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A Study on the Behaviour of Steel Reinforced Concrete Columns and Frames

Une étude sur le comportement des colonnes et des cadres en béton armé

Eine Untersuchung über das Verhalten von Stahlbeton-Stützen und -Rahmen

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1. INTRODUCTION

For ascertaining the safety of a column or a frame under severe earthquake loading it is important to know the load-deflection relation of a column or a frame in the unloading range as well as the loading range under constant vertical and monotonically increasing or alternately repeated horizontal loading(1). Although several experimental investigations have been reported, the theoretical approach to the problem has not been well developed in the field of steel reinforced concrete (SRC).

The behavior of a SRC column under axial force and bending moment is almost the same as that of an ordinary reinforced concrete column in loading range if it is not subjected to severe shear force. However, a SRC column shows some-what different behavior from that of an ordinary reinforced concrete column in the unloading range in the following sence; the covering concrete falls off during the unloading, and the steel flange is not apt to buckle.

As pointed out in the introductory report the computation may become formidable if we start from accurate stress-strain relation. In this paper, starting from idealized hysteretic stress-strain relations and using discrete element approach, moment-curvature relations of a column under constant axial force are calculated. Since, as quoted in the introductory report, consideration of a finite length of the member is unavoidable when the negative slope range of the stress-strain curve of material is to be analyzed, the analysis of a member or a frame is carried assuming the flexural portions with a finite length near the member ends in which the curvature is uniformly distributed.

2. THEORETICAL INVESTIGATION

a. Moment-Curvature Relation: Let us consider a steel-reinforced concrete column section subjected to constant axial force and varying bending moment, in which stress and strain distributions are uniquely determined at a certain loading stage, and suppose that the state of stress at this stage is given by a piecewise-linear function of the state of strain, $\sigma_z = f_z(\varepsilon_z)$, where σ_z and ε_z are the stress and the strain in the fiber in the distance of z from the central axis. The shear force effect is neglected. The increments of axial force, dN, and the bending moment, dM, developed in the subsequent loading are given by

$$0 = dN = \int d\sigma_z \cdot dA = \int f_z' \cdot dA \cdot d\varepsilon_z \tag{1}$$

$$dM = \int d\sigma_z \cdot z \cdot dA = \int f_z' \cdot z \cdot dA \cdot d\varepsilon_z \tag{2}$$

where $d\sigma_z$ and $d\varepsilon_z$ are the stress and the strain increments, respectively, the prime denotes the derivative with respect to ε_z and the integration is carried over the cross sectional area.

If we assume that the plane section normal to the central axis remains plane, $d\varepsilon_z$ is explained by the strain increment on the central axis, $d\varepsilon$, and the curvature increment, $d\kappa$. Employing the definitions,

$$\bar{A} \equiv \int f_z' \cdot dA, \quad \bar{S} \equiv \int f_z' \cdot z \cdot dA, \quad \bar{I} \equiv \int f_z' \cdot z^2 \cdot dA \tag{3}$$

Eqs. (1) and (2) become

$$\bar{A} \cdot d\varepsilon + \bar{S} \cdot d\kappa = 0 \tag{4}$$

$$\bar{S} \cdot d\varepsilon + \bar{I} \cdot d\kappa = dM \tag{5}$$

Eliminating $d\varepsilon$ from Eqs. (4) and (5), the incremental moment-curvature relation is determined as

$$dM = (\bar{I} - \bar{S}^2/\bar{A}) \cdot d\kappa \tag{6}$$

This procedure does not always demand the linearity of the stress-strain relationship, but the nonlinear relationship requires the iterative procedure and the application to the deflection analysis of frames may become difficult. $\bar{A}$, $\bar{S}$ and $\bar{I}$ can be evaluated from Eq. (3) by deviding the cross section into a finite number of strip elements perpendicular to z axis, and by assuming the uniform distrubution of f'_z in a strip.

The hysteretic stress-strain relationship of steel is assumed to be the one shown in Fig. 1(a), taking Bauschinger effect into account. In the figure, σ_y, ε_y, and E_s are the yield stress, yield strain and Young's modulus of steel, respectively. The compressive strength of the covering concrete deteriorates due to the crash at an early stage of the compressive strain. On the other hand, the crashing strain of the confined concrete may be quite large due to the restraining action of steel flanges. From this point of view, two different kinds of the hysteretic stress-strain relations, shown in Figs. 1(b) and (c), are assumed for the confined and covering concrete, respectively. For both of them, the maximum strength σ_B is taken as 75% of the cylinder strength of concrete and the tensile strength is neglected.

<u>b. Deflection Analysis of a Single Column and a Framed Structure</u>: When the shear force effect is neglected, the deflection of a member subjected to bending can be determined by integrating the curvature, if the material is "stable" according to Drucker's postulate, and thus the moment is uniquely related to the curvature. However, as pointed in the introductory report, we must consider that finite lengths of section are governed by the same relationship between moment and curvature, when to deal with the unstable material like concrete.

In this paper, the following mathematical model is considered for simplicity to conduct the deflection analysis of a column or a frame. Let us consider a cantilever subjected to a lateral load P at its tip. When it is assumed that a curvature κ, which corresponds to the bending moment at the end ($=P\cdot\ell$), is uniformly distributed along the length s, and that the remaining portion of the cantilever with length of (ℓ-s) is rigid, the deflection at the tip, δ, can be obtained as

$$\delta = \kappa\cdot s\cdot(\ell - s/2) \qquad (7)$$

In the actual computation, the length s is so determined that the deflection, δ, of this model coincides the exact tip deflection of the entirely elastic cantilever, and it is given as

$$s = (1 - 1/\sqrt{3})\cdot\ell \qquad (8)$$

The incremental load-displacement relation for the cantilever is directly obtained by rewriting Eq.(7) in the incremental form, and using Eq.(6), as follows:

$$dP = \frac{\ell\cdot(\bar{I} - \bar{S}^2/\bar{A})}{s\cdot(\ell - s/2)}\cdot d\delta \qquad (9)$$

This method of approach can be applicable to the deflection analysis of a portal frame subjected to the constant vertical load, N, on columns and monotonic or alternately repeated horizontal load, Q. As conducted in the deflection analysis of a column, the flexural portions are imposed at both ends of the beam and columns, in which uniform curvatures are assumed to be distributed, and all other portions are assumed to show the rigid body motion. A simple computation based on the equilibrium condition at the joints and the geometrical relation gives the linear relation between the load increment and displacement increment.

3. EXPERIMENTS

To ascertain the accuracy of the theoretical treatment, the theoretical results are compared with the experimental ones which have been obtained by the authors(2, 3, 4). A brief explanation on the tests is given here.

The experimental works are composed of six series as shown in Table 1. Shapes and dimensions of specimens and the loading system are given in Fig. 2. In all series, the axial loads, N, are kept constant, and the horizontal or lateral loads, Q, are applied in the monotonic or alternately repeated manner, as identified by the second alphabet of each specimen, M or R, respectively. Numerals appearing in the specimen numbers denote the axial load ratio N/N_0, where N_0 is the ultimate compressive strength of a SRC cross section obtained by the method of superposition.

In SM and SR series, the specimen is subjected to uniform bending, and moment-curvature relations are derived from the deflection data detected at three points along the member axis. The column specimens in CR series are subjected to double curvature bending, and the chord rotation angle, R, is computed from the deflection data detected at the top and bottom of the column. On the other hand, the drift angle, R, of the frame is given by the horizontal displacement, δ, divided by the column height, h, as shown in Fig. 3.

In SM, SR and CR series, the bending cracks are first observed during the tests, the ultimate strength of each specimen is attained due to the yielding of steel and the development of the ultimate compressive strain of the concrete. And finally the covering concrete falls off causing the decrease of resistance.

Damaged portion is located at the center in SM and SR series, and at member ends in CR series. In the frame tests, the bending cracks are observed at beam and column ends, and the final failure occurs at both ends of columns. In all frame specimens, the concrete crash is not observed at the beam ends.

4. DISCUSSION ON THE RESULTS

The theoretical results(solid lines) are compared with the experimental ones(dashed lines) in Figs. 4 to 8. The moment-curvature relations are drawn for SM and SR series in Figs. 4 and 5. For monotonic loading cases, it is observed that the theory well predicts the strength deterioration after the attainment of the maximum strength. The negative slope of the moment-curvature curve becomes steeper with the increase of the axial load. In the large curvature range, the strength seems to converge to the sum of the strengths contributed from the confined concrete and the steel. Since the theory assumes entirely ductile stress-strain relation for the confined concrete, the strengths given by the theory are larger than that by the tests, and the positive slope appears again on the curve in the large curvature range due to the strain-hardening of the steel, except for the case of zero axial load. For repeated loading cases, the theoretical results of SR0 shows a very good agreement with the experimental one. Particularly the stiffning effect due to the closing of cracks are well predicted by the theory, although the stiffness change appears rather sudden in the theory. On the other hand, the large discrepancy is seen between the experimental and theoretical maximum strengths of SR3. However, it is clearly shown in case of SR3, that the strength converges to the sum of the strengths of the confined concrete and the steel.

The hysteresis loops of column specimens subjected to double curvature bending repeatedly are shown in Fig. 6. In general, the theory well predicts the experimental results. As clearly shown in the case of CR0, the discrepancy is seen on the shapes of hysteresis loops; stable spindle type in the test and rather parallelogrammic type in the theory. Since this discrepancy is not obvious in the moment-curvature relation, this is because the theory does not take into consideration the cracks due to bending distributed entire length of the column and the shear deformation. The test result of CR3 shows that the confined concrete already crashed at the final stage of the test.

Shown in Figs. 7 and 8 are the results of the frame analysis. One point that cannot be properly explained yet is the large discrepancy between the theoretical and experimental maximum strengths of FM series. This discrepancy is more or less observed in the results of FR series. It may be said that the accuracy of the theoretical results becomes poorer as the objective structure becomes more complicated. From this point of view, the development of the more refined mathematical model for the deflection analysis is needed. Since the members in CR, FM and FR series are subjected to double curvature bending, the moment curvature relation for such a member may have to be checked by the theory to give an explanation to the discrepancy. The difference in the shapes of the hysteresis loops of the frame is very similar to that observed in case of the columns. The large strength reduction observed in the final experimental loop of FR0 may be caused by the fracture of the steel.

5. CONCLUSIVE REMARKS

The moment-curvature relation of a steel reinforced concrete cross section under the constant axial force and monotonic or alternately repeated bending is computed by separating the cross section into a finite number of strip elements based on the assumption that the plane remains the plane. It can be concluded that it is needed to employ the accurate stress-strain relation of the concrete when to carry out the deflection analysis of the reinforced or steel reinforced concrete members, and its accuracy shows the critical effect on the result of the analysis. When the axial load is zero, the maximum bending moment of the cross section, and thus the maximum strength of the member, depends only on the strength of the bare steel portion including reinforcing bars, and the concrete strength hardly affects on the numerical results. However, when the axial load is present, the estimation of σ_B plays a key role to determine the maximum strength of the member. In the present analysis, σ_B is assumed to be 75% of the cylinder strength, and different types of stress-strain relations are assumed for the covering and confined concrete. The theoretical result shows a good agreement with the experimental one under the loading condition where the mean strain in the confined concrete is small.

The main advantage of the present method of analysis may lie on the point that the hysteretic load-deflection relation of the member of the frame can be directly derived from the moment-curvature relation of the member cross section, by considering the flexural portions with a finite lengths concentrated at the member ends. In general, the deflection analysis of the frames shows the discrepancy between the maximum strength obtained by the present method and that from the test. It seems adequate to impose the rigid portion with a finite length at the member ends, in order to obtain a good agreement with the test result.

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TABLE AND FIGURES

Table 1. Test Specimens and Material Properties

Series	SM				SR		CR		FM		FR	
Loading Type	Monotonic				Repeated		Repeated		Monotonic		Repeated	
Specimen No.	SM0	SM2	SM4	SM6	SR0	SR3	CR0	CR3	FM0	FM4	FR0	FR3
N (ton)	0	30	60	90	0	25	0	35	0	40	0	34.4
N/N_o	0	0.2	0.4	0.6	0	0.3	0	0.3	0	0.4	0	0.3
F_c(ton/cm^2)	.215	.269	.295	.275	.233	.216	.364	.326	.228	.242	.349	.328
${}_s\sigma_y$(ton/cm^2)	3.05	3.12	3.12	3.12	3.42	3.42	3.48	3.48	2.95	2.83	3.46	3.09
${}_r\sigma_y$(ton/cm^2)	3.68	3.68	3.68	3.68	4.18	4.18	3.38	3.91	2.61	2.61	3.87	3.87

N:Axial Load, N_o:Ultimate Compressive Strength, F_c:Cylinder Strength of Concrete, ${}_s\sigma_y$:Yield Stress of Steel, ${}_r\sigma_y$:Yield Stress of Main Reinforcing Bars.

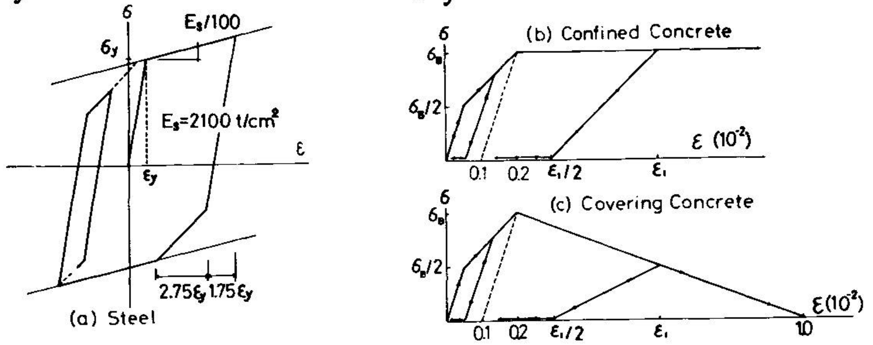

Fig. 1 Stress-Strain Relations

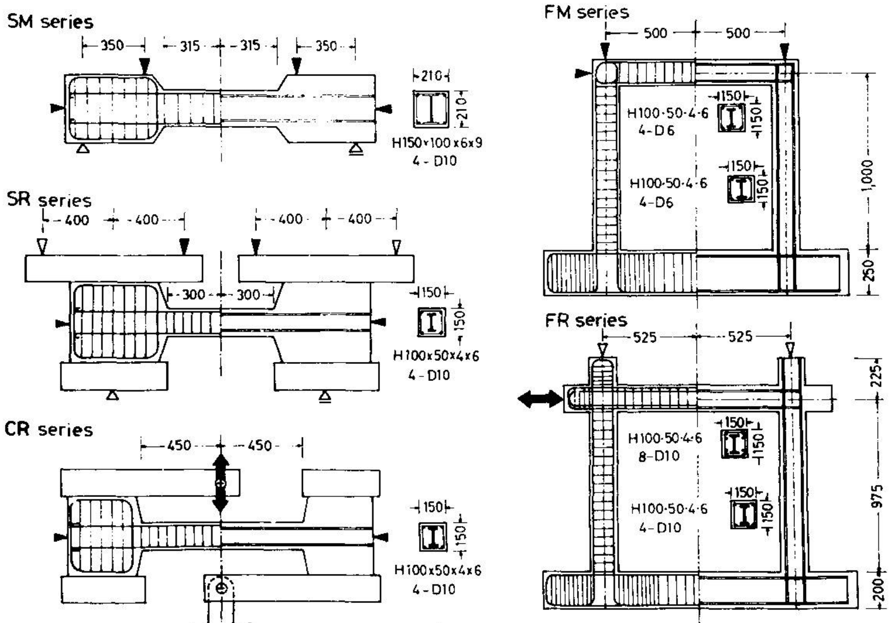

Fig. 2 Test Specimens

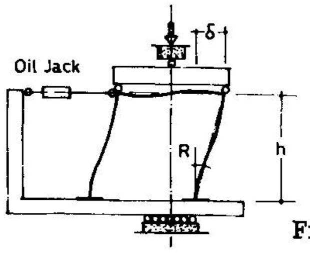

Fig. 3 Loading System of Frame Tests

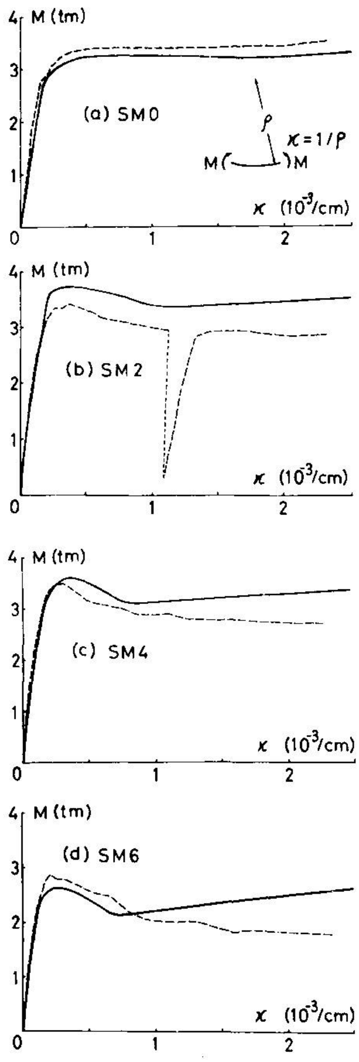

Fig. 4 Moment-Curvature Curves of SM Series

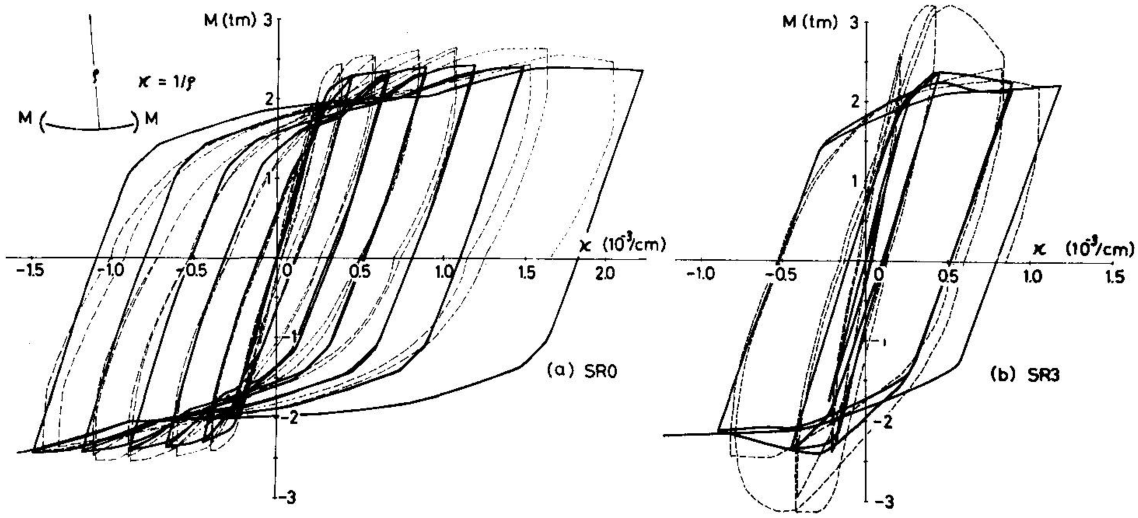

Fig. 5 Moment-Curvature Curves of SR Series

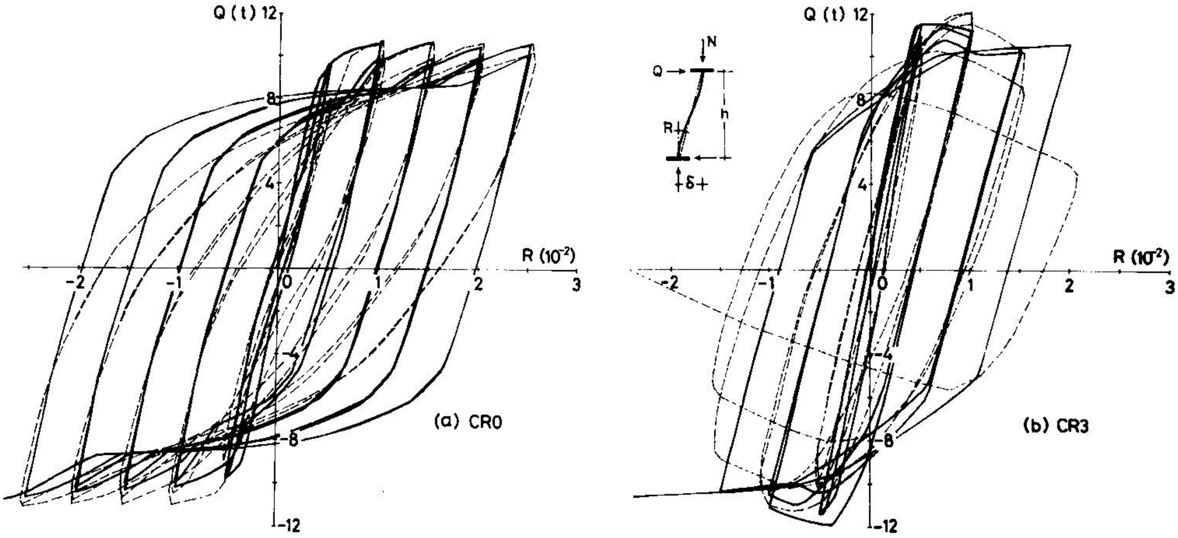

Fig. 6 Load-Deflection Curves of CR Series

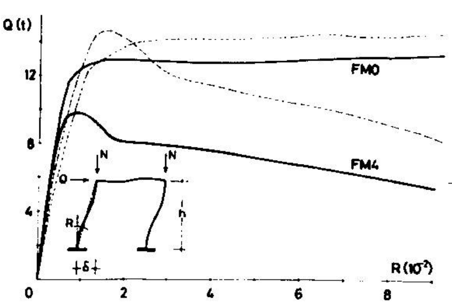

Fig. 7 Load-Deflection Curves of FM Series

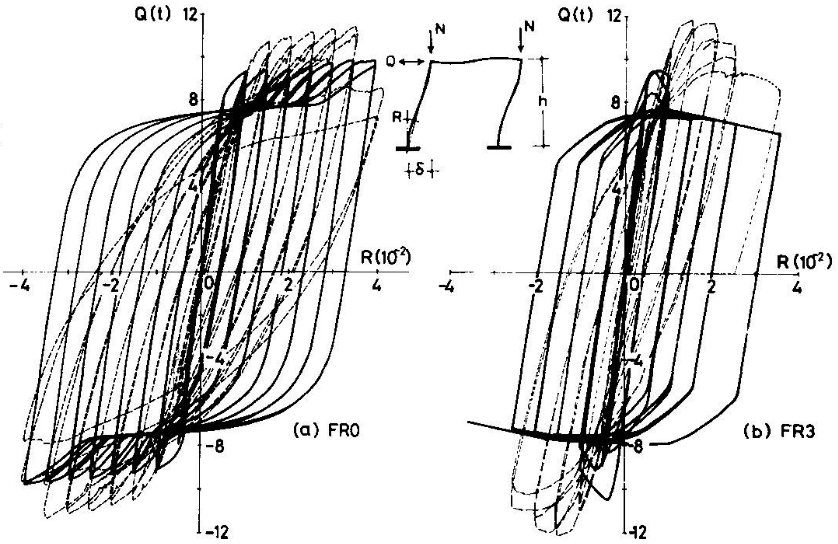

Fig. 8 Load-Deflection Curves of FR Series

SUMMARY

A numerical analysis is carried out to obtain the moment-curvature relationship of the steel reinforced concrete cross section under constant axial force and monotonic or alternately repeated bending moment, based on the idealized stress-strain relations for steel and concrete. A method of the deflection analysis of steel reinforced concrete columns and frames is proposed, introducing a mathematical model. The numerical results are compared with the experimental ones that have been obtained by the authors.

RESUME

On procède à un calcul numérique pour obtenir la relation moment-courbure d'une section en béton armé soumise à une force axiale constante et à un moment de flexion constant ou alterné; on se base sur des diagrammes tension-déformation idéalisés pour l'acier et le béton. On propose une méthode pour le calcul des déformations des cadres et colonnes en béton armé, qui utilise un modèle mathématique. On compare les résultats numériques avec des resultats expérimentaux des auteurs.

ZUSAMMENFASSUNG

Eine numerische Berechnung wird durchgeführt, um die Moment-Krümmungs-Beziehung von Stahlbetonquerschnitten unter konstanter Axiallast und gleichförmig ansteigendem oder wechselndem Biegemoment zu bestimmen. Hierbei werden idealisierte Spannungs/Dehnungs-Beziehungen für Stahl und Beton eingeführt. Eine Methode für die Berechnung der Verformungen von Stahlbetonstützen und -Rahmen wird vorgeschlagen, welche auf einem mathematischen Modell beruht. Die numerischen Resultate werden mit den Ergebnissen von Versuchen der Autoren verglichen.