IV

Strength and Lateral Deformability of Columns of Reinforced Concrete at Shear Failure

Résistance et déformabilité latérale de colonnes en béton armé au stade de la rupture par cisaillement

Festigkeit und horizontale Verformbarkeit von Stahlbetonstützen bei Schubbruch

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A number of reinforced concrete buildings suffered serious damages due to the 1968 Tokachioki earthquake in Japan. The injuries to columns were conspicuous in the damaged buildings and most of them resulted from shear failure. After the earthquake, experimental studies on the shear failure of reinforced concrete columns subjected to lateral forces have been carried out by many researchers in Japan. In the present paper, we propose equations to assess the shear strength and the ultimate lateral deformability at shear failure of reinforced concrete columns on the basis of the investigation of the test data which have been presented in Japan since 1961. The equation for the shear strength of columns is obtained through a modification of an empirical equation for the shear strength of beams, which has been proposed by K.Ohno and T.Arakawa and widely accepted in Japan because of its good applicability.

1. Ohno-Arakawa's equation for shear strength of beams

In order to make shear failure precede flexural yield in a specimen subjected to the simple-beam type of loading, the specimen must be greatly over-reinforced with longitudinal reinforcement. Furthermore, the deformation of the portions of simple beams where shear cracks may occur is in single curvature, whereas the deformation of members in building frames subjected to lateral loads is generally in double curvature. For these reasons, it is open to question whether the test results on simple beams could be directly applied to the prediction on the behaviors of building frames. A special type of loading for shear test was proposed by K.Ohno, one of the authors, at the RILEM Symposium of 1957 in Stockholm. By use of this loading method, the actual conditions of stress and deformation of members in frames subjected to lateral loads can be reproduced in the testing specimens as shown in Fig.1.

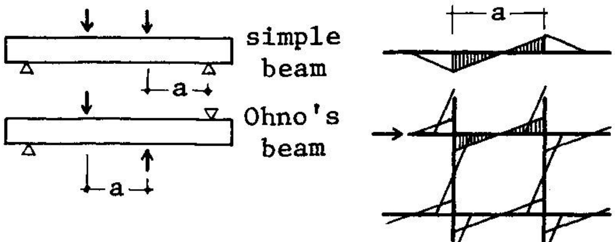


Fig. 1

K.Ohno and T.Arakawa had performed the shear test on 156 beam-specimens with wide variations by adopting the Ohno's method and established the following empirical equation for the shear strength of reinforced concrete beams in building frames, in 1960.

$$\tau_u = \frac{V_u}{bj} = k_u \cdot k_p \frac{0.23\ (f'_c + 180)}{a/d + 0.23} + 2.7\sqrt{p_w \cdot f_y}\ \text{, in kg/cm}^2 \quad (1)$$

where k_u = coefficient for size of specimen.
k_p = coefficient for axial reinforcement ratio.
a = shear span: distance between loading points.
d = effective depth of beam.
f'_c = compressive strength of concrete.
p_w = web reinforcement ratio.
f_y = yield point of web reinforcement.

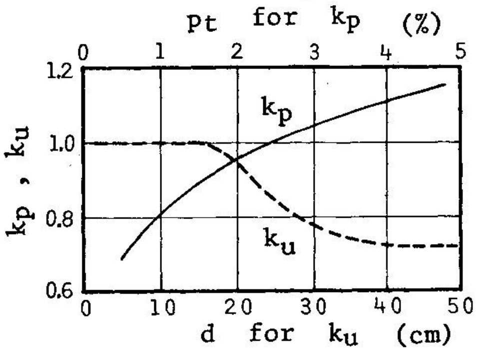


Fig. 2

This equation corresponds not only to the test results by the proposers but also to the test results on restrained beams performed by the groups at Illinois University: K.G.Moody, I.M.Viest, R.C.Elstner, E.Hognestad in 1955 and J.J.Rodriguez, A.C. Bianchini, I.M.Viest, C.E.Kesler in 1959, with good concentration as shown in Fig.3. Recently, the authors recommended an equation for the shear strength of simple beams, that is Eq.(2), which is applicable for only simple beams with web reinforcement and sufficient anchorage of axial reinforcement at the ends of the beams.

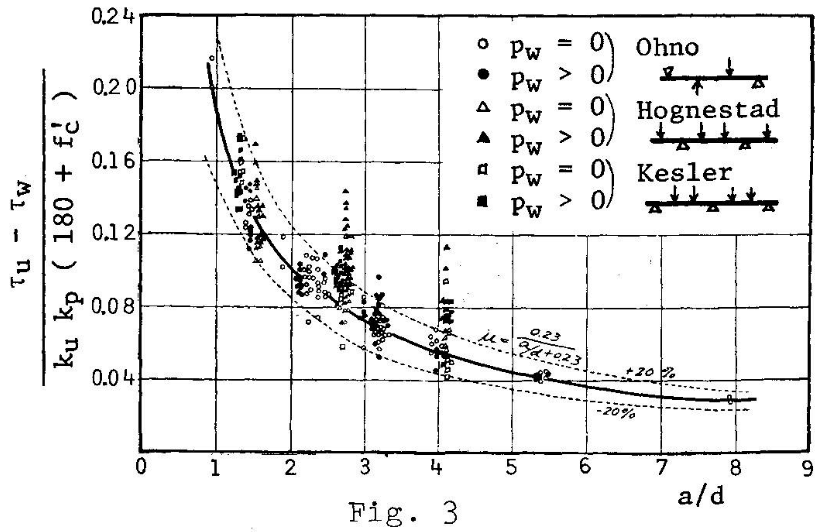


Fig. 3

$$\tau_u = \frac{V_u}{bj} = k_u \cdot k_p \frac{0.23\ (f'_c + 180)}{a/d + 0.23} + 1.4\sqrt{p_w \cdot f_y} \quad (2)$$

where a = shear span: distance between loading point and support, cf. Fig.1.

2. Outline of experimental works on shear resistance of reinforced concrete columns in Japan

Since 1961 in Japan, the shear tests of 378 column-specimens have been carried out by many researchers as shown in Table 1, where the data recognized as flexural failure by the investigator's own judgment are excluded. The types of loading in these investigations can be classified into three categories as follows:

a) A specimen is laterally loaded by Ohno's method after introducing a certain magnitude of axial force in the specimen.
b) A specimen is laterally loaded like a simple beam after introducing a certain magnitude of axial force in the specimen.
c) A specimen of two-story-frame shape is laterally supported at the positions of the top beam and the bottom beam and laterally loaded at the position of the middle beam after introducing a certain magnitude of axial force in each of the columns.

The loading procedures and the number of specimens subjected to each procedure are shown in Table 2.

Table 1

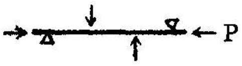
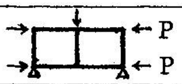
Type of loading	Investigator (Number of specimens)	Total
a) 	H.Aoyama(12), S.Bessho(9), T.Endo(3), M.Hirosawa(9), H.Muguruma(25), T.Nakayama(18), M.Wakabayashi(40), M.Yamada(37), authors(49)	9 (202)
b) 	T.Endo(7), A.Ikeda(104), T.Naka(12), T.Shimazu(5), T.Takeda(27) M.Yamada(1)	6 (156)
c) 	T.Takeda(2), authors(18)	2 (20)
Total		(378)

Table 2

Type of loading	Procedure of loading	Number of specimens		
		Total	For strength	For deforma.
a)	Monotonic increase	156	115	123
	Repeated reversal:			
	increase of peaks	25	9	8
	load limited	8	-	-
	deflection limited	13	-	-
b)	Monotonic increase	47	6	7
	Repeated reversal:			
	increase of peaks	68	27	27
	load limited	27	-	-
	deflection limited	14	-	-
c)	Monotonic increase	9	9	9
	Repeated reversal:			
	increase of peaks	11	9	11
Total		378	175	185

In the present paper, the data subject to one of the following conditions are excluded from the discussion with the intention of concentrating the investigation on the ultimate shear strength and the ultimate lateral deformability at shear failure:(1) the specimen subjected to repeated load by controlling with the load-limit lower than the ultimate shear strength or with the deflection-limit less than the deflection at the ultimate shear strength;(2)the data in which the value of a factor necessary to assess the ultimate shear strength is not shown;(3) the data in which there is no description about the deflection;(4) the specimen with a special type of web reinforcement, e.g. hoop of steel plate;(5) the specimen which ought to be considered to fail in flexure on the basis of the shape of load-deflection curve, the crack pattern and the comparison of the test result with the calculated result on the ultimate strength theory of flexure. Consequently the data available for the discussion on the ultimate shear strength are 175 specimens including 45 subjected to cyclic increasing loads and the data available for the discussion on the ultimate deformability at shear failure are 185 specimens including 46 subjected to cyclic increasing loads. The data cover 105 to 405 kg/cm^2 of concrete strength, 0 to 1.49 % of web reinforcement ratio, 1.0 to 7.5 of ratio of unsupported length to depth of column and 0 to 233 kg/cm^2 of average axial stress.

3. Shear strength equation for reinforced concrete columns

It has been known that the shear strength of columns has a tendency to increase as the magnitude of the introduced axial force increases. The ratios of the test data available for strength to the calculated results from Eq.(1) and Eq.(2) are plotted in Fig.4, where unsupported length of column, h_o, is substituted for 'a' in the equations. In Fig.4, the differences between the monotonic increase and the repeated reversal of loading, and between the simple beam type and the other type of loading are not obviously observed.

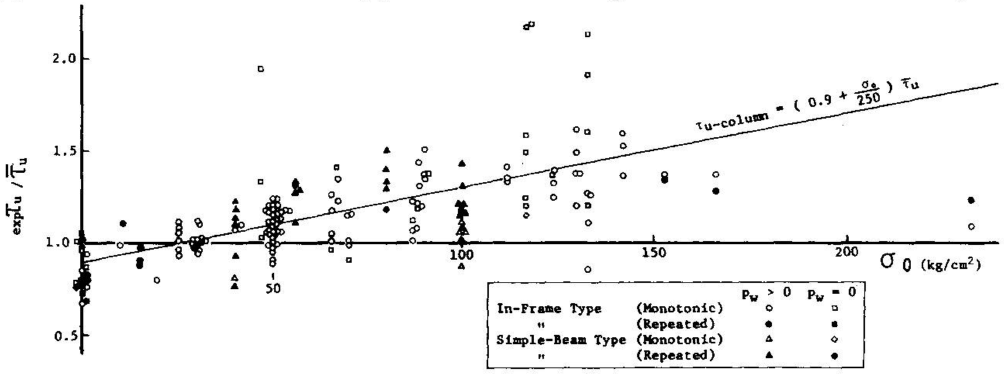


Fig. 4

The following equation is derived from the average line of the plots as an empirical equation for the ultimate shear strength of columns:

$$\tau_{u\text{-column}} = (0.9 + \frac{\sigma_o}{250})\,\bar{\tau}_u \,, \quad \text{in } kg/cm^2 \tag{3}$$

where σ_o = P/bD = average compressive stress of column.
$\bar{\tau}_u$ = calculated value from Eq.(1) or Eq.(2) by substituting unsupported length of column, h_o, for 'a' in the equation.

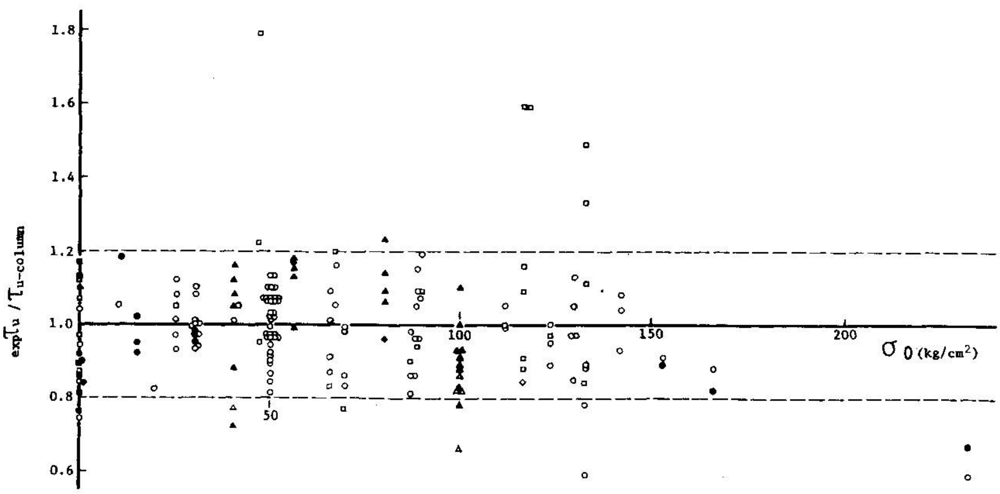


Fig. 5

The comparison of the test data with the calculated results from the proposed equation is shown in Fig.5, where 89.7 % of all the plots falls within the range of ±20 % of the calculated values. The mean value of the ratio $\tau_{exp} / \tau_{u\text{-column}}$ and the standard deviation in each category of loading is presented in Table 3. Far deviated plots in Fig.5 are mostly of specimens without web reinforcement. In practice, columns are usually provided with some amount of web reinforcement and subjected to axial stresses less than about 100 kg/cm^2. Under the limitation of $p_w > 0$ and $\sigma_o \leq 100$ kg/cm^2, the mean of the ratios of the test data to Eq.(3) and the standard deviation are shown in Table 4. The data within the range of ± 20 % from Eq.(3) are 96 % of the total. Considering the variety of the test programs, it may be said that Eq.(3) is satisfactory in accuracy and for the practical evaluation of the ultimate shear strength of columns.

Table 3

Category (whole data)	Number of data	Mean of the ratios	Standard deviation
Total	175	0.991	0.158
Monotonic	130	0.995	0.164
Repeated	45	0.980	0.139
Simple-beam type	33	0.972	0.151
In-frame type	142	0.996	0.159

Table 4

Category ($p_w > 0$, $\sigma_o \leq 100$)	Number of data	Mean of the ratios	Standard deviation
Total	119	0.988	0.111
Monotonic	84	0.982	0.105
Repeated	35	1.003	0.124

4. Lateral deformability of columns at shear failure

The observed deformation consists of two components, flexural and shear, in the test data. As the details of the specimens, the properties of materials and the levels of the acted bending moment at the shear failure are considerably different among the investigators, it is desirable to separate the shear deformation from the flexural deformation, if possible. The flexural deformation cannot, however, be evaluated on the flexural theory of continuum after various patterns of shear cracks have distinctly occurred, or after the bond between axial reinforcement and concrete has been released. For the above reason, it seems rather irrational to distinguish their shares in the observed deformation. The deformability cannot but be treated here as the ability of the whole lateral deformation of columns. The data are widely scattered under any limited condition, so that we proceed with the discussion on the lowest values of the scatters. The effects of four influencing factors, i.e. concrete strength, web reinforcement ratio, ratio of unsupported length to depth of column and average axial stress, are examined respectively, and the following tendencies of the lowest values of the scatters are observed. The lateral deformability of columns will be the smaller,(1) the higher the strength of concrete is,(2) the lower the web reinforcement ratio is,(3) the smaller the ratio of unsupported length to depth of column is,(4) the larger the average axial stress is. The data of 185 specimens are divided into three groups according to the levels of axial stresses, and the equations are obtained as the expressions of the lowest-limit-lines, respectively.

$$\left.\begin{array}{ll} 0 \leq \sigma_o < 40 \text{ kg/cm}^2 & {}_{min}R_{u\,1} = 2p_w(\, h_o/D\,)(\,1 + \frac{300}{f'_c}) \\ 40 \leq \sigma_o < 100 & {}_{min}R_{u\,2} = 1.5p_w(\, h_o/D\,)(\,1+\frac{300}{f'_c}) \end{array}\right\} \quad (4)$$

$$100 \leq \sigma_o \leq 233 \qquad {}_{min}R_{u\,3} = p_w(\ h_o/D\ )(\ 1 + \frac{300}{f_c'}\ )$$

where $R_u = \delta_u/h_o$ = ultimate deflection angle of column.
δ_u = ultimate lateral deflection of column.

On the assumption that these equations correspond to the middle values of the respective ranges of axial stresses, the relation between the minimum deformation and the regarding factors are consolidated into an equation.

$$R_{u\text{-}min} = p_w(\ h_o/D\ )(\ 1+\frac{300}{f_c'})(\frac{500}{\sigma_o+180} - 0.5\ ),\ \text{in } 10^{-3}\ \text{rad.} \qquad (5)$$

The comparison of the test data with Eq.(5) is plotted in Fig.6. Only five plots fall below Eq.(5). The difference between two types of loading, monotonic and repeated, is not obviously observed in Fig.6. In spite of the indistinctness of its physical meaning, Eq.(5) would be useful for assessment of the minimum of the lateral deformability of columns at shear failure for the present until a more rational method is established.

In the design of structural members, it should be intended that the brittle failure such as shear failure might not occur in the ultimate state, so as to insure the ductility of structures. This ought to be brought about by making the flexural strength of members less than the shear strength. The shear strength of columns would be assessed from Eq.(3).

In the case where the shear strength is inevitably less than the flexural strength, the column should be endowed with the sufficient deformability to follow the response lateral deformation of the whole structure. The Eq.(5) would be utilized as a measure for evaluation of the deformability.

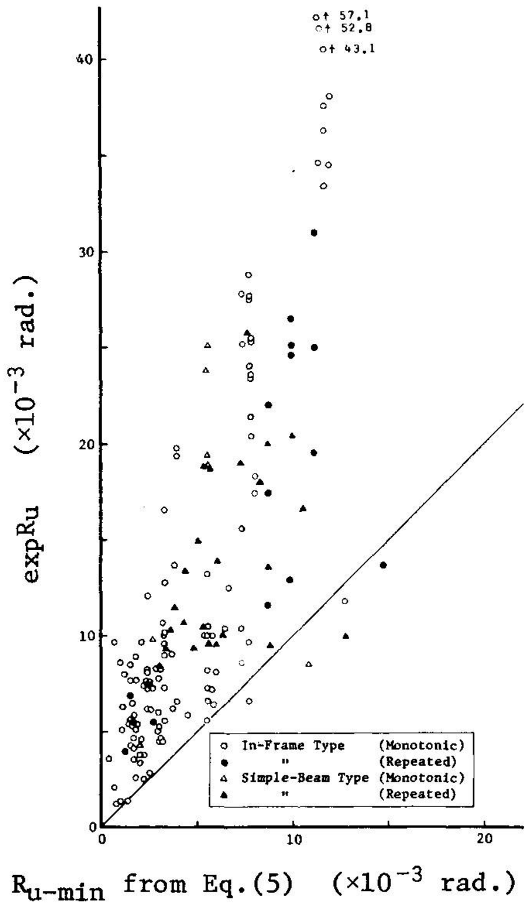


Fig. 6

SUMMARY

The equations to assess the shear strength and the ultimate lateral deformability at shear failure of reinforced concrete columns are proposed. These equations are derived from the investigation of the test data which have been presented in Japan since 1961. The differences between two types of loading, monotonic increase and repeated reversal, are obscure in the shear strength or the ultimate lateral deformability.

RESUME

On propose dans ce travail les équations pour évaluer la résistance au cisaillement et la déformabilité latérale ultime au moment de la rupture par cisaillement de colonnes en béton armé. Ces équations sont dérivées de l'étude des résultats d'essais présentés au Japon depuis 1961. Les différences entre deux types de charge, charge monotone et charge répétée alternée, sont dévoilées par la résistance aux efforts de cisaillement et par la déformabilité latérale ultime.

ZUSAMMENFASSUNG

Es werden die Gleichungen zum Abschätzen des Schubwiderstandes und der horizontalen Auslenkung beim Schubbruch von Stahlbetonstützen vorgeschlagen. Diese Gleichungen werden aus der Untersuchung von Testdaten abgeleitet, die in Japan seit 1961 veröffentlicht werden. Die Unterschiede zwischen zwei Belastungsarten, stetig zunehmend und wiederholt wechselnd, sind hinsichtlich des Schubwiderstandes und horizontaler Bruchverformbarkeit unklar.