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## On the Safety of Steel Members against Buckling under Random, Reversible Loads

De la sécurité des barres métalliques soumises au flambement et à des charges alternées aléatoires

Zur Frage der Sicherheit von Stahlstäben unter zufälliger Wechselbeanspruchung

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### 1. INTRODUCTION

Design rules for compressed steel members are nowadays based on extensive experimental and numerical evidence on the ultimate buckling strength of real, industrially fabricated bars upon virgin loading [1]. If full or partial stress reversal may occur in the member, that is not the only possible ultimate state. As Klöppel and Klee have shown for steel St 37 [2], only 1.5 cycles at a reversed plastic strain of 1 ‰ will lower the yield plateau by 13 %. Beside this strain softening at low strain levels which affects the extreme points of the hysteretic loop, the proportional limit inside the loop is lower than in the virgin loading (Bauschinger effect). At least theoretically, we may thus expect buckling collapse at lower loads after the first plastic excursion in tension. On the other hand, tensile plastic strains may reduce the most important imperfection of the member : initial crookedness ; and sequences of alternating high loads have very small probabilities in most civil engineering structures.

### 2. SIMPLIFYING ASSUMPTIONS

The member considered has an idealized two point section and a sine-shaped initial crookedness (Fig. 1). Its amplitude - known to be 1/1000 or smaller in industrial bars [1] - is used as a fixed parameter.

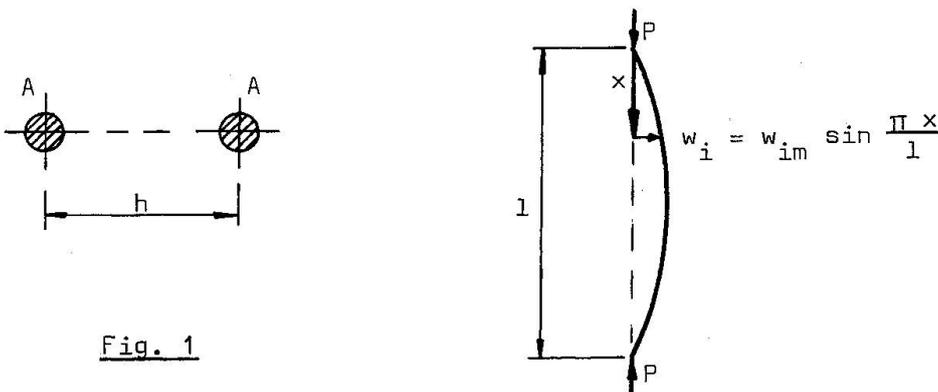


Fig. 1

A bilinear stress-strain relation (Fig. 2) is used for the flanges. Informations on the parameters  $\gamma$  and  $\sigma_p$  are obtained as follows: in the range between 1.5 and 10.5 cycles at plastic strains below 3‰, the extreme point of the hysteretic loop lies on a yield plateau with  $\sigma_c = 0.87 \sigma_y$ , where  $\sigma_y$  denotes the yield stress in the virgin state [2].

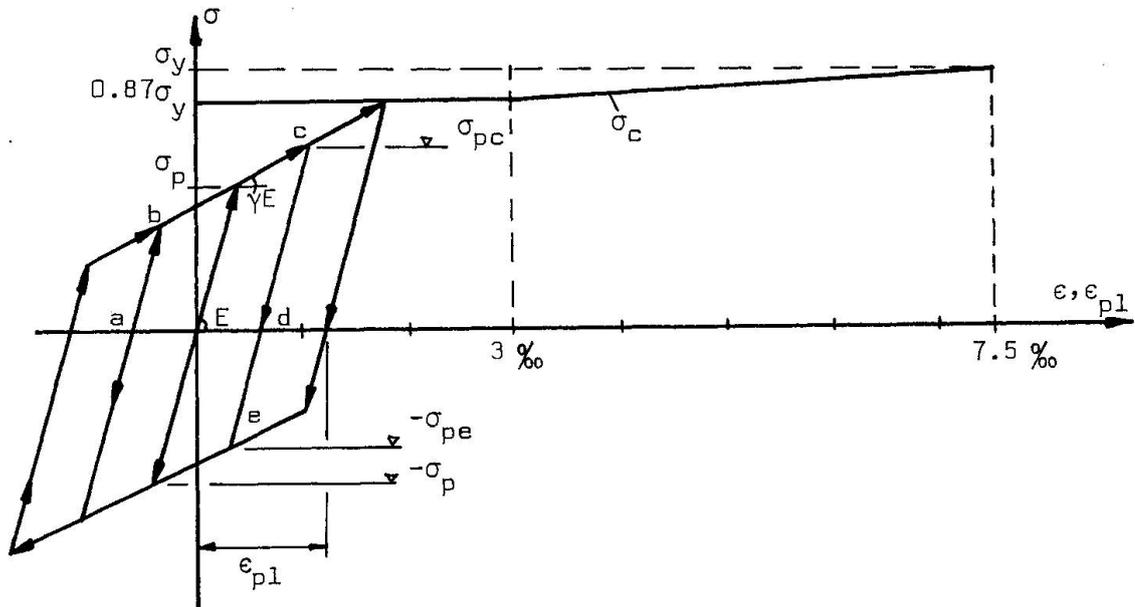


Fig. 2

Following Morrow [3], the dissipation energy spent in one cycle is

$$\Delta A = 4 \frac{1-n'}{1+n'} \epsilon_{pl} \sigma_c$$

where  $n' \sim 0.15$ , independently of the cycle number. In our simplified bilinear loop

$$\Delta A_B = 4 \sigma_p \epsilon_{pl}$$

so that  $\Delta A = \Delta A_B$  leads to

$$\sigma_p = \frac{1-n'}{1+n'} \sigma_c \sim 0.74 \sigma_c$$

The parameter  $\gamma$  is then, by a geometric argument

$$\gamma = \frac{1}{\frac{E \epsilon_{pl}}{\sigma_c - \sigma_p} + 1}$$

Strictly speaking, it would be necessary to iterate, in each load step and at each point of the numerical solution scheme on both flanges, for the correct value of  $\gamma$  associated with the actual  $\epsilon_{pl}$ . Instead, a fixed value of  $\gamma$  is used, based on approximate  $\epsilon_{pl}$  before collapse. The assumed value  $\gamma$  is conservative if no stresses above  $\sigma_c$  occur before collapse.

### 3. NUMERICAL SOLUTION OF THE DETERMINISTIC PROBLEM

Numerical solution follows the well known Engesser-Vianello procedure for each load step. The initial crookedness for a step is the residual deflection from the previous step. If at a point on one of the two flanges stress

history of the previous step was a-b-c-d, (Fig. 2), proportional limits in the bilinear law of the next step will be  $\sigma_{pc}$  and  $-\sigma_{pe}$ , at that point. It can be shown that the Engesser-Vianello procedure converges whenever the absolute value of the longitudinal force in the member is smaller than the ultimate load in compression. Thus, the procedure will not converge - and this is confirmed by numerical experience - if a tensile force of larger absolute value than the ultimate compressive force is applied. A possible solution is to use a backward version of the conventional procedure: given initial deflections  $w_i(x)$  and a first guess  $w_1(x)$  of incremental deflections due to P, find incremental curvature

$$\kappa_1 = -w_1''$$

by numerical differentiation and new excentricities

$$e = e(\kappa_1, P)$$

making use of the material properties. An improved approximation

$$w_2 = e - w_i$$

is so obtained.

#### 4. SIMULATION

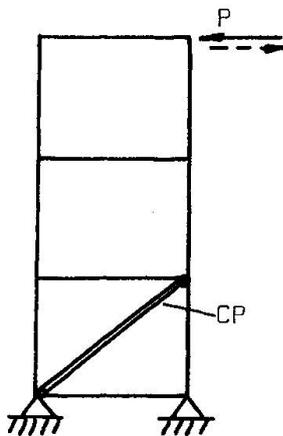


Fig. 3

A stochastic loading scheme containing the probabilities of high absolute values of P and the probability of sign changes is chosen as follows:

$|P|$  has extreme type I distribution

$$\Phi_{|P|} = \exp[-\exp(-\alpha(P-P_0))]$$

The sign of P is subject to a two-state Markov chain with one-step transition probabilities:

	+	-
+	1-Q	Q
-	Q	1-Q

$Q=0$  stands for cases with no sign changes. This is the reference situation of usual design.  $Q=1$  indicates certain stress reversal at each step, as may be the case if P stems from some technical device with fixed operative schedule and random loading.

A value of  $Q=0.5$  may be justified in conjunction with P from wind load extrema. More complex situations, as forces from vibrations or earthquake, with a high correlation between subsequent peaks  $|P|$ , cannot be simulated by this simple scheme, since the values of  $|P|$  are uncorrelated.

Simulation runs for a specific case and  $Q=0, 0.5, 1.0$  are currently in production. Probabilities of failure will be esteemed from a sufficient number of randomly simulated individual histories over a lifetime of 600 load applications (monthly extrema over 50 years), and the differences in failure probability arising between the standart case  $Q=0$  and the case  $Q=0.5$  and  $Q=1.0$  will be tested for statistical significance.

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## SUMMARY

A numerical simulation scheme is set up to test the effect of random stress reversal on the safety against buckling of members with initial out-of-straightness.

## RESUME

On établit un procédé de simulation numérique pour déterminer la probabilité de ruine de barres sujettes au flambement, soumises à des charges alternées aléatoires.

## ZUSAMMENFASSUNG

Es wird ein numerisches Simulationsverfahren angegeben zur Bestimmung der Versagenswahrscheinlichkeit von Knickstäben bei zufällig auftretender Wechselbeanspruchung.