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Representation for Dynamic Loading

Représentation pour les sollicitations dynamiques

Darstellung für dynamische Beanspruchung

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1. Introduction

It is well known that structural behaviour can be qualitatively different for loads with different space distribution or time history. This fact is obvious especially for high stress levels, characterised by non-linear behaviour, and for the nature of limit-states. Failure types like those due to fatigue, to buckling, to strong shocks, etc., illustrate this statement. It could be presumed that for loading patterns that are only slightly different structural behaviour should be in almost cases no more strongly different (a well known exception: behaviour of two identical columns, one of them subjected to pure compression, the second to compression and small lateral forces). A challenge is raised: how to define a general tool for measuring a difference or a "distance" between two loading patterns and between corresponding behaviour modes of a structure? The question is of special interest for dynamic loads, that are basically multi-parameter loads. The fact that several parameters have to be considered in any case of dynamic loading makes this discussion, the main object of which is the use of the above mentioned measuring tool and its implications for design philosophy, especially appropriate for dynamic loading.

2. The space of loadings associated with a structure

The representation of material or structural characteristics or behaviour is often related to quantities like stresses at a point, internal forces at a member section, etc. The loadings will be chosen as basic parameters for this purpose throughout the paper, because they permit to represent characteristics and phenomena which could not be related to previous parameters (stresses, internal forces), especially for high loading levels (e.g. in case of plastic behaviour).

The set [S] of loadings S possibly acting on a structure will be imagined. Two elements of this set, S' and S", can differ by the system of points of application, by the direction and intensity of some forces or of some imposed displacements, by the time history. Two loadings, S' and S", given, their sum, S=S'+S", will be defined by a loading consisting of all the forces of S' and of all the forces of S". The product of a loading S with a scalar number q, q.S, will be a loading having forces q times greater than S. Two loadings, S' and S", given, a linear combination of them, q'S'+q"S", will be also a loading. A general property is put to evidence:

L_1: The set of loadings possibly acting on a structure is a linear space.

A new general property can be also put to evidence:

L_2: Two loadings, S' and S", given, a scalar product of them, (S'.S"), can be defined by the relation

$$(S'.S'') = \frac{1}{T_o} \int_o^{T_o} [\int_V \sum_{i,j}^{1,n} s'_{ij}(x_k,t).e''_{ij}(x_k,t)\, dV(x_k)]\, dt \qquad (1)$$

where T_o represents the length of a time interval that covers the life of the structure dealt with, while $s'_{ij}(x_k,t)$ and $e''_{ij}(x_k,t)$ represent stresses and strains (linearly determined) corresponding to the loadings S' and S".

The set of loadings, [S], that satisfies the properties L_1 and L_2, is a Hilbert space.

The norm of a loading, $\|S\|$, will be given by the relation

$$\|S\| = \sqrt{(S.S)} \qquad (2)$$

and the distance between two loadings, d(S'.S"), will be given by the relation

$$d(S',S'') = \|S' - S''\| \qquad (3)$$

A small distance d(S',S") means that the two loadings are practically not different from the view point of space distribution and time history. n linearly independent loadings, S_i (i=1...n) once given, they can define a base of an n-dimensional Euclidean sub-space, $[S]_n$, of [S]. Any loading of this sub-space can be expressed as a linear combination,

$$S = \sum_i^{1,n} q_i S_i \qquad (4)$$

An arbitrary loading S given, a best approximation of it by elements of the sub-space $[S]_n$ could be defined by the relation

$$\|S - \sum_i^{1,n} q_i S_i\| = \min. \qquad (5)$$

which permits a determination of the coordinates q_i.

The space [S] permits a representation not only of loadings, but also of structural characteristics. If structural behaviour corresponding to a loading (or a point) S is considered, sets of loadings for which structural behaviour is similar are represented by domains B_1 associated with certain states of stress. Boundaries R_1 of such domains are corresponding to limit-states. As and example, the states of stress corresponding to an elastic-plastic frame are represented in fig. 1 Structural behaviour for a given load corresponds to the domain to which the point belongs.

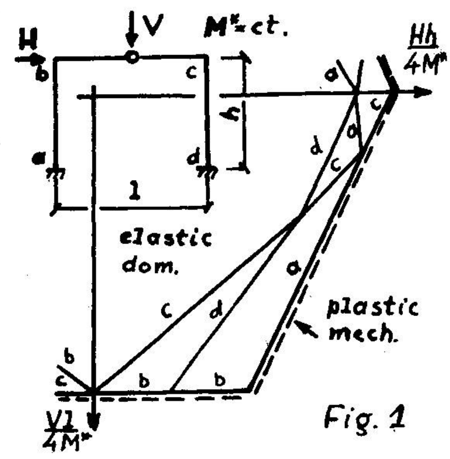

Fig. 1

A random loading is represented by a random point S. In case of the use of an n-dimensional sub-space $[S]_n$, the distribution can be represented by the probability density $g(q_i)$. Random structural characteristics are represented by the fact that the location of domains B_1 is random, i.e. a fixed point S can belong randomly to one or another of the domains B_1. The probabilities $F_1(s)$ of not exceeding the limit-state R_1 for a loading S=s represent structural characteristics from a stochastic view point (they can be dealt with as conditional probabilities: probabilities of not exceeding R_l if $S=s$).

The probabilities H_l, of survival without exceeding the limit-states R_l, have to be determined by means of formulae of total probabilities,

$$H_l = \int F_l(s)\; g(s)\; dV(s) \qquad (6)$$

($dV(s)$: element of volume in the space dealt with).

An elementary structural event is the following: structural characteristics are represented by a system of given (fixed) domains B_l and boundaries R_l, while the loading process is represented by a given point $S=s$. The structural performance depends essentially on the position of the point $S=s$ with respect to the system of domains B_l and boundaries R_l. If both kinds of factors are random, their randomness is different. In case of a one-parameter loading, S, and of one single limit-state, R, the randomness has to be represented on two different axes in the plane. In the general case of multi-parameter loadings, randomness of loadings and of structural characteristics should be represented simultaneously in the cartesian product of the space $[S]$ with itself, $[S]*[S]$. Neglecting this aspect could lead to errors in evaluating structural safety, like in case of using semi-probabilistic techniques.

3. Idealisations

Loadings depend practically in any case on several parameters which can vary randomly. This is especially the case of dynamic loadings. On the other hand, the number of parameters on which loadings acting on a structure depend is theoretically infinite in any case. Safety analysis can be done, even in research activity, only for a moderate number of parameters (say a few parameters of space distribution and a few parameters of time history). The possibilities of practical analysis in design activity are still poorer. This gap between reality and practical possibility of analysis raises an obvious need for idealisation, but for an idealisation made on a consistent basis.

The problem of idealisation could be kept in view like in the following illustrative example. Imagine one wants to analyse a body (or a figure) located in a three-dimensional space. The tool for analysis does not permit more than one single two-dimensional analysis, i.e. projection of the figure on a plane is a primary step, which is to be followed by the analysis. The problem to be solved before performing the two-dimensional analysis is that of an optimum choice of the plane on which the initial problem will be projected. This plane has to be as significant as possible (for example, in case one wants to project an axi-symmetrical ellipsoid the plane has to be parallel to the rotation axis, to provide maximum of information). To have a more realistic image of the problem, the idealisation could be illustrated by another example: how to determine an m-dimensional sub-space that is sufficiently significant for a problem formulated in an n-dimensional space ($m < n$, or even $m \ll n$), and leads to a convenient amount of work in analysis?

The problem of permissible idealisations is sharply raised also in case of testing a structure up to failure (especially for dynamic tests). Failure is actually obtained, in any test, for a well defined loading pattern (space distribution and time history), while research activity should analyse the behaviour up to failure for any possible pattern So one obtains, instead of a whole boundary R_l, only a point of it, located on a radius corresponding to the loading pattern. A test in itself is, from this view point, a kind of poor sampling.

A general tool for evaluating the degree of accuracy of idealisations is the comparison of loading patterns which are the most significant for structural behaviour or for the risk of damage, with their projection on the sub-space adopted for analysis, by means of distances determined according to relations (3) or (5).

4. Design values

The problem of deffining and determining design values for practice has been formulated first in the frame of the semi-probabilistic approach, that represents at present the most advanced tool of wide use in design. More recent and consistent developments related to the adoption of an approximate probabilistic approach are not yet widely used in practice. It seems therefore reasonable, at this time moment, to refer the problem of design values rather to the semi-probabilistic approach. An important feature of the approach recommended for practice is the (implicit) assumption of a single-parameter loading. The (implicit) assumption of a single-limit-state structural behaviour is perhaps less significant. Although the evaluation of the risk of failure is different for the semi-probabilistic and for the approximate probabilistic approaches, some problems raised for the first one could be kept in mind also for the latter one.

In case of a single-parameter loading (this parameter is an intensity-type one) one can define a characteristic value Q_k and a design value Q for the distribution of this parameter. The analysis of structural behaviour under the given (static) loading scheme leads to defining a certain limit-state which is reached for a certain (limit) value of the loading parameter. The randomness of structural characteristics leads to a certain statistical distribution of this limit value and permits to define a characteristic value R_k and a design value R for the parameter dealt with. The semi-probabilistic approach requires that Q does not exceed R.

Imagine now a multi-parameter loading, represented in an n-dimensional space $[S]_n$. The direct generalisation of characteristic value or of design value is in this case a characteristic boundary, or a design boundary, respectively. A representation of such boundaries, for loading and resistance, is given in fig. 2 for a two-dimensional case. The sense of boundaries for R is quite clear. A point of such a boundary is obtained if, for a given direction, the loading intensity corresponding to a limit-state is determined. It is not the same for loadings, because the number of boundaries corresponding to a given global probability of being exceeded is infinite (in fig. 2 there are represented two possible characteristic boundaries, Q'_k and Q''_k).

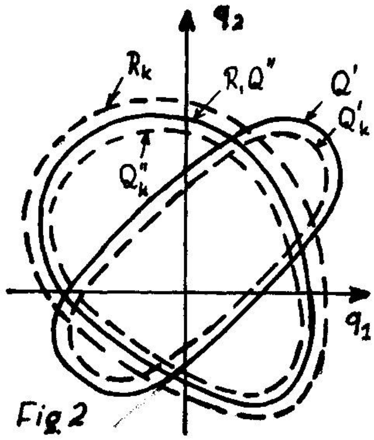

Fig 2

A first problem is the following: how to define a characteristic or a design boundary for loadings? The most natural answer could be: choosing it to be homothetical to the characteristic boundary R_k. But is this answer whenever possible the right one?

Assuming now an accurate definition (leding for example to boundaries Q' and Q'_k of fig. 2) has been adopted, a new problem is raised: how to replace a given boundary by a more elementary one (for example a polyhedral one) in order to make practical computations possible? Should one circumscribe to an elliptic domain a rectangle or an octogone? Should one look for another idea?

* * *

Some problems raised in this discussion might seem too theoretical and sophisticated. Nevertheless they cannot be avoided if a more consistent system of design rules is to be developed. The challenge for an improved approach of multi-parameter loadings is highly actual and this is true especially for dynamic loads, about which safety requirements of design codes say so little.

SUMMARY

The discussion is concerned with problems raised by multi-parameter loadings to which any type of dynamic loading belongs. Some general properties of the set of loadings that can act on a structure are dealt with. Problems of approximation and permissible idealisation are discussed on this basis. Difficulties raised by the definition of design values for multi-parameter loadings are then emphasized.

RESUME

La discussion est consacrée aux problèmes posés par les charges dépendant de plusieurs paramètres, dont les charges dynamiques font part. On présente quelques propriétés générales de l'ensemble des charges qui puissent agir sur une structure. On discute ensuite des problèmes d'approximation et de schématisation admissibles. On met en évidence des difficultés générées par la tentative de définir des valeurs de calcul pour les charges dépendant de plusieurs paramètres.

ZUSAMMENFASSUNG

Der Beitrag befasst sich mit Fragen der Mehr-Parameter Belastungen, zu denen die dynamischen Belastungen immer gehören. Es werden einige allgemeine Eigenschaften der Summe der auf ein Bauwerk möglicherweise wirkenden Belastungen dargelegt, und auf dieser Basis Fragen der Näherungen und der zulässigen Vereinfachungen erörtert. Ferner werden die Schwierigkeiten dargelegt, die sich beim Versuch ergeben, die Berechnungswerte für Mehr-Parameterbelastungen zu definieren.

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