2 are shown for loads up to failure of column 1. The slight decrease in moment at sustained load indicates that PΔ was more than compensated for by reduced distribution of moment to the columns. The changes in EI during sustained loading and as the loads are increased to failure are also shown.

5. CONCLUSIONS

The fact that slenderness and sustained load decrease column capacity is well documented(2,3,6). The results of this study indicate that the provisions of ACI 318-71 to account for these effects do not result in consistent safety factors. Using this accepted design method as a basis for comparison, several aspects of the design are identified for consideration.

1. During sustained load the EI values tend to approach I_sE_s for any of the following conditions; high $^l/r$, low P_ϕ, large $^e/h$, large p_t, and for more moderate combinations of these such as medium $^l/r$ and p_t, or medium P_ϕ and $^e/h$. This behaviour results from the transfer of stress to the steel as the concrete creeps and shrinks. However upon application of short term load to determine failure, the EI values increase. It is suggested that sustained load as a percent of cross section capacity rather than column capacity will provide a more realistic measure of the effects of sustained load.

2. The derivation of the moment magnification formula is based on the concept of including the PΔ effect when calculating the cross section capacity required. However in order to accommodate the possibility of instability failure on the basis of cross section capacity it is necessary to inflate the moment magnification values (increase PΔ) to achieve the appropriate reduction in column strength. Since a common EI is used for both material and instability failure, the added moment (PΔ) for cases with material failure is too large. This study and others(2) show that instability occurs only at small eccentricities and very high $^l/r$ values. Very slender columns are rarely found in practice. Also the possibility of instability failure exists only at the slenderness limit specified by ACI. Therefore designers should use the EI which produces the correct deflection rather than an artificial value to accommodate prediction of failure due to instability.

3. The moments applied to columns in structures braced against sidesway are limited to the moments transmitted by the beams or slabs. In many cases the additional moments due to deflection are largely offset by a redistribution of the applied moment as the column deflects. [The stiffness of a flexural member is not affected by deflection and is not affected to the same extent by creep and shrinkage.] Therefore there may be some benefit in using different moment magnifying procedures depending on whether or not the structure is braced against sidesway.

(Notation is defined or corresponds to the Introductory Report.)

REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-71)", American Concrete Institute.
2. Cranston, W.B., "Analysis and Design of Reinforced Concrete Columns", Research Report No.20, Cement and Concrete Association, London, 1972.
3. Drysdale, R.G. and Huggins, M.W., "Sustained Biaxial Load on Slender Concrete Columns", Journal Structural Division, ASCE, Vol.97, No.ST5, May 1971, pp.
4. Drysdale, R.G., Mirza, M.S., and McCutcheon, J.O., "Recent Research in Deflection of Concrete Structures", Proc. of the Symposium: Deflections of Structural Concrete, Canadian Capital Chapter of ACI, 1971, pp.161-182.
5. Furlong, R.W., "Column Slenderness and Charts for Design", Proc. American Concrete Inst., Vol.68, Jan. 1971, pp.9-17.
6. MacGregor, J.G., Breen, J.E., and Phrang, E.O., "Design of Slender Concrete Columns", ACI Journal, Proceedings, Vol.67, No.1, January 1970, pp.6-28.
7. Portland Cement Association, "Notes on ACI 318-71 Building Code Requirements with Design Applications", PCA, July 1972.
8. Sallam, S., "Design of Reinforced Concrete Columns", M.Eng. Thesis, McMaster Univ., Hamilton, Ontario, 1974.
9. Tan, K.B., "On Short-Term and Sustained Load Analysis of Concrete Frames", M.Eng. Thesis, McMaster Univ., 1972.

SUMMARY

Individual columns were analysed to determine the $P\Delta$ effects for sustained load. The remaining capacity after sustained load compared to the design load using ACI 318-71 gave the actual safety factor. The designs for slender columns and for columns loaded with small eccentricities were quite conservative. Some general comments on column safety are based on results for behaviour of columns in frames.

RESUME

Des colonnes isolées ont été examinées afin de déterminer les effets du $P\Delta$ sous l'influence de charges soutenues. La résistance effective après application de charges soutenues divisée par la charge utile selon ACI 318-71 donne le facteur de sécurité. Le dimensionnement de colonnes élancées et colonnes avec de petites excentricités, donne des résultats prudents. Quelques commentaires généraux sur la sécurité sont basés sur les résultats du comportement de colonnes dans des cadres.

ZUSAMMENFASSUNG

Es werden Einzelstützen untersucht hinsichtlich ihres Verhaltens 2. Ordnung unter Dauerlasten. Der Sicherheitsfaktor er-

gibt sich aus der nach Einwirkung der Dauerlast noch verbleibenden Tragkapazität, bezogen auf die Bemessungslast nach der ACI-Norm 318-71. Die Bemessung von schlanken Stützen bzw. Stützen mit kleinen Exzentrizitäten liegt auf der sicheren Seite. Einige allgemeine Bemerkungen zur Sicherheit von Stützen beziehen sich auf das Verhalten von Stützen in Rahmen.

Leere Seite
Blank page
Page vide