Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 13 (1973)

Artikel: Non-linear analysis for K-type braced steel frames

Autor: Wada, Akira / Suto, Fukuzo / Fujimoto, Morihisa

DOI: https://doi.org/10.5169/seals-13743

Non-linear Analysis for K-Type Braced Steel Frames

Analyse non-linéaire des cadres en acier à structure en K

Nichtlineare Berechnung von Stahlrahmen mit K-Ausfachung

Akira WADA
Staff
Computer Division
Nikken Sekkei Ltd.

Fukuzo SUTO
Dr. Eng., Chief
Computer Division
Nikken Sekkei Ltd.
Japan

Morihisa FUJIMOTO
Dr. Eng., Professor
Tokyo Institute
of Technology

1. Introduction.

The architectural and civil engineering construction in Japan must be built up in the natural environments under frequently experienced great earthquakes and typhoons that visit our country at random or in autumn every year. Particularly in high storied buildings, since they are subjected to large repeated horizontal forces due to the earthquake and wind, there are several occasions that braces are set into frames as the structural elements against the external forces. (See Fig. 1)

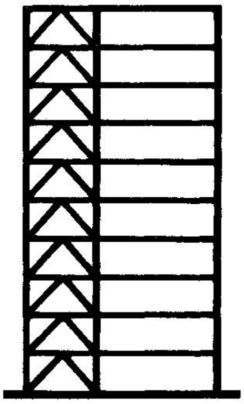

Fig.1 K-type braced frames

Since, however, the brace bears a large axial force, it is needed to secure the rigidity and ultimate strength, so that a large cross-sectional members must be used. As a result the slenderness ratio (L/r) becomes comparatively small in the order of 30~70, and consequently a brace on the compression side is apt to break down by plastic buckling.

In order to clarify the mechanical behavior of a structure having such characteristics analytically, this paper deals with a nonlinear analysis taking consideration of the nonlinearity due to the yield of materials and the nonlinearity based on the finite deformation theory unable to be neglected in the analysis of buckling, and thereby performs the analysis as mentioned below.

For the purpose, the behavior of compressive members with different slenderness ratio was first analyzed for repeated loadings, and after making various checkings of the results (omitted in this paper), the analysis of the behavior of 1 story 1 span frame provided with K-type braced as shown in Fig. 9 is made for the case in which a repeated horizontal force is applied.

Following is a review of selected recent studies conducted in our country on this topic. Yamada, Tsuji, and Takeda and Wakabayasi, Nonaka, Ogi, and Yamamoto studied the behavior of braces being subjected to repeated axial forces in the elasto-plastic range by using a small experimental model. On the other hand, Matsui, Mitani, and Tsumatori have studied the behavior of braces with various slenderness ratios in the state of buckling towards collapse state, taking propagation of plastic region in axial direction of material into consideration and using a numerical analysis of C-D-C method.

However, no exact analysis of braced frames has been reported yet , because it is extremely difficult to analyze the elasto-plastic behavior under the condition of repeated loading accompanied by such an instability phenomenon as buckling of braces.

2. Several assumptions taken for this analysis.

a. The residual stress and the deflection in the initial period due to welding etc. are not taken into account.

b. The member is assumed to be of such type that the beam theory is applicable. But, in order to approximate the curves of each member after deformation, the member is divided into 5~15 elements (Fig. 2 & 9).

c. In each element, only the bending deformation and the axial deformation is considered and the shearing deformation is neglected. As a result, the displacement functions used in this analysis are expressed by Eq. (1) (See Fig. 2).

$$\begin{Bmatrix} \triangle u \\ \triangle v \\ \triangle \theta \end{Bmatrix} = \begin{bmatrix} 1 & x & o & o & o & o \\ o & o & 1 & x & x^2 & x^3 \\ o & o & o & 1 & 2x & 3x^2 \end{bmatrix} \cdot {}_m\alpha \quad \ldots\ldots (1)$$

$$\text{where} \quad {}_m\alpha = \{ {}_m\alpha_1 \; {}_m\alpha_2 \; {}_m\alpha_3 \; {}_m\alpha_4 \; {}_m\alpha_5 \; {}_m\alpha_6 \}^T \quad \ldots\ldots (2)$$

The relation between incremental displacement $\triangle_m u$ and $_m\alpha$ of element m is expressed by Eq. (3) by substituting coordinates $x = \pm c/2$ at both ends i and j of the element into Eq. (1). (c : length of element)

$$\triangle_m u = {}_m T \cdot {}_m\alpha \qquad \text{namely, } {}_m\alpha = {}_m T^{-1} \cdot \triangle_m u \quad \ldots (3)$$

$$\text{where} \quad \triangle_m u = \{ \triangle_m u_i \, \triangle_m v_i \, \triangle_m \theta_i \, \triangle_m u_j \, \triangle_m v_j \, \triangle_m \theta_j \}^T \quad \ldots\ldots (4)$$

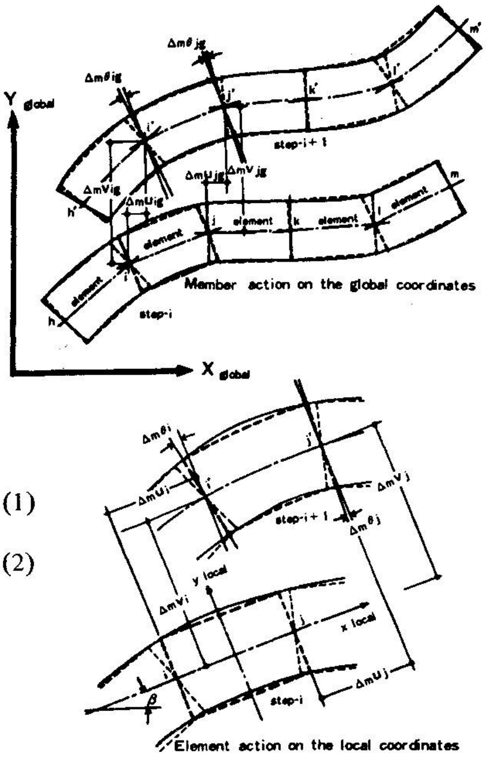

Fig. 2 Deformation & coordinates

d. The strain ($\triangle\epsilon_x$) in the element can be expressed by Eq. (5), taking a large deformation into consideration and using the assumption that its cross section remains plane after deformation;

$$\triangle\epsilon_x = \frac{d\triangle u}{dx} + \frac{1}{2}\left(\frac{d\triangle v}{dx}\right)^2 - \left(\frac{d^2\triangle v}{dx^2}\right) y \quad \ldots\ldots\ldots (5)$$

Derivatives of $\frac{d\triangle u}{dx}, \frac{d\triangle v}{dx}$ and $\frac{d^2\triangle v}{dx^2}$ in Eq. (5) can be obtained by differentiating displacement function Eq. (1).

e. Following the assumptions c & d, only ϵ_x and σ_x are considered in each element, and yielding occurs according to their magnitude.

f. As for the stress-strain relation of material, a hysteresis loop which is shown in Fig. 3 is used, considering the strain-hardening and the Bauschinger-effect.

g. The distribution of stress intensity and strain in the cross section are considered by dividing the cross section into 20 layers as shown in Fig. 4, and the values of stress and strain in each layer are considered by the values at the each center of gravity.

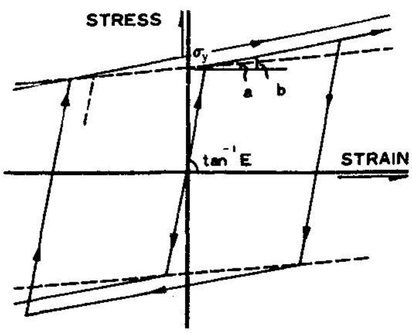

Fig. 3 Relation of stress-strain

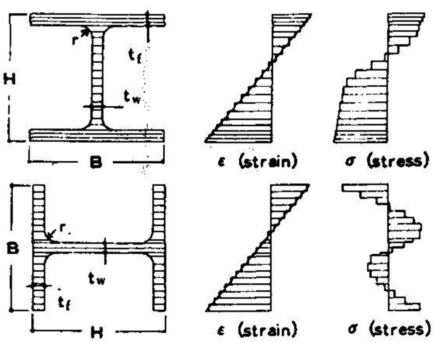

Fig. 4 Distribution of stress & strain

3. Nonlinear analysis based on the stationary principle of potential energy.

The strain energy $_mU$ which is stored in the element m by incremental displacements can be developed as follows,

$$_mU = {}_mU_1 + {}_mU_2 \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (6)$$

where

$$\left[\begin{aligned} {}_mU_1 &= \iiint_V \sigma_x \cdot \Delta\epsilon_x \, dx\,dy\,dz \\ &= \int_C \int_A \sigma_x \left\{ \frac{d\Delta u}{dx} + \frac{1}{2}\left(\frac{d\Delta v}{dx}\right)^2 - \frac{d^2\Delta v}{dx^2} y \right\} dA\,dx \\ &= {}_m\alpha^T {}_ma_1 + \frac{1}{2} {}_m\alpha^T_m A_1 {}_m\alpha - {}_m\alpha^T {}_ma_2 \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (7) \\ {}_mU_2 &= \iiint_V \frac{1}{2} E (\Delta\epsilon_x)^2 \, dx\,dy\,dz \\ &= \int_C \int_A \frac{1}{2} E \left\{ \frac{d\Delta u}{dx} - \frac{d^2\Delta v}{dx^2} \right\}^2 dA\,dx \\ &= \frac{1}{2} {}_m\alpha^T_m A_2 {}_m\alpha - \frac{1}{2} {}_m\alpha^T_m A_3 {}_m\alpha + \frac{1}{2} {}_m\alpha^T_m A_4 {}_m\alpha \quad \cdots\cdots\cdots \quad (8) \end{aligned}\right.$$

(herein, term $\frac{1}{2}\left(\frac{d\Delta v}{dx}\right)^2$ in $_mU_2$ is dropped because it is considered negligible.)

Symbols of $_ma_1$, $_ma_2$, and $_mA_1 \sim {}_mA_4$ in Eq. (7) and Eq. (8) are vectors and matrices given as follows;

$${}_ma_1 = \int_C\left(\int_A \sigma_x dA\right) \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} dx \quad \cdots\cdots \quad (9) \qquad {}_ma_2 = \int_C\left(\int_A \sigma_x \cdot y\,dA\right) \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 6x \end{Bmatrix} dx \quad \cdots\cdots\cdots\cdots \quad (10)$$

$${}_mA_1 = \int_C\left(\int_A \sigma_x dA\right) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2x & 3x^2 \\ 0 & 0 & 0 & 2x & 4x^2 & 6x^3 \\ 0 & 0 & 0 & 3x^2 & 6x^3 & 9x^4 \end{bmatrix} dx, \quad \cdots (11) \qquad {}_mA_2 = \int_C\left(\int_A E\,dA\right) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} dx \quad \cdots (12)$$

$${}_mA_3 = \int_C\left(\int_A E \cdot y\,dA\right) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 6x \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 6x & 0 & 0 & 0 & 0 \end{bmatrix} dx, \quad \cdots (13) \qquad {}_mA_4 = \int_C\left(\int_A E \cdot y^2 dA\right) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 12x \\ 0 & 0 & 0 & 0 & 12x & 36x^2 \end{bmatrix} dx \quad \cdots (14)$$

Equations from (9) to (14) include $\int_A \sigma_x dA$, $\int \sigma_x \cdot y dA$, $\int E dA$, $\int E \cdot y dA$ and $\int E \cdot y^2 dA$ which are calculated on the both ends of divided elements according to the method which is described in the assumption (g) and are integrated inside of the elements assuming linear change in the respective axial directions of them. Here at the point situated in plastic region which σ_x is beyond the yield point, E is set as E_{st} in the calculation.

After carrying out calculation mentioned above, the strain energy stored in an element due to incremental displacement is given in Eq. (15).

$$\begin{aligned} {}_mU &= {}_m\alpha^T ({}_ma_1 - {}_ma_2) + {}_m\alpha^T ({}_mA_1 + {}_mA_2 - {}_mA_3 + {}_mA_4)\, {}_m\alpha \\ &= \Delta_m u^T {}_mT^{-1T} ({}_ma_1 - {}_ma_2) + \Delta_m u^T {}_mT^{-1T} ({}_mA_1 + {}_mA_2 - {}_mA_3 + {}_mA_4)\, {}_mT^{-1} \Delta_m u \\ &= \Delta_m u^T {}_mf_{in} + \frac{1}{2} \Delta_m u^T {}_mK \Delta_m u \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (15) \end{aligned}$$

The potential energy of Π_{i+1} for the whole structure in $(i+1)$ state is shown Eq. (17) by summing up strain energies of all elements and works done by external forces. The displacement components of all elements must be transformed into a global coordinate system using Eq. (16). (See Fig. 2)

$$\Delta {}_m u_g = L \Delta {}_m u \qquad \text{namely, } \Delta {}_m u = L^{-1} \Delta {}_m u_g \qquad \cdots\cdots \quad (16)$$

where $\Delta {}_m u_g$: incremental displacement vector of element m in global coordinate.

$$\Pi_{i+1} = \Pi_i + \frac{1}{2} \Delta u_g^T K \Delta u_g + \Delta u_g^T f_{in} - \Delta u_g^T f_{ex} \qquad \cdots\cdots \quad (17)$$

where $K = \sum_m {}_m L^{-1T} {}_m K {}_m L^{-1}$

$f_{in} = \sum_m {}_m L^{-1T} {}_m f_{in}$

As the potential energy is stationary in the equilibrium condition, Δu_g can be obtained by the variational principle. Namely, $\delta\Pi_{i+1} = 0$ for any arbitrary $\delta\Delta u_g$ results in the following equation.

$$K \Delta u_g + f_{in} - f_{ex} = 0 \qquad \cdots\cdots \quad (18)$$

Vector Δu_g is obtained by solving the simultaneous equation given in Eq. (18). The coordinate of each joint-point can be adjusted using Δu_g .

The calculation of one step is completed when incremental strain $\Delta\epsilon_x$ is obtained by Eq. (5) and with it the strain and stress at both ends of each element are adjusted.

The planned analysis is made possible by accumulating the calculation described above by the step-by-step method and iterating technique.

The flow chart of this numerical analysis is shown in Fig. 5.

4. The elasto-plastic behavior of braced steel frames.

4-1) Objects of analysis.

The elasto-plastic behaviors of braced steel frames are analyzed numerically and compared with the experimental results, assuming the condition under which a K-type braced steel frame is subjected to repeated horizontal forces as shown in Fig. 9.

The divided state of elements is shown in Fig. 9, and it is taken into consideration such effects that a large deformation could occur and complicated distribution of deformation in plastic region could be followed.

Further, behaviors under repeated loading are studied by increasing or decreasing displacements of loading-point in a stepwise method to the ultimate state. In order to compare with the experimental results the following values of yield-stress of steel members are used for the calculation.

column and beam:	(H-194x150x6x9)	$\sigma_y = 3.04 \text{ton/cm}^2$
brace:	(H-100x100x6x8)	$\sigma_y = 2.86 \text{ton/cm}^2$

4-2) The results of the analysis and discussions.

Fig. 6 shows the relation between horizontal external force and horizontal deformation. This figure shows that the overall behavior agrees sufficiently well with the experimental result except scattered differences. As for the largest load the analytical result gives a little larger value than the experimental result. This is probably due to incompleteness of the test specimen, namely the influence of initial deformation and residual stress.

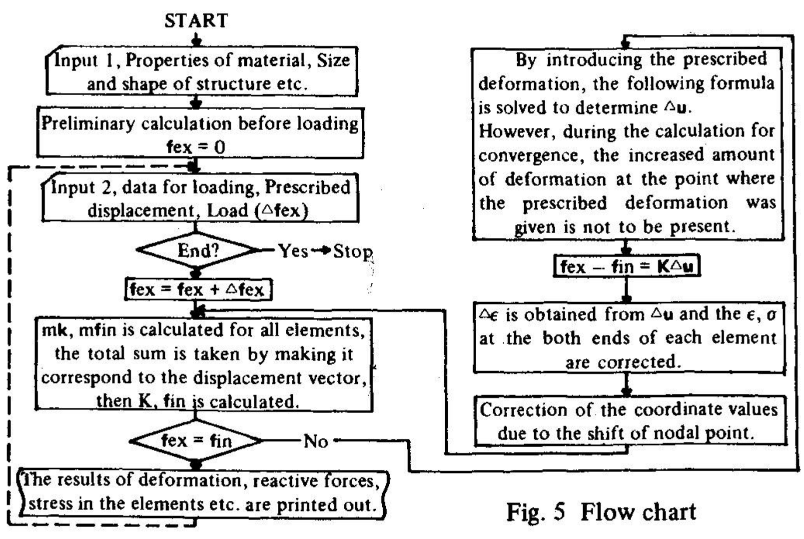

Fig. 5 Flow chart

The decrease of load after maximum loading (post-buckling of brace) is attained more suddenly in the analytical result than in the experimental result. The reason for this is that the analysis is made for the model which is replaced by beams with lengths which are spanning between nodal points, in other words, that some larger slenderness ratio is estimated in the analysis than in the test specimen.

Fig. 7 shows corresponding deformations to the state marked with heavy dots in Fig. 6. The scale of the deformation-figure is half of that of the model. The comparison of the final deformation form with Fig. 10 shows that they both are of very similar pattern. Fig. 8 shows the change of the yielded area until buckling is first observed on the brace of compressed side and also how yielded places occur as the deformation mode of braces changes.

5. Conclusion.

The numerical analysis of elasto-plastic behaviors of braced steel frames which are subjected to repeated horizontal forces and its comparison with experimental results has ascertained usefulness of this analytical method. It can be thought that this method of analysis generally makes it possible to analyze the elasto-plastic behaviors accompanied by instability phenomena, which hitherto was considered to be difficult, and also possible to acquire the distribution of stress inside of members, which is difficult to obtain from experiments.

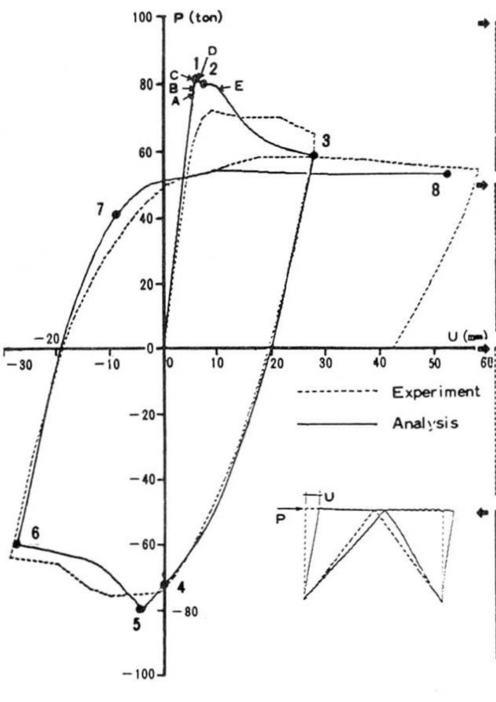

Fig. 6 Relation of load-deflection

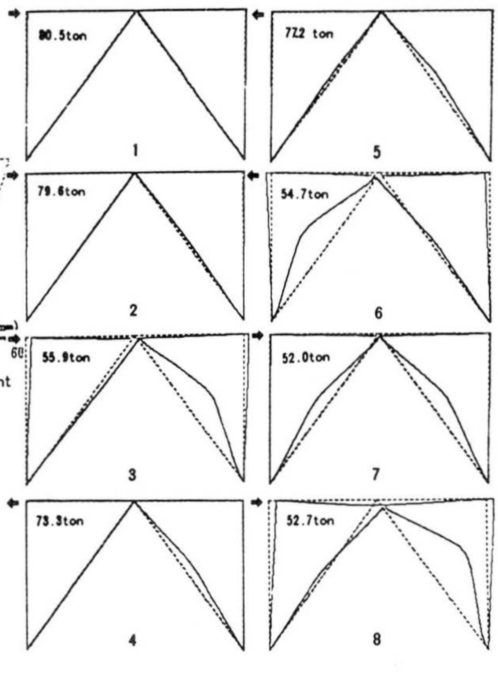

Fig. 7 Deformation of frames

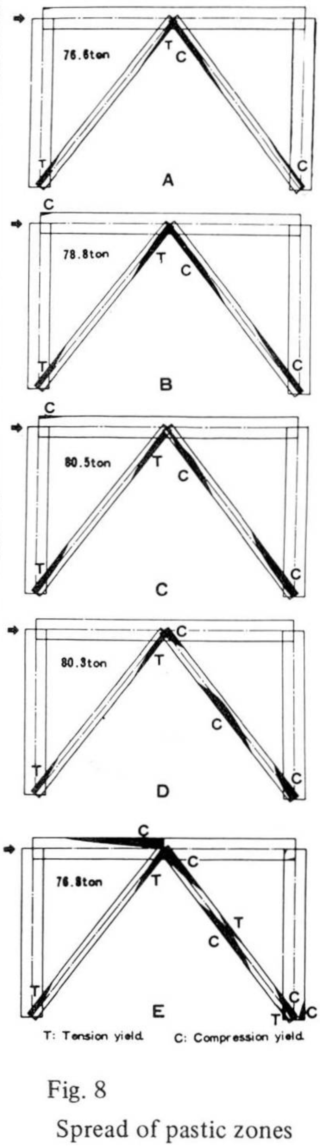

Fig. 8
Spread of pastic zones

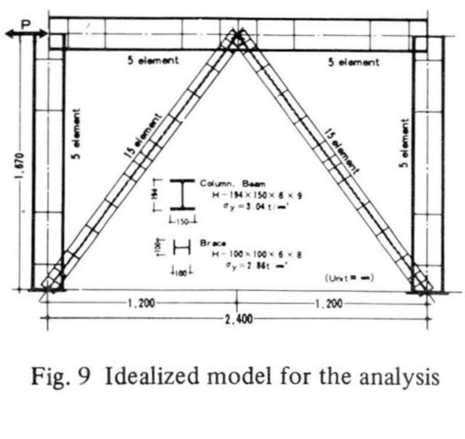

Fig. 9 Idealized model for the analysis

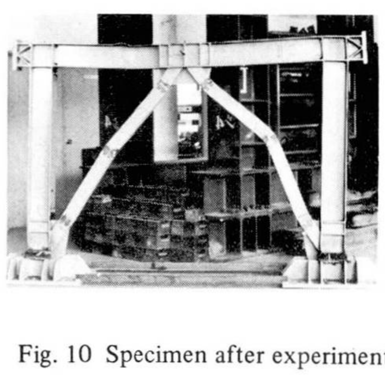

Fig. 10 Specimen after experiment

6. Acknowledgement.

We would like to express our gratitude to Dr. Eng. H. HOKUGO and Mr. K. MORIYA of Nippon Telegraph and Telephone Public Corporation for providing with experimental data, enabling us to proceed in this study. We also wish to thank Mr. K. SHIRAKATA and Mr. R. KOSUGI, who were under graduate course of master of the Tokyo Institute of Technology when this research was being carried out.

7. Bibliography

a. "The Introductory Report of IABSE Symposium 1973" autumn, 1972.

b. Fujimoto, M., Hokugo, H., Hashimoto, A., Moriya, K.,: "Study on elasto-plastic behabior of braced rigid steel frame subjected to alternately repeated horizontal load. (No. 4) Trans. of Annual Meeting of A.I.J. Oct. 1972.

c. Igarashi, S., Inoue, K., Kibayashi, M., Asano, M.:"Hysteretic characteristics of steel braced frames", Trans. of A.I.J. June, 1972.

d. M.J. Turner, E.H. Dill, H.C. Martin, R.J. Melosh: "Large deflections of structures subjected to heating and external loads", A.I.A.A. Feb. 1960.

SUMMARY

This paper deals with a nonlinear analysis of K-type braced steel frames taking consideration of both the nonlinearity due to the yield of materials and the nonlinearity based on the finite deformation theory unable to be neglected in the analysis of instability problems.

RESUME

Ce rapport traite l'analyse élasto-plastique des cadres en acier avec contreventement en K en considérant la non-linéarité due au fluage du matériau et celle basée sur la théorie des déformations finies qui ne peut être négligée dans les problèmes d'instabilité.

ZUSAMMENFASSUNG

Dieser Bericht befasst sich mit einer numerischen Untersuchung des elasto-plastischen Knickproblems in der Fachwerkebene von K-Fachwerkständern aus Stahl unter der Einwirkung statisch wechselseitig wiederholter Horizontalkraft nach der Theorie endlich grosser Verschiebungen, nebst Vergleich mit den Versuchsergebnissen.