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## Cyclic Stress-Strain Response of 2024 Aluminium under Biaxial States of Stress

Courbe cyclique tension-déformation de l'aluminium 2024 dans un état de tension biaxiale

Zyklische Spannungs/Dehnungs-Reaktion von Aluminium 2024 unter zweiaxigem Spannungszustand

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### INTRODUCTION

Although the stress-strain behaviour of metals under states of combined stress has been studied experimentally for monotonically-increasing loads, research into elastic-plastic material response for loadings of a repeated nature has been mainly concerned with uniaxial stress states [1,2]. Apart from a few recent investigations into low-endurance biaxial fatigue [3-5], there is very little test data for cyclic loading involving multiaxial stress states. Consequently, the major objective of the work initiated in this paper will be to determine cyclic stress-strain characteristics for metals under various states of biaxial stress in the plastic range. It is clear that such experimental work is essential to the formulation of adequate material models for the analysis of structural components subjected to complex, repeated loads [6,7].

In this preliminary investigation, cyclic biaxial response is determined for 2024-T351 aluminum by subjecting thin-walled cylinders to repeated axial loading combined with alternating torsion. Thus, the fundamental variables studied are axial stress  $\sigma$ , shear stress  $\tau$ , axial strain  $\epsilon$  and shear strain  $\gamma$ . For convenience, we introduce the quantities

$$\eta = \frac{\sigma}{\tau} \quad \rho = \frac{\epsilon}{\gamma} \quad (1)$$

which define a stress ratio and strain ratio, respectively. Furthermore, as a scalar measure of stress and strain under biaxial states, the following conventional definitions of "effective stress"  $\bar{\sigma}$  and "effective strain"  $\bar{\epsilon}$  are adopted:

$$\bar{\sigma} = (\sigma^2 + 3\tau^2)^{\frac{1}{2}} \quad \bar{\epsilon} = \left( \frac{3\epsilon^2 + \gamma^2}{3} \right)^{\frac{1}{2}} \quad (2)$$

In this paper, results are presented for a series of fully-reversed strain-controlled tests in which the amplitude of the effective strain,  $\Delta\bar{\epsilon}$ , is the controlling parameter and a constant strain ratio  $\rho$  is maintained during a given test. Particular emphasis is given to the "steady-state" response of the material under cyclic biaxial loading.

## EXPERIMENTAL INVESTIGATION

### a) Specimens and Apparatus

The thin-walled tubular specimens employed in this investigation were machined from  $1\frac{1}{2}$  in. (38 mm.) diameter solid bars of 2024-T351 aluminum to the dimensions shown in Fig. 1. The material was used as supplied and, after grinding, the internal and external surfaces of the cylinders were polished in order to prevent premature fatigue failures. A description of the pertinent mechanical properties of the aluminum alloy is given in Table 1.

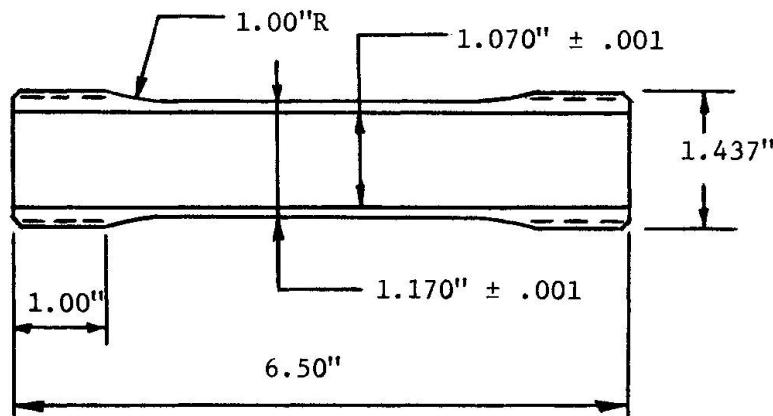


Fig. 1

TABLE 1

### Tensile Properties

Modulus of Elasticity	Yield Strength	Ultimate Strength	% elongation
10,200 ksi (717,000 kg/cm <sup>2</sup> )	50.8 ksi (3,570 kg/cm <sup>2</sup> )	68.6 ksi (4,820 kg/cm <sup>2</sup> )	22

All experiments were conducted using a specially-designed apparatus, which operates in a standard close-looped (MTS) servo-controlled electro-hydraulic testing system. This testing facility, which is described in detail elsewhere [8], permits simultaneous application of repeated axial loading and cyclic torsion to the tubular specimens. The axial load and torsion are controlled independently, and either the applied loading itself or the output ( $\epsilon, \gamma$ ) of the transducers measuring the strain may be selected as controlling parameters.

### b) Experimental Programme

Due to possible frequency effects and the presence of "cyclic creep" under load-controlled conditions, a series of fully-reversed constant-amplitude strain-controlled tests was conducted. Cyclic stress-strain response was thus determined under biaxial stress states for the following prescribed strain ratios:  $\rho=0, \frac{1}{2}, 1, 2, \infty$ ; where  $\rho=0, \infty$  correspond to pure torsion ( $\sigma=0$ ) and axial stress ( $\tau=0$ ), respectively.

For each predetermined value of  $\rho$ , two tests were performed: one for an

effective strain range  $\Delta\bar{\epsilon} = \pm 0.75\% = 1.5\%$ , and a second test for which  $\Delta\bar{\epsilon} = \pm 1.0\% = 2.0\%$ . All tests were run at a frequency of 0.1 cycles per second and during each test, axial load-deformation and torque-twist hysteresis loops were recorded periodically using X-Y plotters. Although the prime purpose of the experiments was to investigate material response prior to and including steady-state conditions, the cyclic loading programme in each case was applied until failure of the specimen occurred.

## TEST RESULTS AND DISCUSSION

Results of a typical fully-reversed strain-controlled test are shown in Fig. 2 where hysteresis loops obtained for combined axial load and torsion ( $\rho=1$ ) are reproduced for various cycles of deformation. For purposes of comparison, the monotonic stress-stress curve under axial tension is also included in this figure. As cycling proceeds a progressive increase in stress amplitude is evident, which indicates that this metal exhibits cyclic strain-hardening characteristics under biaxial loading. Furthermore, although the results for a given cycle show that the stress limits in torsion are essentially equal, one observes that the stress limit in compression for the axial stress  $\sigma$  becomes greater than the tensile limit during cycling. This behaviour, termed "mean stress relaxation", is apparently related to the "cyclic creep" phenomenon, which should occur in a corresponding load-controlled test [1,2].

In Fig. 3, the stress limits in torsion and axial stress are plotted against the number of cycles for the effective strain amplitudes  $\Delta\bar{\epsilon} = 1.5\%$ ,  $2.0\%$  and various strain ratios. These curves indicate that, during an initial transitory period of cyclic strain-hardening under combined stress, the stress-strain behaviour asymptotically approaches some limiting "steady-state" cyclic response. In addition, from the difference in stress limits for a particular cycle the amount of mean stress relaxation is readily obtained.

In order to characterize the material response under steady-state conditions, "steady-state yield-interaction" curves have been constructed (Fig. 4). Using the 0.2% proof stress as the definition of yielding, the yield stress components  $(\sigma_y, \tau_y)$  obtained from the steady-state hysteresis loops are plotted and compared to the initial yield curve. From these results it appears that, in addition to the cyclic strain-hardening phenomenon mentioned above, another type of hardening accompanies cyclic deformation - a hardening which is manifested by an increase in yield stresses with increasing number of cycles. Yet, in contrast to cyclic strain-hardening (where the stress amplitude increases with increasing strain amplitude) the amount by which these steady-state yield stresses increase is less for the higher value of  $\Delta\bar{\epsilon}$ . However, in view of the number of tests conducted in this preliminary investigation, and the particular definition of yield adopted (0.2% offset), more experimental data must be generated and the implication of other yield definitions examined [8] in order to confirm this trend. Further experiments are also required to determine "steady-state yield curves" for a variety of strain-controlled and load-controlled conditions and to assess whether interaction curves, such as those given in Fig. 4, provide useful information for the analysis and design of structural elements subjected to repeated loads.

In conclusion, the numbers of cycles to "failure"  $N_f$  are listed in the following table for the various applied strain amplitudes and prescribed strain ratios.

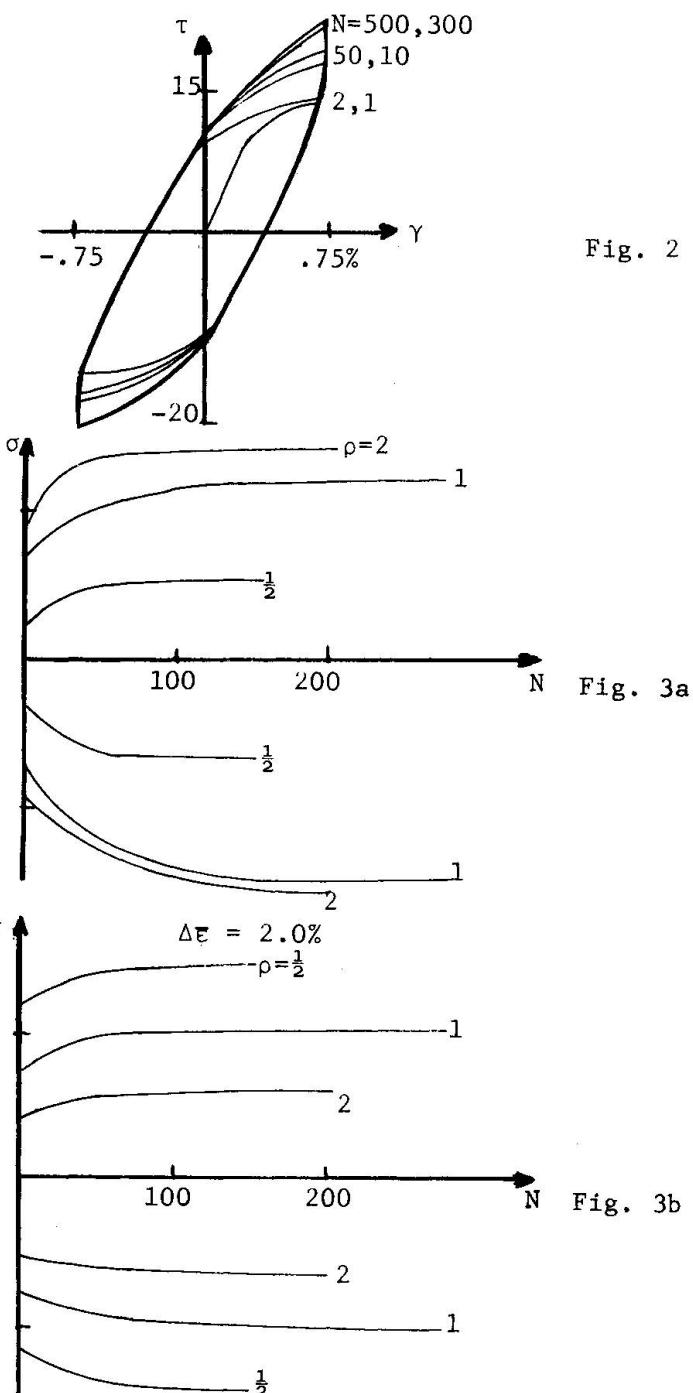
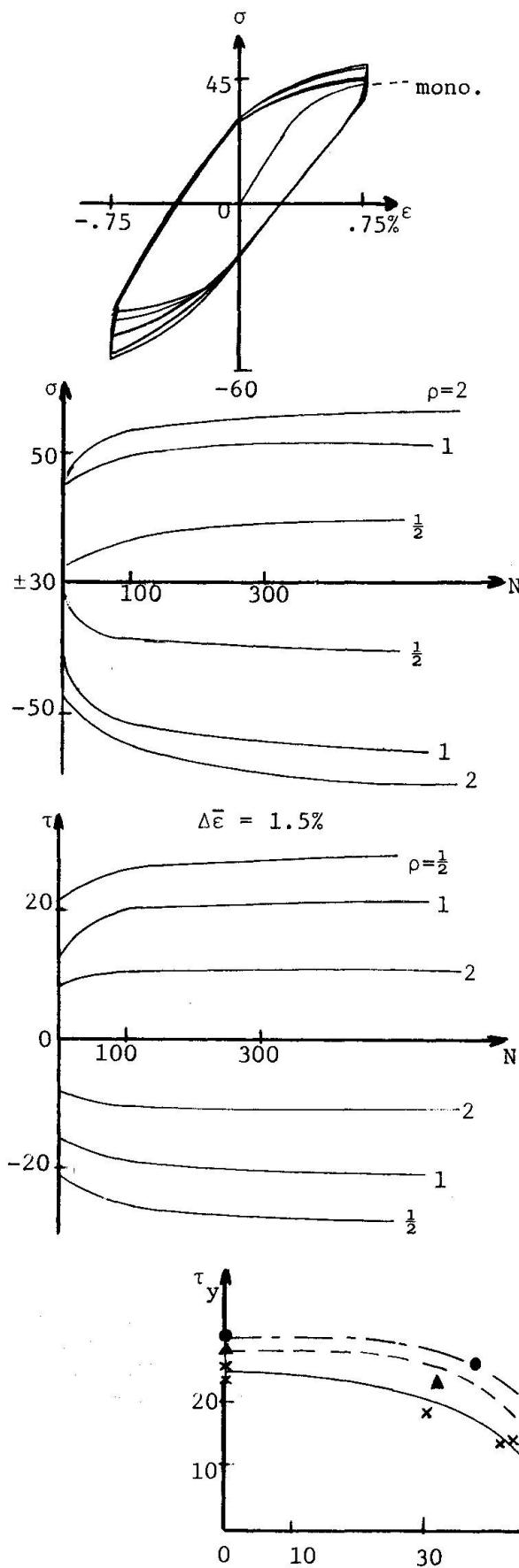


Fig. 2

Fig. 3a

Fig. 3b

Fig. 4

- $\times$  initial
- $\blacktriangle$   $\Delta\bar{\epsilon} = 2.0\%$
- $\bullet$   $\Delta\bar{\epsilon} = 1.5\%$

TABLE 2

$\Delta\varepsilon$	$\Delta\gamma$	$\Delta\bar{\varepsilon}$	$\rho$	$N_f$
$\pm 1.00\%$	0%	2.0%	$\infty$	197
$\pm 0.96$	$\pm 0.48$	2.0	2	219
$\pm 0.87$	$\pm 0.87$	2.0	1	299
$\pm 0.65$	$\pm 1.31$	2.0	$\frac{1}{2}$	166*
0	$\pm 1.73$	2.0	0	43*
$\pm 0.75$	0	1.5	$\infty$	377
$\pm 0.72$	$\pm 0.36$	1.5	2	621
$\pm 0.65$	$\pm 0.65$	1.5	1	662
$\pm 0.49$	$\pm 0.98$	1.5	$\frac{1}{2}$	570
0	$\pm 1.30$	1.5	0	324

\* failed by buckling

It should be noted that, although most specimens exhibited fatigue failures, buckling occurred for the higher amplitudes of torsion. Certain aspects of this delayed-buckling phenomenon under cyclic torsion were briefly discussed in [9].

## ACKNOWLEDGEMENTS

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## SUMMARY

The main objective of the work initiated in this paper is to determine stress-strain characteristics for metals under various states of repeated biaxial loading in the plastic range. In this particular investigation, the results of a series of constant-amplitude, fully reversed strain-controlled tests are reported for 2024-T351 aluminum under cyclic axial stress combined with alternating torsion. The observed cyclic strain-hardening and steady-state response are discussed, and the notion of a "steady-state yield-interaction" curve is introduced.

## RESUME

L'objectif principal de l'analyse amorcée dans ce travail est de déterminer les caractéristiques tension-déformation des métaux soumis à différents états de charges répétées biaxiales dans le domaine plastique. Dans cette étude particulière, on présente les résultats d'une série d'essais sur l'aluminium 2024-T351 soumis à des tensions axiales variant cycliquement, combinées avec des efforts de torsion alternés, les déformations totalement inversées restant d'amplitude constante. On discute ensuite l'allure des courbes pour des déformations cycliques dans le domaine d'écrouissage et pour l'état stationnaire et on introduit la notion de courbe d'interaction des tensions de fluage à l'état stationnaire.

## ZUSAMMENFASSUNG

Das Hauptziel der vorliegenden Arbeit ist die Bestimmung von Spannungs-Dehnungs-Charakteristiken für Metalle unter verschiedenen Zuständen wiederholter zweiachsiger Beanspruchung im plastischen Bereich. In dieser speziellen Untersuchung werden die Resultate einer Serie mit konstanter Amplitude und vollständig umgekehrter dehnungs-kontrollierter Versuche für Aluminium 2024-T351 unter zyklischer axialer Spannung kombiniert mit wechselseitiger Torsion, aufgezeigt. Die beobachtete zyklische Verfestigung und die Reaktion aus stetiger Belastung werden diskutiert und der Ausdruck einer "stetigen Belastung-Fliessen-Interaktion" eingeführt.