Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 16 (1974)

Artikel: An experimental-analytical study of complete load-deformation characteristics of concrete compression members subjected to biaxial bending

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DOI: https://doi.org/10.5169/seals-15719

An Experimental-Analytical Study of Complete Load-Deformation Characteristics of Concrete Compression Members Subjected to Biaxial Bending

Etude analytique et expérimentale de la relation charge-déformation de pièces comprimées en béton armé soumises à une flexion biaxiale

Eine rechnerische und versuchstechnische Untersuchung über die Beziehungen zwischen Last und Verformung von Druckgliedern unter schiefer Biegung

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INTRODUCTION

This investigation was aimed at studying the "complete" behaviour of short reinforced concrete pin-ended columns subjected to biaxial bending moments as the applied compressive loads were increased from zero until failure which was defined as the stage at which spalling of concrete took place accompanied by buckling of the steel reinforcing bars. This paper discusses briefly the mathematical formulation leading to the computer program besides reviewing the experimental procedure. The experimental and computed load-deflection curves are compared for a symmetrically loaded column subjected to biaxial bending.

THEORETICAL ANALYSIS

A numerical analysis was developed by the writers (Ref.1,2) to determine strain and curvature distributions in any structural concrete section subjected to biaxial bending moment and axial compression. This analysis can account for any given section geometry and material properties. Member cross-section is divided into several small elements and the stress resultants P, M_x and M_y on this section can be expressed as function of ϕ_x, ϕ_y and ε_p given by the following equations (see Fig.1):

$$P = P(\phi_x,\phi_y,\varepsilon_p) \quad (1a), \qquad M_x = M_x(\phi_x,\phi_y,\varepsilon_p) \quad (1b),$$

and $$M_y = M_y(\phi_x,\phi_y,\varepsilon_p) \quad (1c)$$

where P = axial force
M_x = bending moment about the x-axis
M_y = bending moment about the y-axis
ε_p = uniform direct strain due to an axial load P
ϕ_x = the curvature produced by the bending moment component M_x and is considered positive when it causes compressive strains in the positive y-direction, and
ϕ_y = the curvature produced by the bending moment component M_y and is considered positive when it causes compression in the positive x-direction.

The strain ε_k across any element k can be assumed to be uniform and since plane sections remain plane during bending

$$\varepsilon_k = \varepsilon_p + \phi_x y_k + \phi_y x_k \qquad (2)$$

where x_k and y_k are coordinates of the centroid of the element k.

Having established the strain distribution across the cross-section, the axial force P and the bending moment components M_x and M_y can be calculated using the following equations:

$$P_{(c)} = \sum_{k=1}^{n} f_k a_k \quad (3a), \quad M_{x(c)} = \sum_{k=1}^{n} f_k a_k y_k \quad (3b), \quad M_{y(c)} = \sum_{k=1}^{n} f_k a_k x_k \qquad (3c)$$

Subscript (c) indicates values of P, M_x and M_y calculated in an iteration cycle, and a_k is the area of element k. The values of P, M_x and M_y can be estimated using the Taylor's theorem from the values of $P_{(c)}$, $M_{x(c)}$ and $M_{y(c)}$ from equations (3) as follows:

$$P = P_{(c)} + \frac{\partial P_{(c)}}{\partial \phi_x}\delta\phi_x + \frac{\partial P_{(c)}}{\partial \phi_y}\delta\phi_y + \frac{\partial P_{(c)}}{\partial \varepsilon_p}\delta\varepsilon_p \qquad (4a)$$

$$M_x = M_{x(c)} + \frac{\partial M_{x(c)}}{\partial \phi_x}\delta\phi_x + \frac{\partial M_{x(c)}}{\partial \phi_y}\delta\phi_y + \frac{\partial M_{x(c)}}{\partial \varepsilon_p}\delta\varepsilon_p \qquad (4b)$$

$$M_y = M_{y(c)} + \frac{\partial M_{y(c)}}{\partial \phi_x}\delta\phi_x + \frac{\partial M_{y(c)}}{\partial \phi_y}\delta\phi_y + \frac{\partial M_{y(c)}}{\partial \varepsilon_p}\delta\varepsilon_p \qquad (4c)$$

The values $\delta\phi_x$, $\delta\phi_y$ and $\delta\varepsilon_p$ are increments in ϕ_x, ϕ_y and ε_p required to produce changes δP, δM_x and δM_y respectively. The partial derivatives $\frac{\partial P_{(c)}}{\partial \phi_x}$, ...etc. are the rates of change of P, M_x and M_y with ϕ_x, ϕ_y and ε_p.

These partial derivatives in equations (4) are replaced by the corresponding difference quotients; and by suitably incrementing each deformation quantity at a time, the rates of change can be evaluated and substituted in equations (4). The resulting simultaneous equations (4) are solved for $\delta\phi_x$, $\delta\phi_y$ and $\delta\varepsilon_p$ and these increments are added to the initial deformations and the process is repeated using the new deformation values until convergence is obtained. For more details, the reader may refer to Reference 2 and 3.

The central deflections δ_{2x} and δ_{2y} along x and y-directions respectively (see Fig.2) were calculated using the following equation derived from a suitable modification of the moment-area theorems to account for behaviour non-linearities.

$$\delta_{2x} = \frac{\phi_y l^2}{8} \quad (5a) \qquad \delta_{2y} = \frac{\phi_x l^2}{8} \quad (5b)$$

The axial load P_3 for each loading step was calculated using the value of P computed and modified for the influence of mid-span deflections using the following equations:

(a) Loading Condition I (see Fig.3a)

$$P_3 = \frac{P(e_x^2 + e_y^2)^{1/2}}{[(e_x + \delta_{2y})^2 + (e_y + \delta_{2x})^2]^{1/2}} \quad (6a)$$

(b) Loading Condition II (see Fig.3b)

$$P_{3x} = \frac{Pe'_y}{2(e'_y + \delta_{2x})} \quad (6b) \qquad P_{3y} = \frac{Pe'_x}{2(e'_x + \delta_{2y})} \quad (6c)$$

and $P_3 = P_{3x} + P_{3y}$ (6d)

More details can be found in Reference 3.

EXPERIMENTAL PROCEDURE

The test specimens were designed as short, tied columns with a square under-reinforced section (see Fig.4). Eleven specimens were manufactured and tested to study the influence of the variation of eccentricities e_x and e_y and the total longitudinal steel percentage (p + p'). Details of the symmetrically loaded test specimen U-3 and the loading arrangement are shown in Fig.3(b) and 4. The stress-strain curves for the concrete and the reinforcing steel wire D5 are shown in Figs.5 and 6, respectively.

Hard rubber blocks were incorporated in the loading frame to dissipate some of the energy of the loading system. Beyond the ultimate load, this arrangement permitted the application of further deformations which were accompanied with a decrease in the applied loads measured using calibrated load cells.

The average curvatures along the mid-span section of the column were evaluated using two sets of experimental data - one from strain gauges installed on the concrete and the reinforcing steel wire and the other from the demec gauges suitably arranged near the section. Deflections along the x- and y-axes were measured using dial gauges with a least count of 0.001 in. Details of the test procedure, material properties, etc. along with the experimental data can be found in Reference 3.

CURVATURE EVALUATION

Experimental observation shows that the strain distributions across the mid-span section along the x and y-direction are very nearly linear, therefore curvature in either direction is given by

$$\phi = \frac{\varepsilon_c + \varepsilon_s}{d} \tag{7a}$$

where ε_c and ε_s are strains in the concrete and the reinforcing steel respectively and d is the distance between the points where ε_s (steel strain in tension reinforcement in x- or y- directions) and ε_c (the extreme concrete compressive strains in x- or y-directions) are measured. After significant cracking, it was observed from the strain gauges that the strain distribution in the compression block became non-linear. Similarly demec gauge results indicated a non-linear strain distribution across the entire section. Average curvature can be approximated by the equation

$$\phi = \frac{\varepsilon_c}{kd} \tag{7b}$$

where kd is the distance between the point where ε_c is measured and the point of zero strain and in x- or y-directions.

COMPARISONS OF EXPERIMENTAL AND ANALYTICAL RESULTS

The cross section was idealized as shown in Fig.7. The theoretical and experimental biaxial moment-curvature curves across the mid-span sections for the symmetrically loaded specimen U-3 (Fig.4) are shown in Fig.8. (e'_x = 17.78 cm. e'_y = 17.78 cm.) The experimental moment-curvature relationships were obtained until either strain gauges became damaged or demec points became dislodged while the theoretical values were computed until the maximum moment capacity. Fig.9 shows the axial load P_3 - central deflection relationship at mid-span of the column. The theoretical values were computed up to the stage when the maximum moment capacity was attained while the experimental values were measured up to the collapse or buckling of the reinforcement. More test data on symmetrically and unsymmetrically loaded specimens can be found in Reference 13.

CONCLUSIONS

The above theoretical analysis indicates that it is possible to predict biaxial moment-curvature and load-deflection curves up to the maximum moment capacity of the column specimen. Also the use of hard rubber blocks in column compression tests makes it possible to measure the complete biaxial moment-curvature and load-deflection curves up to the failure stage as defined earlier. The analysis in this paper has resulted in the evaluation of the flexural rigidity coefficients for members of three-dimensional structural concrete frames and can be incorporated without much difficulty into the existing computer programs for analysis of three-dimensional framed structures.

ACKNOWLEDGEMENTS

The writers gratefully acknowledge the financial support of the Department of Education, Government of Quebec, the National Research Council of Canada and the Canada Emergency Measures Organization to continuing studies of Structural Concrete Systems at McGill University. The results reported in this paper form a part of Mr. Hsu's Ph.D. thesis.

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SUMMARY

The above theoretical analysis indicates that it is possible to predict biaxial moment-curvature and load-deflection curves up to the maximum moment capacity of the column specimen. Also the use of hard rubber blocks in column compression tests makes it possible to measure the complete biaxial moment-curvature and load-deflection curves up to the failure stage as defined earlier. The analysis in this paper has resulted in the evaluation of the flexural rigidity coefficients for members of three-dimensional structural concrete frames and can be incorporated without much difficulty into to existing computer programs for analysis of three-dimensional framed structures.

RESUME

L'étude théorique rend possible la détermination des courbes moment-courbure et charge-déformation jusqu'à la charge ultime de la colonne considérée. L'utilisation d'éléments en caoutchouc durci lors d'essais de colonnes comprimées rend possible la mesure intégrale des courbes moment biaxial-courbure et charge-déformation jusqu'à l'état de rupture mentionné plus haut. L'étude présentée ici a permis l'évaluation des coefficients de rigidité à la flexion d'éléments de cadres tridimensionnels en béton armé; elle peut être incorporée sans grandes difficultés aux programmes existants d'ordinateur pour l'étude des structures tridimensionnelles.

ZUSAMMENFASSUNG

Die vorliegende Untersuchung zeigt, dass es möglich ist, die Beziehungen zwischen Biegemoment und Krümmung sowie zwischen Last und Verformungen bis zur Biegetragfähigkeit der Versuchsstützen auch für schiefe Biegung theoretisch vorauszusagen. Die Verwendung von Gummiblöcken für die Lagerung der Versuchsstützen gestattet die vollständige Beobachtung dieser Beziehungen bis zum Bruch. Das vorgelegte Berechnungsverfahren führt zu Koeffizienten für die Ermittlung der Biegefestigkeit von Bauteilen in räumlichen Tragwerken und kann leicht in bestehende Computer-Programme eingebaut werden.

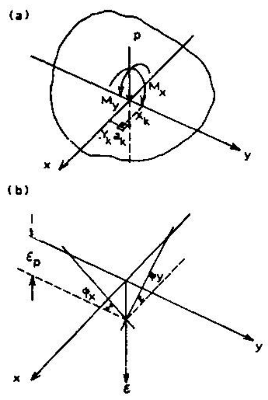


FIG.1 IDEALIZATION OF A CROSS-SECTION SUBJECTED TO BIAXIAL BENDING AND AXIAL COMPRESSION

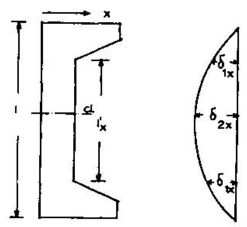


FIG.2 BIAXIALLY LOADED COLUMN

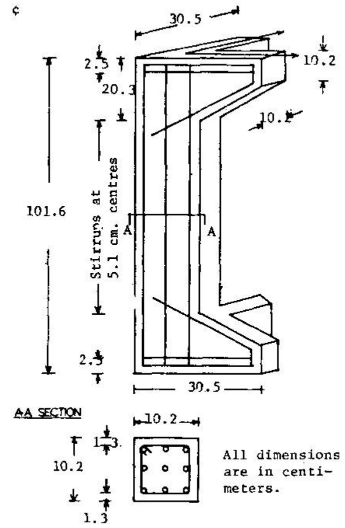


FIG.4 DETAILS OF SPECIMEN U-3

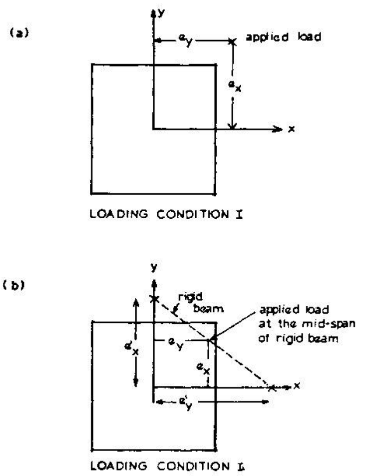
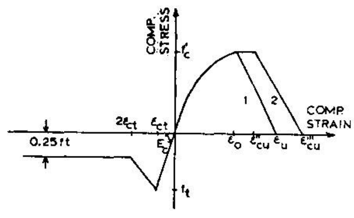


FIG.3 LOADING CONDITIONS FOR BIAXIALLY LOADED SHORT COLUMN



1: Unconfined concrete
2: Confined concrete

FIG.5 IDEALIZED STRESS-STRAIN CURVE

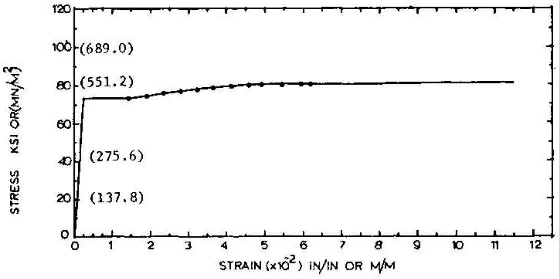


FIG.6 STEEL STRESS-STRAIN CURVE FOR D-5 DEFORMED BAR

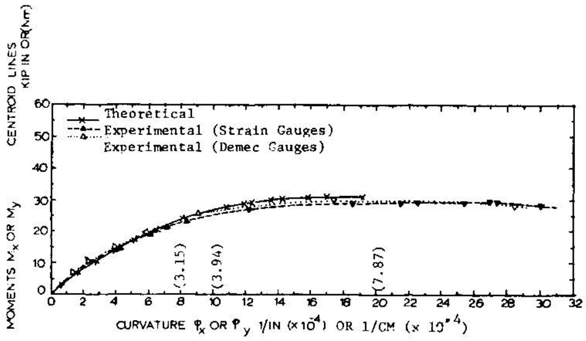


FIG.8 BIAXIAL MOMENT-CURVATURE CURVES FOR SPECIMENT U-3

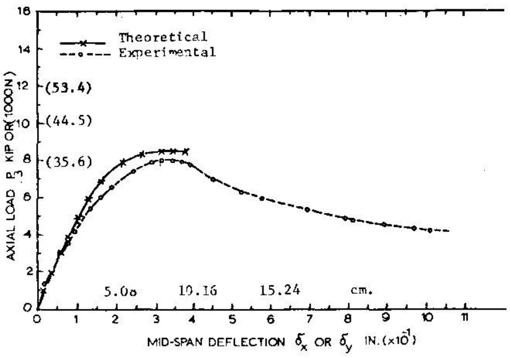


FIG.9 LOAD-DEFLECTION CURVES FOR SPECIMEN U-3

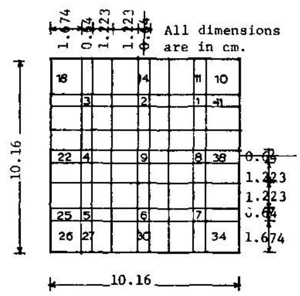


FIG.7 BIAXIALLY LOADED COLUMNS: SPECIMEN U-3