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### III

#### **Repeated Loading Tests on Continuous Beams subjected to Both Axial and Vertical Loading**

Essais de chargement répété sur poutres continues soumises simultanément à des charges axiales et verticales

Wiederholte Belastungsversuche an Durchlaufträgern mit gleichzeitig wirkender axialer und vertikaler Last

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#### **1. Introduction**

The phenomenon of incremental collapse in ductile structures subject to repeated loading has been understood for many years<sup>(1)</sup>. Incremental collapse occurs when cyclic application of alternative load combinations results in a progressive build up of deflections as restricted plastic flow takes place on each application of load. The threshold load level, above which incremental collapse can occur, is termed the shakedown load.

When designing structures using plastic theory, this effect is not usually considered. It is argued that a given structure is more likely to be rendered useless by a single overload sufficient to cause plastic collapse than by the build up of deflections as a series of somewhat smaller overloads cause incremental collapse. It is also noted that this build up of deflections is very much restricted by strain hardening.

It is necessary to reconsider this conclusion when a framed structure which exhibits frame instability is subject to loads likely to cause incremental collapse<sup>(2)</sup>. A single overload that is insufficient to cause failure can significantly weaken the resistance of a structure to subsequent cycles of repeated loading. Furthermore, at load levels above the shakedown load, successive cycles of load at a given level are accompanied by progressively increasing increments of deflection until unstable collapse takes place, often after a few cycles of load. It is clear that the probability argument used to avoid the necessity of considering variable repeated loading in plastic design breaks down in the presence of this "acceleration to collapse".

The phenomenon is convincingly demonstrated by the results of the tests described in this paper and is shown to be amenable to analysis but only when strain hardening is included.

#### **2. Experimental Investigation**

Tests were carried out using two and three-span continuous beams of 12.7mm

( $1/2$ in) square cross section. The general arrangement for these tests is shown in Figs 1 and 2. Repeated application of these loading patterns leads to the incremental collapse mechanisms indicated by plastic hinge positions in the figures.

As well as cyclic loading tests carried out using a frequency of 20 minutes per complete cycle, static loading tests were also carried out using the most severe of the two load cases which is shown in each case as that for the first half cycle.

Similar apparatus was used for both series of tests and is shown in Figs 3 and 4. It was mounted on a large lathe bed and loading was applied through chain and lever system. Vertical load was applied to the test beams through knife edges at the level of the centre line of the beams so that no secondary moments could be induced as the beams deflected.

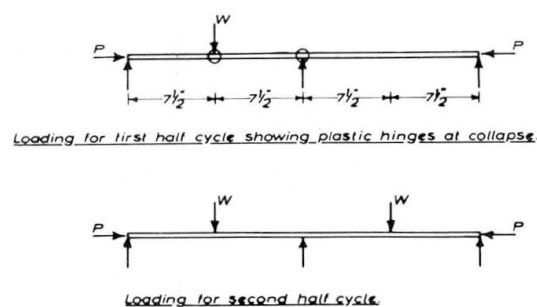


Fig 1. Loading cycle for two-span beams

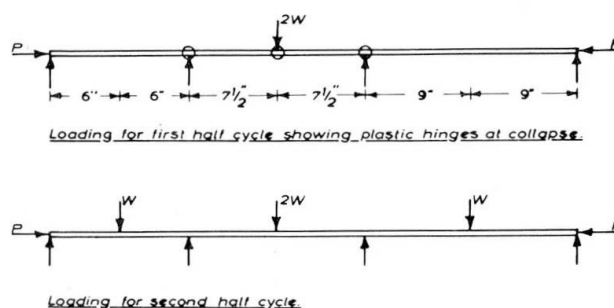


Fig 2. Loading cycle for three-span beams

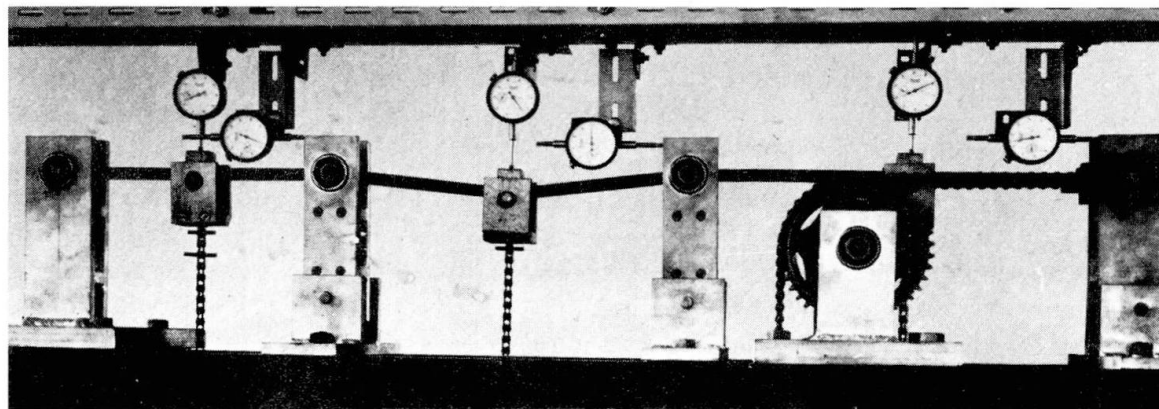


Fig 3. Apparatus for three-span tests

One of the end supports was firmly bolted to the rigid base and was capable of supplying both horizontal and vertical reactions to the beams. This support had a pair of 12mm diameter ball journals at the level of the centre line of the beams. The remaining supports allowed horizontal movement at beam level, having soft metal bushes for an axle at the lower end and ball journals at the level of the test beams. At the internal supports, the beams were lightly clamped between half-round pieces of steel in a specially designed clamp with stub axles which fitted into the journals on the supports. This arrangement provided a simple support capable of resisting vertical movement up and down without interference to the axial loads in the beams.

Steel blocks were welded to the ends of the test beams and accurately drilled to accommodate spindles which provided the end supports. One end of each beam was thus provided with a pinned connection to the rigid support, the other end with a pinned connection to a support free to move in the axial direction. It was through the spindle providing this latter connection that the axial thrust was applied.

The ends of this 25mm dia spindle were shaped to form knife edges on the centre line of the spindle. Axial load was applied through two chains, linked to these knife edges, and passing over large chain wheels before being connected to the loading lever.

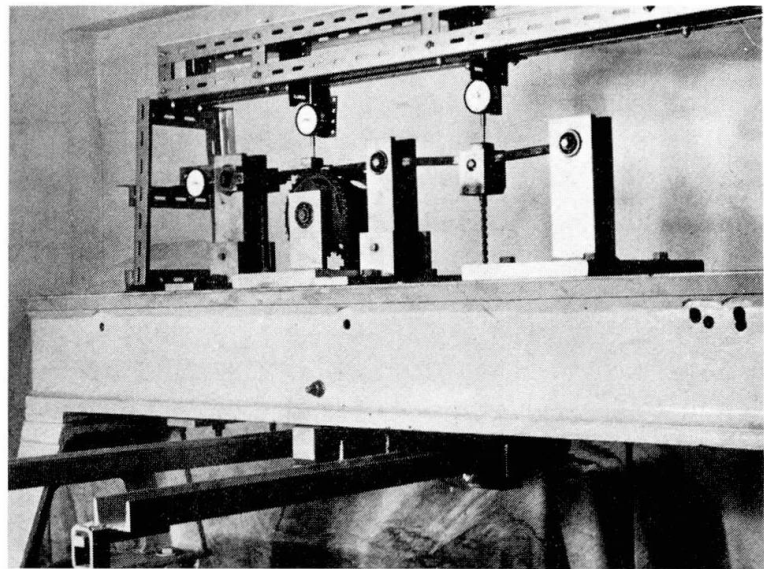


Fig 4. Apparatus for two-span tests

For the two-span tests, cyclic loading was achieved automatically by means of a hydraulically operated jack acting under one of the loading levers. This jack, which is not shown in Fig 4, was driven by a pulsator unit the frequency being controlled by a clock and the range by two micro-switches. When the jack was extended, the loading lever was disconnected from the structure which was therefore subjected to the single load from the second lever. When the jack returned to its lower position, the load was automatically replaced and both spans were loaded.

For the tests on three-span beams, the pulsator operated two jacks in parallel which simultaneously removed and replaced the loads on the outer spans of the beams.

### 3. Control Tests

The material properties of the steel were determined by means of tests on simply supported beams of 300mm span subjected to a central point load. These tests were carried well into the strain hardening range so that the flexural rigidity, full plastic moment and coefficient of strain hardening<sup>(3)</sup> were obtained for five specimens cut from each 3m length of material.

### 4. Experimental Results - Two-Span Beams

Both static loading and repeated loading tests were carried out by first applying an axial load, which was thereafter maintained at a constant level, and then applying increasing vertical load until collapse occurred or the deflections became excessive. Four levels of axial load were considered, namely zero, 0.974kN (219lb), 1.962kN (441lb) and 2.976kN (661lb).

For the static collapse tests, each load level was maintained until the creep rate had fallen to an acceptably low value. Failure was defined in the case of unstable behaviour as the load actually causing failure and in the case of stable behaviour as the load at which the last plastic hinge formed obtained from the load-deflection curve.

For the repeated loading tests, it was desired to investigate both the value of the shakedown load and the response to cycles of load **above** the shakedown load. For each test, the structure was subject to several cycles of load at a given level as the load on one span was taken on and off at regular intervals. If the deflection stabilised, the load level was increased and further cycles of load were applied, if not, cyclic loading was continued until unstable collapse took place.

A most critical factor in the response to cycles of load is the "stability balance" value of the axial load<sup>(3)</sup>. This is the axial load at which strain hardening exactly equalises the 'PΔ' effect and the curve of deflection versus cycles of load is theoretically linear. Here strain hardening and axial load effects were predicted to balance at an axial load of 1.000kN (225lb). Figs 5, 6 and 7 show both experimental (full lines) and theoretical (broken lines) loading curves for axial loads of zero, 0.974kN (219lb) and 1.962kN(441lb).

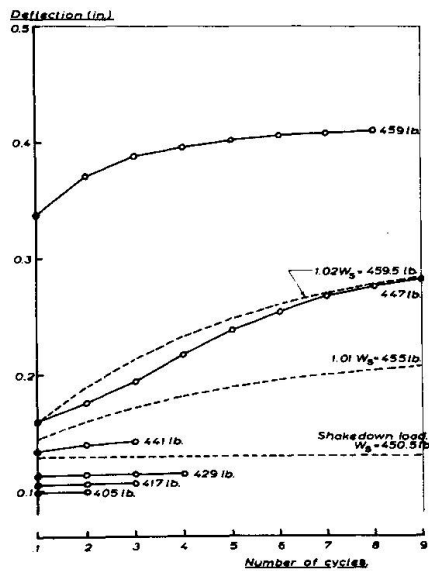


Fig 5. Two-Span Beam. Axial Load zero

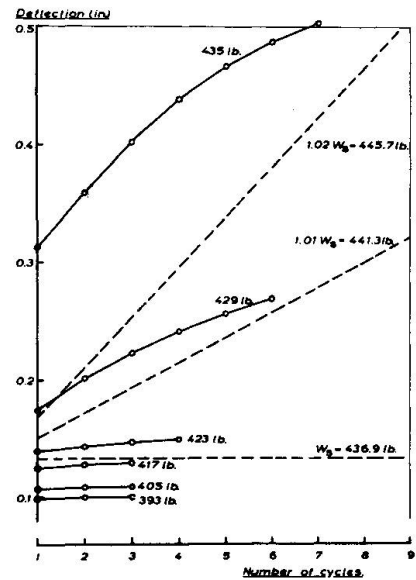


Fig 6. Two-Span Beam. Axial Load 219lb

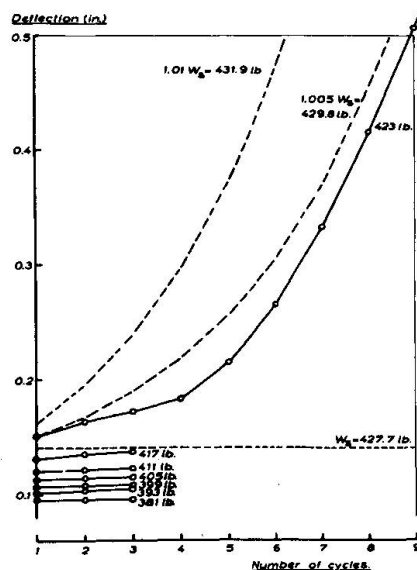


Fig 7. Two-Span Beam. Axial Load 441lb

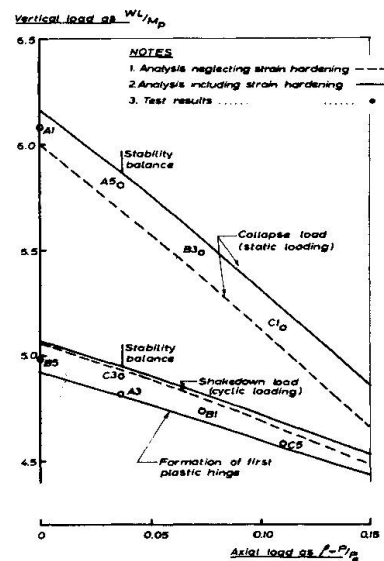


Fig 8. Two-Span Beams - Failure Loads

The derivation of the theoretical curves follows the method given previously (2). The transition from stable to unstable behaviour is very clear and closely follows the theoretical predictions. At an axial load of 2.976kN (661lb) the acceleration to collapse was even more rapid than in the curves shown and failure took place on the 6th cycle of load which was precisely what was predicted for a load 1% in excess of the shakedown load.

The failure loads obtained are summarised in Fig 8 where they are plotted against the Euler ratio  $P/P_e$ . The slightly low values of the shakedown load are probably due to a certain amount of dynamic loading as the load was automatically taken on and off. This was, to a large extent, cured for the three-span tests.

### 5. Experimental Results - Three-Span Beams

The procedures and general pattern of results were similar to those for the two-span tests though, with three plastic hinges participating in the collapse mechanism, the influence of strain hardening was found to have increased. Figs 9 and 10 show representative curves of deflection versus number of cycles of load and Fig 11 summarises the experimental and theoretical failure loads.

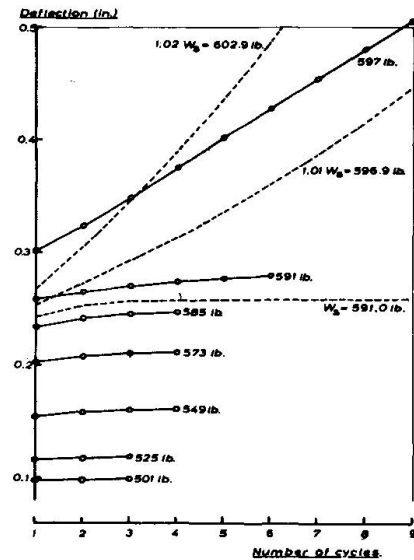


Fig 9. Three-Span Beam.  
Axial Load 441lb

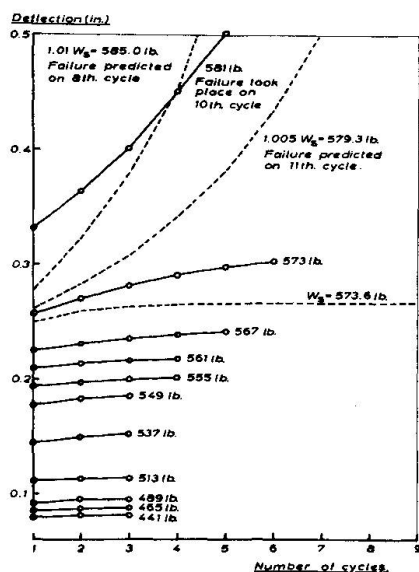


Fig 10. Three-Span Beam. Axial Load 669lb

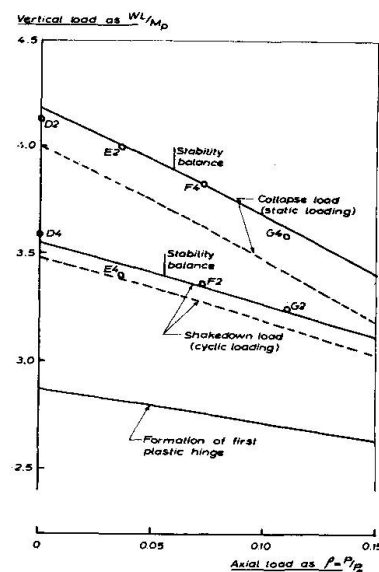


Fig 11. Three-Span Beams. Failure Loads

## 6. Conclusion

It has been demonstrated by tests on axially loaded continuous beams subjected to repeated cycles of vertical load that there is a transition from a stable to unstable pattern of deflection response at quite low values of axial load. A valid theoretical comparison is also demonstrated. A similar pattern of behaviour has been shown, by theoretical analysis only, <sup>(2)</sup> to exist in frames. It follows that incremental collapse is a much more serious problem in structures subject to frame instability and variable repeated loading effects cannot be readily dismissed by arguments based on probability.

## 7. Bibliography

1. B G Neal "The plastic methods of structural analysis.  
Chapman and Hall, London 1956.
2. J M Davies "Variable repeated loading and the plastic  
design of structures". Struct. Engr. Vol 48, May 1970.
3. J M Davies "Frame instability and strain hardening in  
plastic theory". Proc. A.S.C.E., J. Struct. Div. ST 3  
June 1966.

## SUMMARY

Tests are described in which axially loaded continuous beams were subjected to both static and cyclic loading. When the effect of the axial compressive load predominated over strain hardening, the static failure was unstable and repeated loading eventually led to "acceleration to collapse". At load levels above the shakedown load each cycle of load resulted in an increased increment of deflection until failure took place after a few cycles of load. A valid theoretical comparison is demonstrated.

## RESUME

On décrit des essais au cours desquelles des poutres continues ont été soumises à des charges axiales statiques et dynamiques. Lorsque l'effet de la charge de compression axiale l'emportait sur celui de l'écrouissage, la rupture statique était instable, et une charge répétée précipitait éventuellement la rupture. A des niveaux de charges supérieurs à la charge ultime, chaque cycle de charge provoquait une augmentation toujours plus grande de la courbure, jusqu'à la rupture qui se produisait après quelques cycles de charges. On démontre une comparaison théorique valable.

## ZUSAMMENFASSUNG

Es werden Versuche beschrieben, bei denen axial belastete Durchlaufträger statischer und zyklischer Belastung ausgesetzt sind. Wenn der Effekt des axialen Druckes die Verfestigung überwog, war das statische Versagen instabil und die wiederholte Belastung führte schliesslich zum "beschleunigten" Kollaps. Bei Belastungsstufen über der "Shakedown"-Last resultierte aus jedem Lastzyklus eine Zunahme der Auslenkung, bis der Bruch nach einigen Zyklen eintrat. Ein brauchbarer theoretischer Vergleich wird angeführt.