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## The Resistance of Prestressed Concrete to Dynamic Loading. Its Fatigue Resistance; Miner's Hypothesis

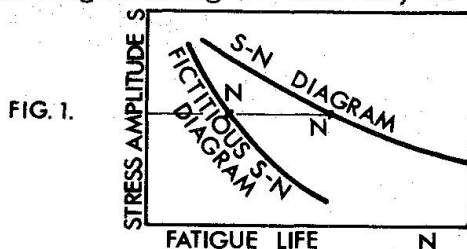
La résistance du béton précontraint à la charge dynamique. Sa résistance de fatigue; hypothèse de Miner

Die Festigkeit von vorgespanntem Beton gegenüber dynamischer Belastung. Seine Ermüdungsfestigkeit; Hypothese von Miner

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### 1. Different Kinds of Dynamic Loading

There are three kinds of dynamic loading: impact, vibration and fatigue which are mostly interconnected. With impact the structure should be very flexible to dissipate energy. In the case of vibration some rigidity is essential and it is important that the natural frequency of the member differs from that of the vibration in order to avoid resonance. Obviously fatigue occurs at the same time. Ferry Borges<sup>1</sup> showed in Fig.9 the Wöhler diagram (known as S-N curve) and in Fig.10 the Goodman diagram (which is based on constant load range). The latter is usually of little importance because of the varying load spectrum, and Miner's hypothesis for the cumulative loading is often considered. Ferry Borges has clearly stated that "Basic safety concepts differ with the various branches of engineering". However, he mentioned in eq.12 that instead of the value "1"



only "0.3" may apply, which is based on experience with aircraft. In this case random loading with heavy impact takes place and, instead of the S-N curve applying to ordinary fatigue, a fictitious curve (Fig.1) ought to be considered, as shown by Freudenthal and Heller<sup>2</sup>, referred to in (1), with smaller values.

### 2. The different load spectra, occurring in Civil Engineering

Generally the impact effect is taken into account in the magnitude of the loading, as e.g. with bridges, and the cumulative effect of the load spectrum during the expected life can be assessed by means of Miner's hypothesis. With road bridges there are many millions of relatively light loadings and a limited amount of heavy loadings<sup>3</sup>. Wind loading is based on the wind speed and the dynamic pressure, dependent on various factors. The cumulative effect due to frequency and magnitude of the wind gusts during the expected life must be considered. Based on the data available, it is a question of calculated risk to assume the suitable safe return period of maximum wind speed during the expected lifetime of a structure. It may also be necessary to consider excitations due to vortex oscillations. Special hurricanes or tornadoes are usually ignored, since their locations, maximum speed and duration are unknown. The design of a

structure subjected to wind can be based on the fatigue resistance of the members, provided the structure is sufficiently stiff to avoid excessive vibration. It is important to investigate aerodynamically the behaviour of a model in a wind tunnel for various wind speeds up to the maximum and to examine the possibility of excessive vibration due to vortex shedding, as the frequency due to a possible excess loading should differ from the natural frequency to avoid resonance. This might not be possible if the structure is too light, but it might be achieved by the provision of suitable dampers when the amplitude of the vibration caused by wind excitation can be reduced to a satisfactory amount by means of energy dissipation. Examples are mentioned under heading 6.

With impact, great flexibility is required and in earthquake districts impact forces acting in opposite direction must be considered with a possibility of dissipating energy, the magnitude of the forces being usually not known. Sufficient amount of displacement must be allowed, which may be reduced by dampers. The cumulative effect in fatigue may be similar to the random loading in aircraft engineering with a reduced Miner's value.

### 3. The behaviour of prestressed concrete

The behaviour of prestressed concrete varies to a great extent, dependent on the relative magnitude of the prestressing force. The Author has shown in 1950<sup>4</sup> that the load deflection curves, shown diagrammatically, comprise 3 stages (except for type 1): elastic rigid, elastic flexible and plastic range, starting at PD (permanent deformation). With ordinary reinforced concrete, the permanent deformation occurs earlier. Different types of under-reinforced beams of the same section may be obtained (Fig.2). This was based on the distinction between full and partial prestressing with relatively large and reduced prestressing forces. Fig.2 also shows deflection curves. It was assumed that the ultimate loads of the 5 types, of gradually increasing displacements, are slightly reduced with decreasing prestressing forces. However, it is now known that they are almost equal. A structure between type 1 (at which cracking and failure may occur simultaneously) and type 2 is most suitable for a structure which must be stiff with minimum deformation (e.g. a turbo foundation to resist heavy vibration). Fig.2 has been incorporated in the Appendix of the "First Report on Prestressed Concrete" of the Institution of Structural Engineers, London 1951<sup>5</sup>. Successful impact tests on flexible masts were reported by the author in 1956<sup>6</sup>.

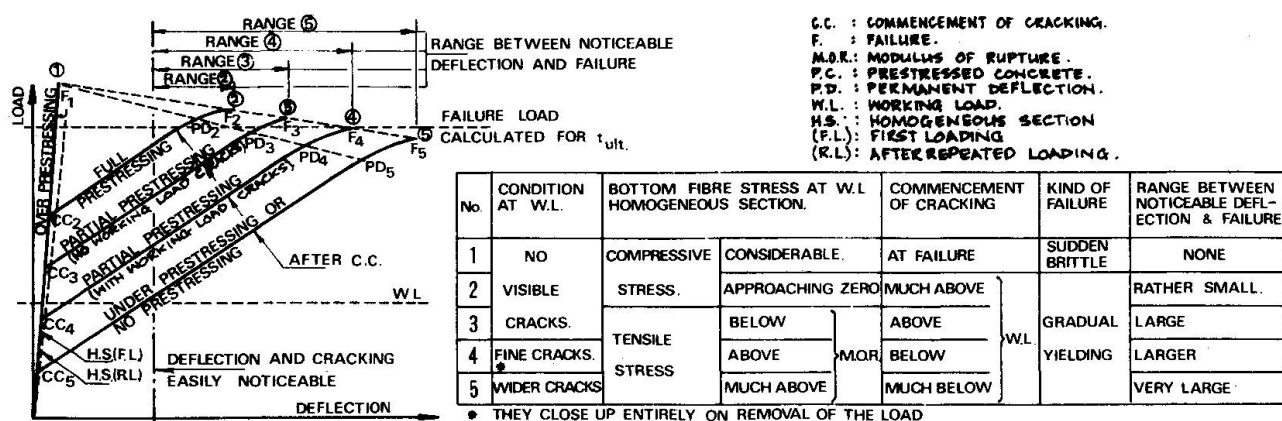


FIG 2: LOAD DEFLECTION CURVES FOR FIVE UNDER-REINFORCED P.C. BEAMS OF THE SAME SECTION.

### 4. The Fatigue Resistance of Prestressed Concrete

The author was actively associated with extensive research from 1951-1969. First the effect of cracking on the fatigue resistance was studied. At tests in Liège 1951, it was ascertained that the static failure load was not reduced by 3 million previous load cycles when cracks had opened and closed 3 million times<sup>6</sup>.

This is illustrated in Fig.3 in which the 3 loading ranges with increasing upper load ranges are seen. Although there is a difference in stiffness between the static load deflection curves before and after each million of load cycles, as seen in (a), this difference is insignificant compared with the general behaviour up to failure, as seen in (b). Fig.4 shows another example of a test result of 1954<sup>7</sup>. First a static loading was carried out until cracks became visible and static loadings were repeated after 583,000 and 747,000 cycles. The permanent sets were very small, but the displacements substantial. Thus the displacements capable of dissipating energy were mainly of elastic nature which is of great advantage at vibrations and earthquake resistance. It should be noted that in both examples tensioned and non-tensioned prestressing wires were provided. In another test almost 9 million load cycles were successfully applied with gradual increasing load range up to 68% of the failure load<sup>8</sup>. In all these cases only the cumulative effect of fatigue was investigated in association with British Railways<sup>9</sup>.

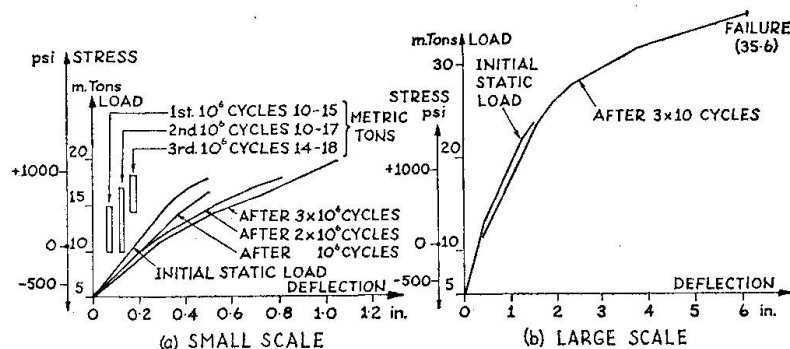


Fig. 3 — Load-deflection Diagram: Fatigue Test, Composite Slab, Liège, 1951.

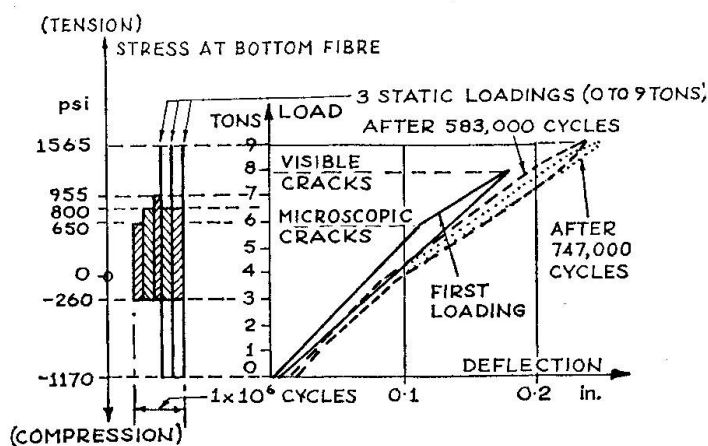


Fig. 4. — Load-deflection Diagram: Fatigue Tests, 1954.

The author was associated with further research at the DUKE University, USA 1965-69. It was the purpose to obtain S-N curves of the fatigue resistance of various prestressed concrete beams, subjected to constant load ranges<sup>10</sup>. A relatively very high fatigue resistance was obtained for constant load ranges between 30% and 70 to 90% of the static failure load, based on the assumption that 30% corresponds to the dead load and the load factor of safety is 1.5 for the dead load and 2.5 for the live load, as was required for bridges, corresponding to a service load of 52% of the static failure load. Also in this case tensioned and non-tensioned tendons (but in this case strands) were provided. Failure commenced due to brittle fracture of some wires of the tensioned strands, as long as the

upper limit did not exceed 85% of the static failure load. With a higher upper limit crushing of the concrete occurred. After the brittle failure of some wires, the beams were capable of taking up an appreciable number of further load cycles before final failure.

Further subject of research was to compare the results obtained from constant load ranges with corresponding stress ranges of the steel itself (tested in the air) and to investigate the applicability of Miner's hypothesis. Some of the test results were presented in 1972<sup>12</sup>. Fig.5 shows the basis of comparison, as derived by Hu. The investigations by Warner and Hulsbos<sup>13</sup>, Tide and Van Horn<sup>14</sup> and Hilmes and Ekberg<sup>15</sup> agree very well and these results can be presented by a simplified S-N curve, shown in Fig.6. It is seen that for a small stress range of 10% (between 40 and 50% of the strength) 10 million load cycles can be

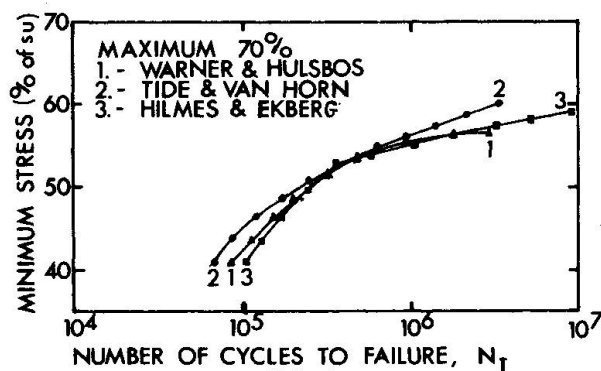


FIG. 5.

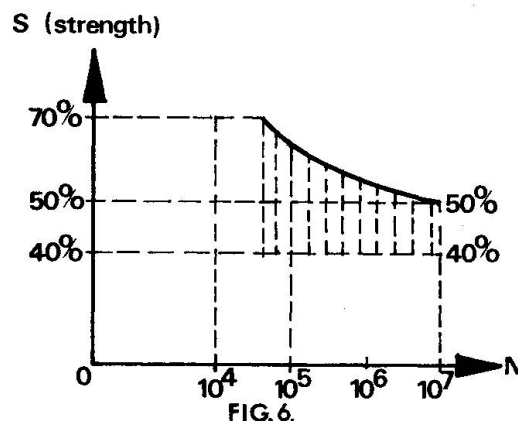


FIG. 6.

expected. However, a corresponding beam test for a constant load range between 30 and 52% of the static failure load withstood only about 2.5 million cycles before failure. Some of the test beams showed results which were relatively much higher than those corresponding to the appropriate stress ranges in the steel, whereas others were much lower. This was apparently the consequence of unsatisfactory bond, which became evident from the crack distribution (great spacing and upper forking)<sup>12</sup>. The test specimens were produced in a prestressing

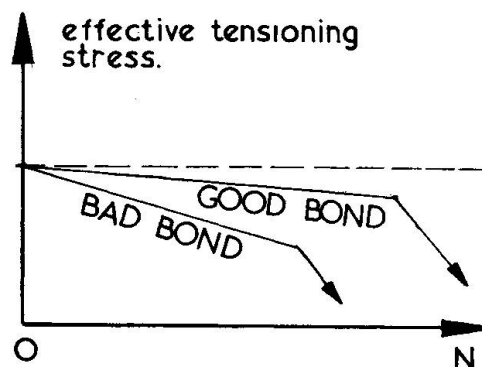


Fig. 7 Reduction in Tension stress during Fatigue.

plant without constant supervision, whereas all the previous specimens had been carefully produced at the Laboratory of the University. From a number of investigations at which strain measurements were made at intermediate static loadings it was ascertained that a gradual loss of the effective prestress had taken place with increasing number of cycles. This loss was greater at the beams with bad bond; at a certain state a rapid reduction in the initial tensioning stress took place, as indicated in Fig. 7. This is a very important result, since it shows that with bad bond the effective prestress decreases, thus in spite of a constant load range increasing stress ranges have to be considered.

Venuti<sup>14</sup> carried out tests which he wanted to base on statistical considerations. He tested e.g. 18 beams to 50% of the static failure load in which case there ought not to have occurred any failure after 5 million cycles, because the stress range in question corresponded to about 10 million cycles. However only 11 beams survived the loading after 5 million cycles and with two of the 7 beams which failed earlier, compression failure took place. At another group of beams the load was increased to 60% of the failure load. In this case one beam sustained 3 million cycles, whereas the remaining beams failed after much shorter fatigue life, the smallest number being only 53,000 cycles (also with compressive failure). These results can be explained similarly to some of the latest tests at Duke University as being caused by insufficient bond, quite different from the excellent behaviour of the previous beam tests<sup>11</sup>.

It would obviously be quite unsatisfactory to have to consider the possibility of such uncertainty, since a prestressed concrete beam with pretensioned tendons must have sufficient bond. Otherwise a completely unsuitable material would be obtained which would be worse than reinforced concrete with usually well anchored steel. However, it may be stated that small test specimens require a more careful vibration than larger members and that generally prestressed concrete members are well produced. Nevertheless the manufacture of prestressed concrete members should be well supervised.



## 5. Miner's Hypothesis

The simple formula by Miner can obviously not be taken as a strict rule but only as a hypothesis. From Fig.1 it is seen that the S-N curve on which it is based needs a revision in the case of heavy impact combined with random loading, as with aircraft loading. However, this does not seem to apply to structural members used in civil engineering, exposed to wind pressure or bridge loading; provided that the effect of impact is duly taken into account in the loading. Obviously, if the bond is insufficient and the effective prestress is gradually reduced owing to loss of bond (Fig.7), it cannot be expected that Miner's value remains valid, since even during constant load ranges the stress range gradually would increase.

The tests<sup>12</sup> have indicated that with satisfactory bond the known test results about the fatigue resistance of the steel in the air can be used as a basis for assessing the fatigue resistance of a prestressed member subjected to the corresponding stress ranges. It has even become evident that with good bond, the fatigue resistance of the prestressing steel in a prestressed concrete member is better than that of the steel in the air. This depends on the design. With good crack distribution it appears understandable that the resistance of the prestressing steel embedded in the concrete is higher than that of the steel in the air. It is hereby assumed that failures at testing of prestressing steel with fractures at the anchorage are excluded, where Miner's rule is investigated. Thus it may be stated that the original value for Miner's hypothesis seems generally to be applicable to fatigue of prestressed concrete members in civil engineering. However, a fictitious S-N curve might have to be considered according to Fig.1, or the Miner value may have to be reduced considerably (say to 0.3) where heavy impact occurs in connection with random oscillations as is the case with aircraft. This might also be applicable to earthquakes.

## 6. Some design notes (based on well-manufactured prestressed concrete)

Bridges: Present design criteria are rather safe. This would allow substantial increase in maximum loads or revision of future design loading (i.e. increase of live load), if based on a combined limit state of serviceability and collapse (i.e. avoiding fatigue failure and excessive deformation).

Structures subjected to wind: Because of the possibility of adjusting its rigidity prestressed concrete is very suitable. Model tests in wind tunnels and artificial dampers are preferably provided as discussed in section 4. Three examples (with two of which the author was associated) are described in the paper by Bobrowski and Cramer<sup>17</sup>) under Theme II.

Where earthquake resistant constructions are required: It is obviously impossible to deal with these problems in a few lines. Research has clearly shown that suitably designed prestressed concrete is capable of large displacement and dissipating energy due to impact forces with little permanent set. In columns it would be necessary to provide tendons at opposite sides and to allow sufficient displacement.

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(It seems to be surprising that the author's most telling research presented some twenty years ago (e.g. see Figures 3 and 4) has apparently been completely ignored in the basic reports of the Symposium).

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### SUMMARY

Resistance of prestressed concrete to dynamic loading is discussed. Great resilience against impact is important, but stiffness is essential for heavy vibrations. Miner's hypothesis seems satisfactory at repeated loading with limited impact. At random loading with heavy impact, as with aircraft and possibly also for earthquakes, Miner's value ought most likely to be reduced.

### RESUME

On discute dans ce travail la résistance du béton précontraint soumis à des charges dynamiques. Une grande résilience contre les chocs est importante, mais la rigidité est essentielle pour les vibrations à basse fréquence. L'hypothèse de Miner semble satisfaisante en cas de charge répétée et de chocs limités. En cas de charge stochastique accompagnée de chocs importants, tel que les chocs produits par les avions et probablement aussi pour les tremblements de terre, la valeur de Miner devrait très probablement être réduite.

### ZUSAMMENFASSUNG

Es wird der Widerstand von vorgespanntem Beton gegen dynamische Belastung diskutiert. Grosse Elastizität gegen Stoss ist wichtig, doch ist die Steifigkeit ausschlaggebend für starke Vibrationen. Die Hypothese von Miner scheint bei wiederholter Belastung mit begrenzter Stärke der Stösse zu genügen. Bei beliebiger Belastung mit starken Stössen, wie bei Flugzeugen oder eventuell Erdbeben, sollte der Wert von Miner sehr wahrscheinlich reduziert werden.