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## Time Effects in the Shakedown of Reinforced Concrete Beams

Effets du temps sur l'orientation de poutres en béton armé

Zeiteinwirkungen beim Sich-Einspielen von Stahlbetonträgern

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### Introduction

The subject of shakedown on reinforced concrete beams, to which Professor Park's survey paper made reference, has been clouded by conflicting evidence; on one hand, early researchers (1) claimed that the elastic-perfectly plastic theory of incremental deformations was well able to predict the observed behavior of concrete beams under variable repeated loading. On the other hand, later work (2) indicated very little relationship between the predictions of the classical theory, and experimental results, and attributed this discrepancy variously to the effects of bond deterioration (3) and strain-hardening of the steel (4).

In a further effort to clarify the matter, we consider in this contribution the effect of concrete creep on the response of concrete beams subject to alternating repeated loads, and on the occurrence of shakedown. The approach consists of the following phases:

1. Development of a general analysis for concrete beams under arbitrary load histories, incorporating the effect of the time-dependent response of the concrete.

2. Verification of the analytical predictions by appropriate tests.

3. Use of the analysis to perform a study of the way in which creep affects the incremental deformations of concrete beams under cyclic loading.

The steps, and the resulting conclusions, will be discussed shortly in the following.

### General Analysis for Arbitrary Load History

Following earlier work (5), time-dependent moment-curvature relations were established, based on classical concrete beam theory of plane sections and the actual properties of the constituent materials, of which steel was of the elastic-plastic-strain hardening type, and the concrete was of stress-strain-time relationships measured directly in the laboratory.

With moment-curvature-time relations known for all beam sections, any framed structure can be analyzed by adjusting the distribution of moments at any point in time so as to satisfy equilibrium and compatibility. Because of the non-linearity of the system, an interactive solution scheme was used, which incorporates the necessary checks for elastic or inelastic loading, elastic unloading at any section, as well as attainment of the elastic limit which shifts in magnitude according to the maximum moment reached in prior load cycles (4). A computer program incorporating these features enables prediction of moments, curvatures, and deflections at any stage of arbitrary load histories applied to continuous beams. Effects of shear deformations or shear failure, or bond or anchorage deterioration, were not considered in the analysis.

### Experimental Verification

It is obvious that an analysis as full of gross assumptions and simplifications as the one outlined must be checked by comparison of theoretical and experimental results. For this purpose, two-span continuous concrete beams were tested under a variety of load histories consisting of cyclic and sustained loading (6). The case presented here is typical of results obtained.

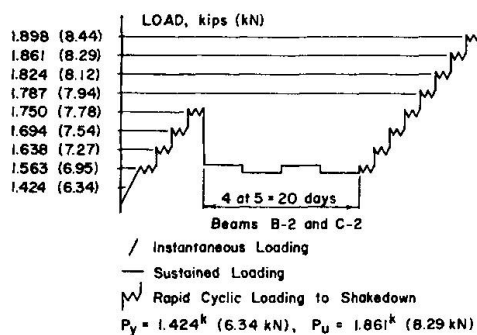


Figure 1

Fig. 1 shows the load history which consisted of a first stage of rapid cyclic variation of load at increasing levels, followed by a 20-day period of two load levels sustained alternately for five days each, and concluding with another sequence of cyclically varying loads to failure.

Figs. 2, 3, and 4 show the analytical and observed load-deflection curves for the rapidly-cycled loading stages, and the deflection-time-curves for the sustained loading stage. Good correlation is observed at all stages up to 98 per cent of the predicted plastic limit load

for the two-span beam. By that time over 70 cycles of high loads had been applied, and some evidence of bond deterioration was apparent. Nevertheless, the observed deflections were less than those predicted up to failure, which occurred under a load slightly larger than the fully-plastic limit load.

It is concluded from this and other comparisons that the analysis is well capable of predicting the response of such beams to applied load histories.

### Effect of Creep on Shakedown of Concrete Beams

Once the validity of the method of analysis of load history effects was confirmed experimentally, it was utilized with some confidence in a systematic exploration of the effects of creep on the incremental deformations of continuous concrete beams under repeated variable loading.

In tests, shakedown, or deflection stabilization of a structure is presumed to have occurred whenever the deformation increment between corresponding stages of two successive load cycles vanishes. Any change of displacement due to creep will give the

appearance of incremental deformations, unless considered in the theory, or eliminated by appropriately fast rates of load cycling

The previously established analysis was used to investigate the effect of creep on the displacements of a two-span continuous beam of the same properties as that of the previous analysis, under load cycles applied at various rates, and Fig. 5 summarizes the results. The applied load, of magnitude  $P_{\max} = 0.92 P_u$ , is

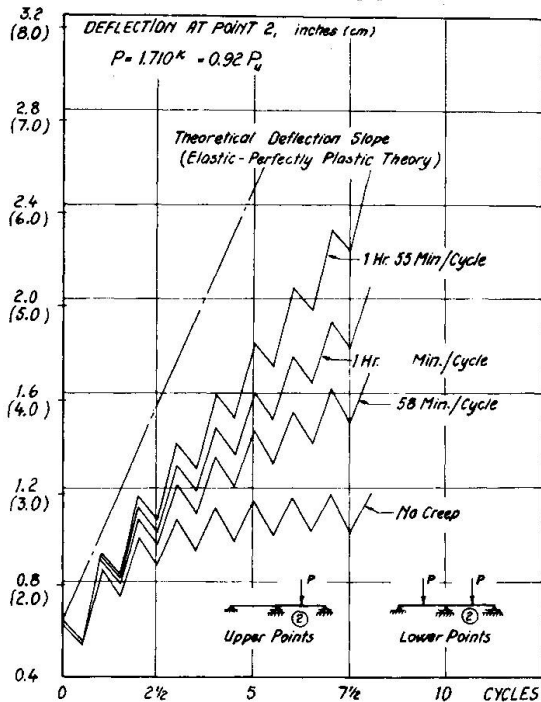


Figure 5. Theoretical Shakedown Including Creep

above the elastic-perfectly plastic shakedown load, and the dash-dotted line shows the linearly-increasing displacements predicted by classical theory of incremental deformations assuming perfect plasticity. Because of strain-hardening, the beam will actually shake down in the absence of creep, as shown by the curve labeled "No Creep." Intermediate curves indicate the predicted increase in deflections for different rates of load cycling, and demonstrate that the effect of creep will tend to give the appearance of perfectly-plastic incremental deformations.

It may be concluded that both strain-hardening and creep should be included for a valid assessment of the occurrence of incremental deformations in concrete structures subject to cyclic overloads.

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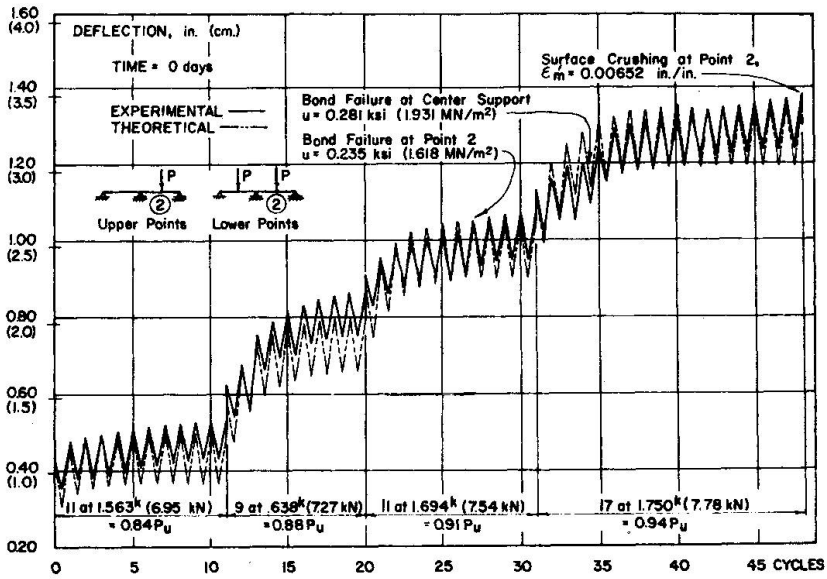


Figure 2

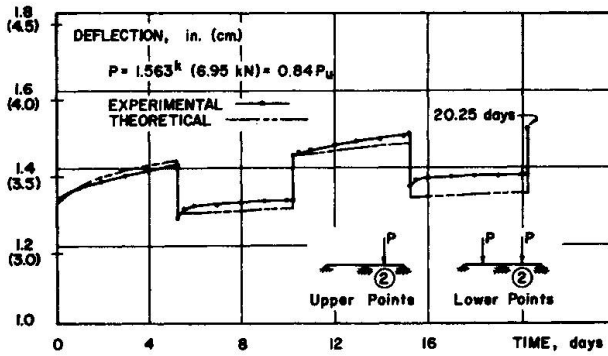


Figure 3

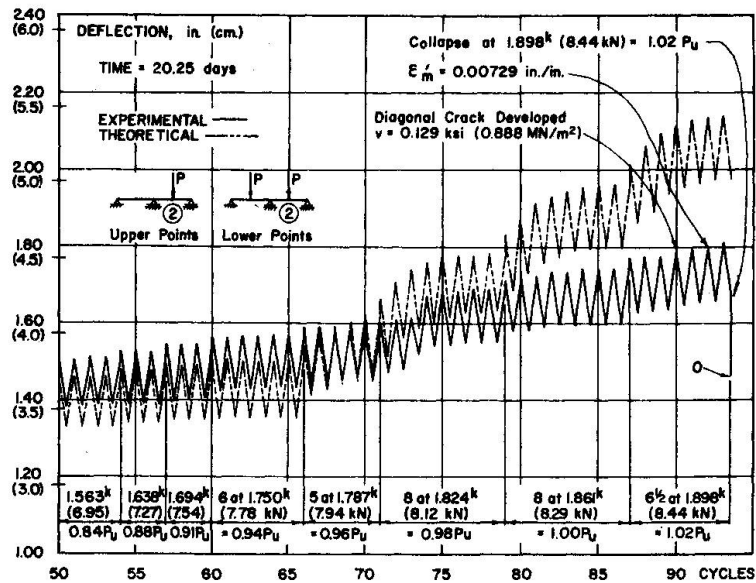


Figure 4

## SUMMARY

A general analysis for the flexural response of reinforced concrete beams to arbitrary load and time histories is presented, and verified by comparison with experimental results. This analysis is then used to evaluate the effect of creep on the incremental deformations of concrete beams under variable repeated loading. It is concluded that creep should be considered for a valid assessment of theories of incremental deformation.

## RESUME

Ce travail présente une méthode d'analyse générale pour l'étude du comportement de poutres en béton armé soumises à la flexion, à un instant arbitraire du processus de charge. Les résultats sont vérifiés par une série d'essais. Cette méthode est ensuite utilisée pour évaluer l'influence du fluage sur les déformations additionnelles de poutres en béton armé soumises à des charges répétées variables. Il en résulte que le fluage devrait être pris en considération pour une conception valable des théories sur les déformations additionnelles.

## ZUSAMMENFASSUNG

Eine allgemeine Berechnungsmethode für das Biegeverhalten von Stahlbetonbalken unter beliebiger Last und zeitlichem Verlauf wird dargestellt und durch Vergleich mit experimentellen Resultaten bestätigt. Diese Methode wird dazu verwendet, den Effekt des Kriechens auf die zunehmenden Deformationen von Betonbalken unter variabler wiederholter Belastung zu ermitteln. Daraus wird gefolgert, dass das Kriechen beim brauchbaren Abschätzen der zunehmenden Deformationen berücksichtigt werden sollte.

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