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Method of Analysis for Cyclically Loaded R.C. Plane Frames Including Changes in Geometry and Non-Elastic Behavior of Elements under Combined Normal Force and Bending

Méthode d'analyse de cadres plans en b.a. chargés cycliquement, comprenant les variations de géométrie et le comportement non-élastique d'éléments soumis à effort normal et à flexion composés

Untersuchungsmethode für zyklisch belastete ebene Stahlbeton-Rahmen einschliesslich der Geometrie-Änderungen und des nicht-elastischen Verhaltens von Elementen unter zusammengesetzten Axial- und Biegungskräften

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INTRODUCTION. The advances gained by structural analysis, coupled with the availability of large capacity computers, could lead to the idea that today the "exact" solution of any structural problem exists, and its obtainment is only a matter of assembling in a program the appropriate ingredients, all of them already well established.

This is not the case for reinforced concrete structures in non linear range: neither the constitutive laws of the materials nor the behavior of the structural elements can be said to be conveniently clarified. The method that will be briefly exposed in the following (°) includes most of the ingredients necessary to be defined as "general", but nevertheless its classification as "exact" is justified only within certain simplifying hypotheses that are listed below without comment:

- the constitutive laws of the materials are independent of time
- the contribution of concrete in tension is disregarded
- linear distribution of strain along the depht of section is assumed, excluding bond slip during all the loading history, and local buckling of steel bars
- properties of the materials are assumed not to deteriorate after repeated stressing, while Bauschinger effect on steel is considered
- actions of shear stresses are disregarded

GENERAL DESCRIPTION OF THE METHOD. The procedure follows an incremental way. Each step requires solution of the set of equilibrium equations:

$$|K|_s \{\Delta S\}=\{\Delta P\} \qquad (1)$$

The vectors $\{\Delta S\}$ and $\{\Delta P\}$ contain the increments of nodal displacements and external loads: both have $3n$ (n=number of nodes) dimension and are referred to a global coordinate system x,y .

(°) A more extended illustration is contained in ref. |11|

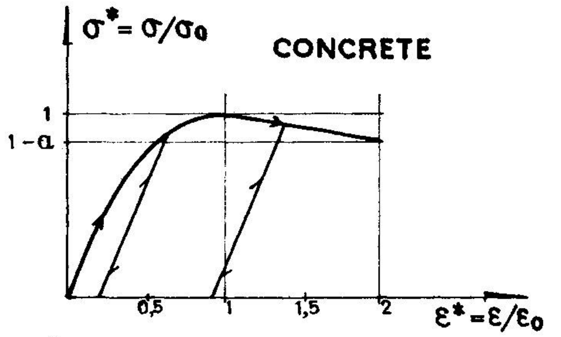
The geometrical and mechanical behavior of the structure in the course of each step is described by the stiffness matrix $|K|_s$, which is the result of the assembly of the stiffness matrices $|K|$ determined for every individual member in which the structure has been discretized.

The essence of the method lies evidently on the calculation of matrix $|K|_s$ for each step. The procedure requires an iteration, which ends when two coincidental successive solutions $\{\Delta S\}$ are obtained for the same $\{\Delta P\}$ from Eqs. (1).

Two causes of non-linearity are contained in $|K|_s$: behavior of materials and variations in the geometry of the structure. They can be analized separately, and so will they be presented.

NON LINEARITY DUE TO INELASTIC BEHAVIOR OF THE ELEMENTS.

Constitutive laws of the materials. The calculation of the stresses σ for given ε is performed in two separate subroutines so that any particular law can be inserted simply; figs.1 and 2 show the laws presently adopted.



$$\begin{cases} \sigma^* = \varepsilon^*(2-\varepsilon^*) & \varepsilon^* \leq 1 \\ \sigma^* = \sigma^*(1-\alpha\varepsilon^*+\alpha) & \varepsilon^* > 1 \end{cases}$$

FIG.1

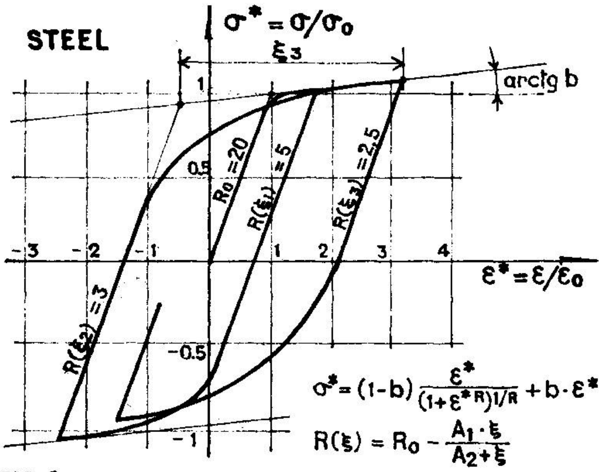
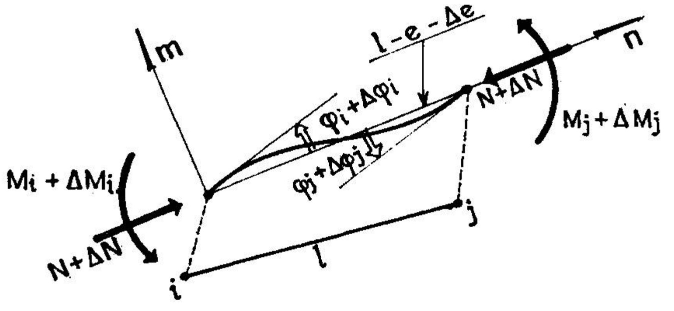


FIG.2

The diagram in fig.1 reproduces the well known Hognestad formula, extended to general stress paths with straight unloading or reloading lines, parallel to the initial tangent. The law for the steel is described in detail elsewhere |7|. It can reproduce with good approximation the behavior of different types of steel: the constant b defines the slope of the hardworking line; the exponent R, which varies after every inversion, affects the curvature of the diagram, to represent the Bauschinger effect.

Stiffness matrix of an element. A matrix $|K_n|$ defines the relationship between the three nodal force components (incremental), and the three nodal "deformations" $\{\Delta S_n\}$. This relationship is established in the intrinsic coordinate system m,n. Elements must be such, that 2° order effects within them are negligible (see fig.3).

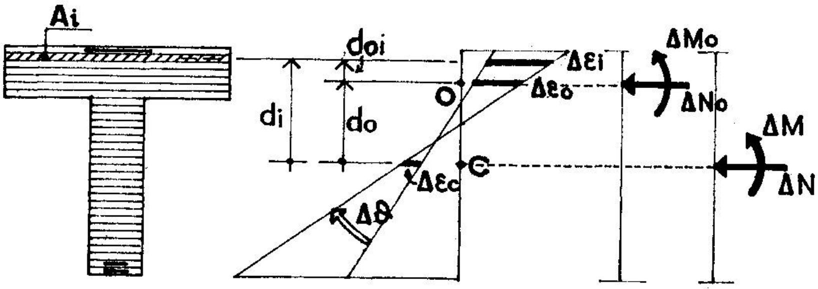
First the flexibility matrix of the element is determined. The columns of the three-by-three $|K_n|^{-1}$ contain the increments of nodal deformations versus the increments of the components of $\{\Delta F_n\}$. The coefficients are calculated by numerical integration along the length of the element, applying separately the three components of $\{\Delta F_n\}$ but considering the deformability due to their simultaneous application. The matrix $|K_n|^{-1}$ is a linearization of a non-linear step: its coefficients are the exact ones, only if at the end of the step the values of the nodal increments $\{\Delta F_n\}$ result to be equal to those applied in calculating the matrix itself.



$$\{\Delta F_n\} = \begin{bmatrix} \Delta N \\ \Delta M_i \\ \Delta M_j \end{bmatrix} \qquad \{\Delta S_n\} = \begin{bmatrix} \Delta e \\ \Delta \varphi_i \\ \Delta \varphi_j \end{bmatrix}$$

FIG.3

To perform the integration along the element, the current deformability of a suitable number of internal cross-sections must be determined, which is defined by the relationship: $\{\Delta\varepsilon\}=|E|\ \{\Delta\Sigma\}$ (see fig.4)



$$\{\Delta\varepsilon\} = \begin{Bmatrix} \Delta\varepsilon_c \\ \Delta\theta_c \end{Bmatrix} \qquad \{\Delta\Sigma\} = \begin{Bmatrix} \Delta N \\ \Delta M \end{Bmatrix}$$

$$\{\Delta\varepsilon_o\} = \begin{Bmatrix} \Delta\varepsilon_o \\ \Delta\theta_o \end{Bmatrix} \qquad \{\Delta\Sigma_o\} = \begin{Bmatrix} \Delta N_o \\ \Delta M_o \end{Bmatrix}$$

FIG.4

where matrix $|E|$ of the section is generally variable from step to step. The cross-section is subdivided into a number of concrete and steel areas: the path of every area in the space σ-ε must be recorded, because it determines the behavior of the area for the load history.

Provided that $\Delta\sigma_i$ and $\Delta\varepsilon_i$ are the actual stress and strain variations due to application of combined ΔM and ΔN on the i-th concrete or steel area, its secant modulus E_i for the current step will be defined: $E_i = \frac{\Delta\sigma_i}{\Delta\varepsilon_i}$. Thus, the cross-section can be treated as if composed by elastic parts with varied moduli. The following relations can be written, referring to the current homogenized cross-section centroid 0:

$$\begin{vmatrix} \Delta\varepsilon_o \\ \Delta\theta_o \end{vmatrix} = \begin{vmatrix} \frac{1}{\Sigma A_i E_i} & 0 \\ 0 & \frac{1}{\Sigma A_i E_i d_{oi}^2} \end{vmatrix} \cdot \begin{vmatrix} \Delta N_o \\ \Delta M_o \end{vmatrix} \qquad (2)$$

which, transferring the vectors $\{\Delta\varepsilon_o\}$, $\{\Delta\Sigma_o\}$ to the fixed geometric center C, become:

$$\begin{vmatrix} \Delta\varepsilon_c \\ \Delta\theta_c \end{vmatrix} = \begin{vmatrix} (\frac{1}{A}+\frac{S}{A^2 J}) & -\frac{S}{AJ} \\ -\frac{S}{AJ} & -\frac{1}{J} \end{vmatrix} \cdot \begin{vmatrix} \Delta N \\ \Delta M \end{vmatrix} \qquad \begin{aligned} A &= \Sigma A_i E_i \\ S &= \Sigma A_i E_i d_i \\ J &= \Sigma A_i E_i d_i^2 - \frac{S^2}{A} \end{aligned} \qquad (3)$$

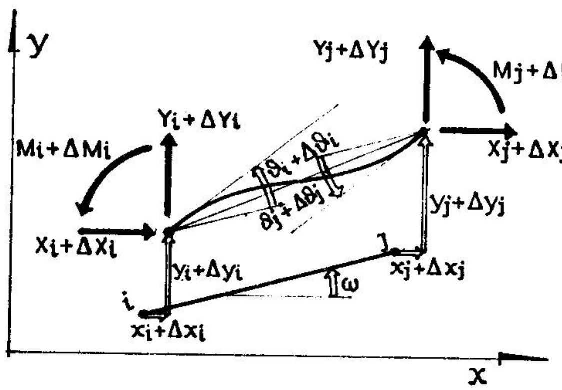
During the step, a "general" iteration is performed to obtain the solution of equations (1). Within each cycle of it, for each element every cross-section is given a vector$\{\Delta\Sigma\}$, corresponding to the proposed $\{\Delta F_n\}$, and, by "internal" iterations, relations (3) are solved, yielding matrix $|E|$. Thus, the general iteration proceeds dealing always whith deformabilities updated with actual state and history of stress.

It may be noted that matrices $|E|$are symmetrical. The same property consequently extends to all $|K_n|^{-1}$,$|K|$and to $|K|_s$ matrices.

NON LINEARITY DUE TO CHANGES IN GEOMETRY. Finite deflections of elastic plane frames have been the object of recent extensive studies|8| |10|,|13|so that only a brief account of the relations employed will be reported.

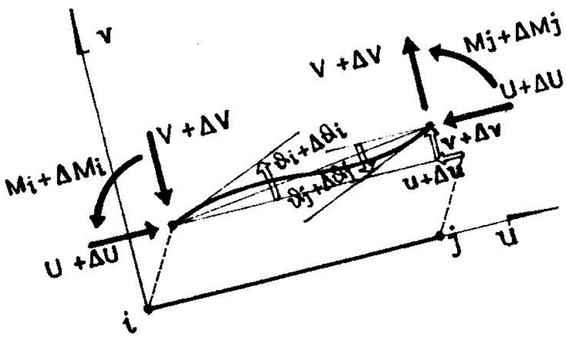
Prior to their assembly into the overall matrix $|K|_s$, matrices $|K_n|$of individual elements must be trasferred from the local system m,n to a global system x,y. When the displacements of the elements during the load history cannot be neglected, the transformation from m,n to x,y is non linear. The technique of linearization adopted is the approximation of a first order differential expansion with finite increments.

In order to pass from the intrinsic to the global system, an intermediate coordinate system u,v is employed.



$$\{\Delta F_x\} = [\Delta X_i \;\; \Delta Y_i \;\; \Delta M_i \;\; \Delta X_j \;\; \Delta Y_j \;\; \Delta M_j]^T$$

FIG.5 $$\{\Delta S_x\} = [\Delta x_i \;\; \Delta y_i \;\; \Delta\vartheta_i \;\; \Delta x_j \;\; \Delta y_j \;\; \Delta\vartheta_j]^T$$



$$\{\Delta F_u\} = [\Delta U \;\; \Delta V \;\; \Delta M_i \;\; \Delta M_j]^T$$

FIG.6 $$\{\Delta S_u\} = [\Delta u \;\; \Delta v \;\; \Delta\vartheta_i \;\; \Delta\vartheta_j]^T$$

For finite displacements, the relationships between the components of the vectors$\{S_u\}$and$\{S_n\}$are:

$$e= l-\sqrt{(l-u)^2+v^2} \qquad \phi_i= \theta_i-arctg\frac{v}{l-u} \qquad \phi_j= \theta_j-arctg\frac{v}{l-u} \qquad (4)$$

By differentiation, the preceding relations can be given the linear form: $\{\Delta S_n\}=|A|\ \{\Delta S_u\}$, with increments in place of differentials.

The relationship between the vectors of nodal forces in the two systems is: $\{F_u\}=|A|'\{F_n\}$; and, by diffentiating:

$$\{\Delta F_u\}= |A|'\{\Delta F_n\}+|D|\ \{\Delta S_u\} \qquad (5)$$

The matrices $|A|$ and $|D|$ have the expressions:

$$|A| = \begin{vmatrix} \frac{l-u}{l-e} & \frac{-v}{l-e} & 0 & 0 \\ \frac{-v}{(l-e)^2} & \frac{-(l-u)}{(l-e)^2} & 1 & 0 \\ \frac{-v}{(l-e)^2} & \frac{-(l-u)}{(l-e)^2} & 0 & 1 \end{vmatrix} \tag{6}$$

$$|D| = \begin{vmatrix} d_{11} d_{12} & 0 & 0 \\ d_{22} & 0 & 0 \\ (symm) & 0 & 0 \\ & & 0 \end{vmatrix} \quad \begin{aligned} d_{11} &= \frac{1}{(l-e)^4} \,|v^2(l-u)\cdot U + \{2(l-u)^2+v^2\}vV| \\ d_{12} &= \frac{1}{(l-e)^4} \,|-v^3U+(l-u)^3V| \\ d_{22} &= \frac{1}{(l-e)^4} \,|(l-u)\{(l-u)^2+v^2\}U - v(l-u)^2V| \end{aligned}$$

Both matrices contain in their coefficients the components of the displacements accumulated in the previous history; the "geometric"matrix $|D|$ contains also the accumulated force components U and V.

The transformation of displacements and forces from u,v to x,y is accomplished by a matrix with constant coefficients $|T|$, defining the initial position of the element (through its angle ω with x axis).

Making substitutions, the stiffness relation $|K_n|\ \{\Delta S_n\}=\{\Delta F_n\}$ becomes: $|K|\ \{\Delta S_x\}=\{\Delta F_x\}$, with:

$$|K| = (|T|'\ |A|'\ |K_n|\ \ |A|\ \ |T|) + |T|'\ |D|\ \ |T| \tag{7}$$

The matrix $|K|$contains in $|K_n|$the linearization of the mechanical behavior, and in $|A|$and $|D|$the linearization of the kinematical behavior. It has to be noted a difference between the two criteria of linearization. The matrix $|K_n|$has a "secant" character, in what its coefficients are checked with the situation at the end of each step. Taking into account possible inversions of strain variations in the areolas from step to step, this matrix could not be expressed as "tangent". As much as $|K_n|$is concerned, the lenght of the step would be only limited by the possibility of missing strain inversions during the step itself.

The matrices $|A|$and $|D|$on the other hand, by the way they have been obtained, are "tangent initial" since their coefficients are calculated with the values of variables at the beginning of each step. Therefore the lenght of the step has to be commensured to the importance of the effects of changes in geometry.

EXAMPLE. It has been chosen as example a frame tested by Ferguson and Breen (Ref.|5|). Here the frame has been subdivided in 12 elements and the loading paths of 3960 concrete and 264 steel areas were recorded. The constants adopted for the materials were (kg/cm^2): concrete: σ_0= 280, initial modulus E=250000, $\sigma(3,8‰)=0.85\cdot\sigma_0$; steel: σ_0=3850, in E= 2050000, b=0.02, R_0=20. In fig. 7 experimental and theoretical curves are compared: the agreement is excellent. The curve(c) shows the first-order non elastic analysis of the same problem.Ultimate stress diagrams of two cross-sections appear in fig.9, while fig.8 shows the calculated deflected shape in case (b).

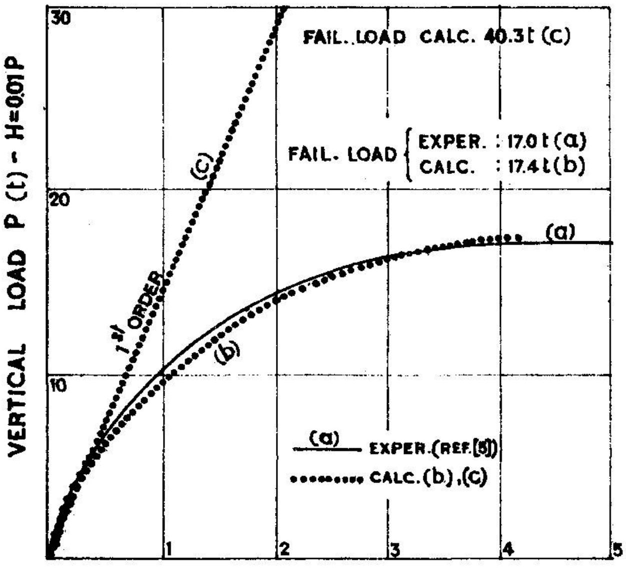


FIG.7 DEFLECTION - NODE 5 (cm)

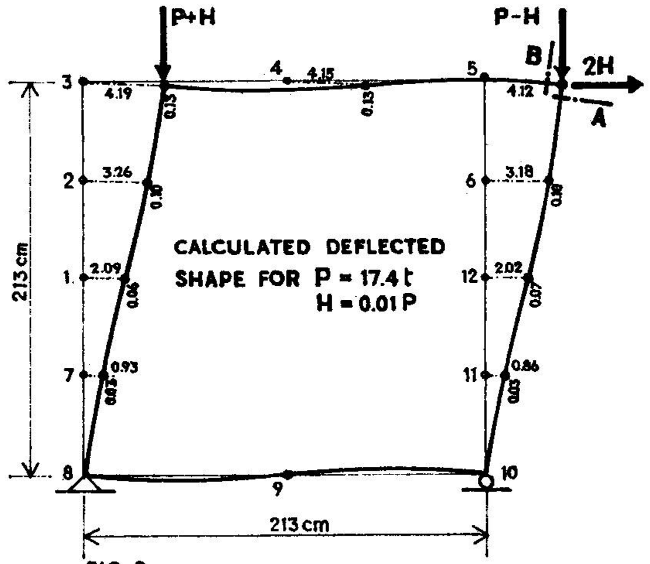


FIG.8

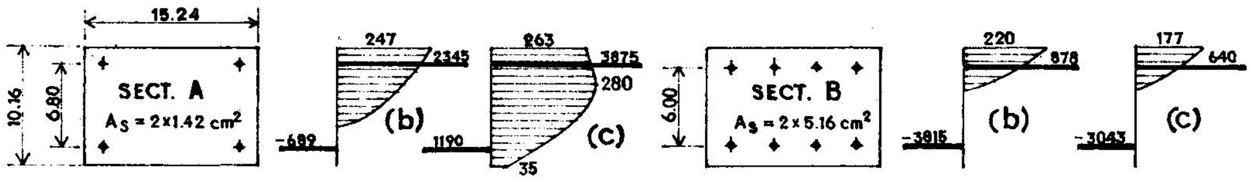


FIG.9 STRESS DIAGRAMS AT FAILURE (CALC.) Kg/cm²

The same frame has been submitted to two cyclical horizontal loadings,(case *d*): *P*=o, case *e*): *P*=6t). The influence of *P* is relevant both in($2H$-δ)and(M-θ)diagrams. Case *e*) has higher moments but overall plasticization is very low and the frame fails for instability.The collapse in case *d*)is due to attainment of limit strains on concrete.

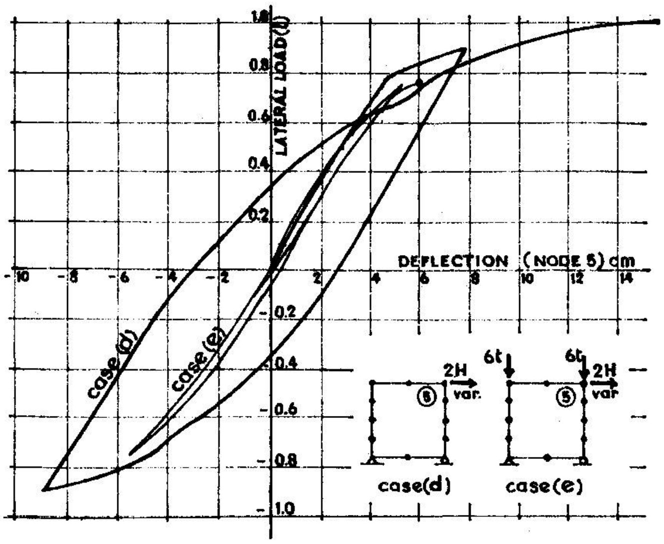


FIG.10 - HOR. LOAD - DEFLECTION (NODE 5)

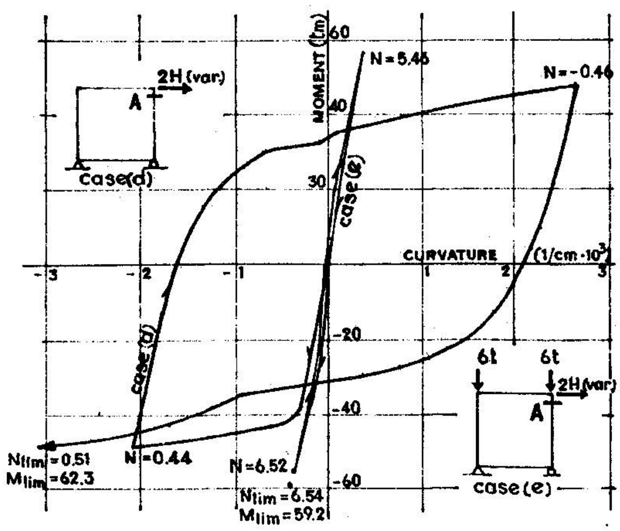


FIG.11 - MOMENT - CURVATURE (SECT. A)

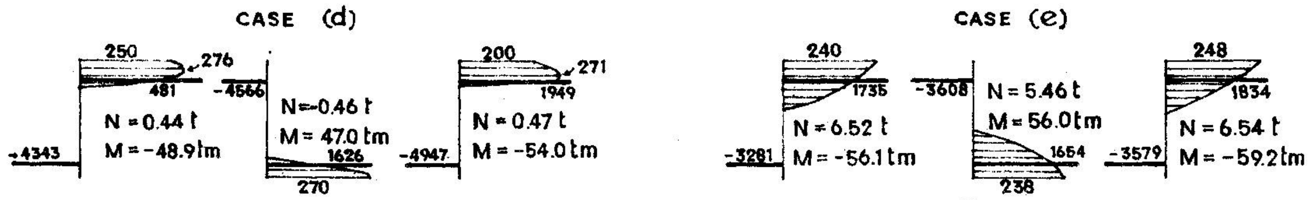


FIG. 12 HISTORY OF STRESSES ON SECTION A (kg/cm²)

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SUMMARY

The procedure is based on the stiffness method in incremental way, and has been programmed for computer. The solution is obtained by the calculation of a linearized stiffness matrix for every increment. The element matrices derive from an analysis of the behavior of several internal cross-sections: these are discretized into concrete and steel areolae, for each of which the loading path is recorded, following any non-holonomic constitutive law. Geometric effects are included in the overall matrix by adjusting the projection of elements displacements and forces at every step.

RESUME

Le procédé se base sur la méthode des déformations appliquée par incréments, et a été programmé pour ordinateur. La solution s'obtient en calculant une matrice de rigidité linéarisée à chaque incrément. Les matrices des éléments dérivent de l'analyse du comportement d'un certain nombre de sections: celles-ci sont discretisées en aréoles de béton et d'acier, dont l'histoire de charge est mémorisée, suivant une loi constitutive quelquonque. Les effets géometriques sont inclus dans la matrice d'ensemble, par le réglage de la projection des déplacements et des forces des éléments à chaque pas.

ZUSAMMENFASSUNG

Das Verfahren stützt sich auf die Steifigkeitsmethode auf inkrementalem Wege und wurde auf dem Computer programmiert. Die Lösung ergibt sich durch Berechnung einer linearisierten Steifigkeitsmatrix für jeden Zuwachs. Die Elementmatrizen folgen aus der Analyse des Verhaltens einer bestimmten Anzahl innerer Querschnitte: diese sind in Beton- und Stahlareolen diskretisiert, deren Belastung nach einem konstitutiven Gesetz aufgezeichnet wurde. Die geometrischen Aenderungen sind in der Gesamtmatrix durch Anpassung der Projektion der Elementverschiebungen und -kräfte bei jedem Schritt inbegriffen.