Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 13 (1973)

Artikel: Theoretical prediction of the load-deflexion relationship of steel members and frames

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DOI: https://doi.org/10.5169/seals-13742

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I

Theoretical Prediction of the Load-Deflexion Relationship of Steel Members and Frames

Prédiction théorique de la relation charge-déformation d'éléments en acier et de cadres

Theoretische Voraussage der Last-Deformations-Beziehung von Stahlteilen und Rahmen

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1.INTRODUCTION

Failure modes of steel members and frames subject to repeated and reversed loading can be classified into three categories,that is,low-stress high-cycle fatigue,high-stress low-cycle fatigue and failure due to instability by the accumulation of deformation. For aerodynamic excitation associated with wind turbulence,it is usual to design the structures to resist elastically since the duration of it be substantially long and the response associated with resonance will tend to unlimited,accordingly high cycle fatigue problem may be paramount for such a situation. On the contrary,the duration of the earthquake ground motion is very short and the expected return period of the occurence of catastrophic earthquake is very long(say more than 100 years),though variable in different zones. For such a large but short term loading with very small expectation of occurence,it will be reasonable to allow the structures to undergo substantial plastic deformation provided that the human lives should not be lost by the collapse of the structures. So far as available experimental results and actual experiences of earthquake response indicated,it is very likely that the numbers of deformation response accompanying large plastic strain will be limited to the order of ten,thence low-cycle fatigue will not take place if severe buckle and stress concentration are carefully avoided.

In this discussion,it is assumed that all kinds of buckling are properly prevented until enough plastic deformation will develop. Under this condition, principal objective is forcussed to the prediction of the load-deformation relationship of members and frames under repeated and reserved loading,which forms the essential part of the dynamic response analysis of structures in inelastic region, and of the evaluation of the incremental collapse load(overturning by instability) of structures by earthquake. It commences with the description of the stress-strain characteristics of steel material under cyclic and reversed loading and then reports on a theoretical method for determining the moment-curvature and load-deflexion characteristics of steel members under monotonic loading. The explanation of a technique to construct the load-deflexion relationship for cyclic and reversed loading directly from the obtained monotonic loading curve will be succeeded. Finally, several experimental results of members and frames are compared with this theoretical prediction.

2.MATERIAL BEHAVIOR

Load-deformation(or $\sigma-\epsilon$) relationship of a steel bar subject to alternative tension and compression in plastic region is shown in Fig.1(1). Let unloading point at i.cycle C_i and let the corresponding load point on the curve of i+1.cycle C_i'. Segment $C_i'C_{i+1}$ is transfered rightward horizontally until C_i' coincides with C_i. If this maneuver is worked out for i=1~n, a fictitious load-deformation curve is obtained. This fictitious curve is compared with the monotonic load-deformation curve of the same specimen in Fig.2. It can be seen both are almost identical. Based upon this finding,the curve for cyclic loading can be constructed from the monotonic load-deformation(or $\sigma-\epsilon$)curve in a reverse order as is illustrated in Fig.3. The Baushinger effect cannot be included in this technique. Detailed discussion on material behavior including Baushinger effect will be done elswhere of this symposium(2).
Significant feature of the $\sigma-\epsilon$ relationship for cyclic and reversed loading is that the loop enlarges for every additional cycle. Obviously this phenomenon comes from the effect of strain-hardening.

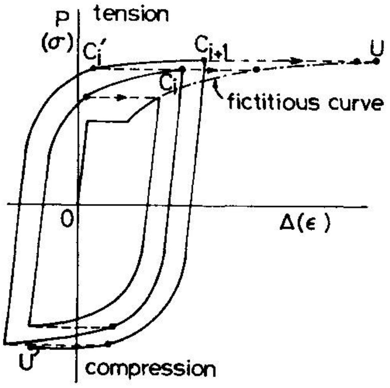

Fig.1 Stress-Strain Curve for Steel for Repeated Loading

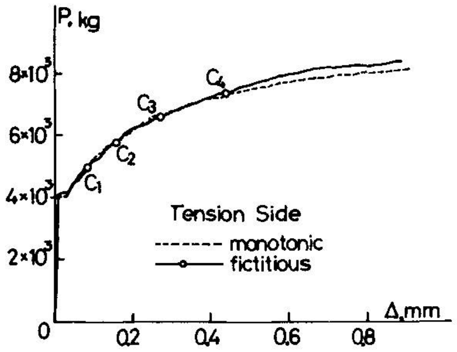

Fig.2 Comparison with Monotonic and Fictitious Curve

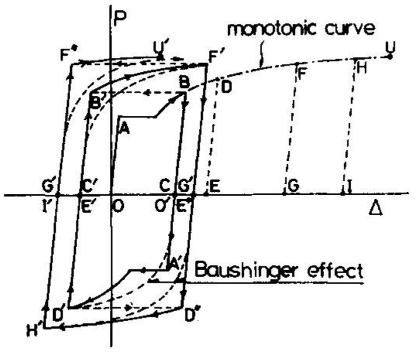

Fig.3 Construction of Cyclic Curve from Monotonic Curve

3.MOMENT-CURVATURE RELATIONSHIP

The load-deflexion characteristics of structural steel members are mainly dependent on the moment-curvature ($M-\phi$)relationships of the sections since most of the deformation arises from strains associated with flexure.
Based on $\sigma-\epsilon$ relation shown in Fig.4, $M-\phi$ relationship for H-shaped section is numerically calculated and depicted in Fig.5(3),(4). Dashed lines in the figure

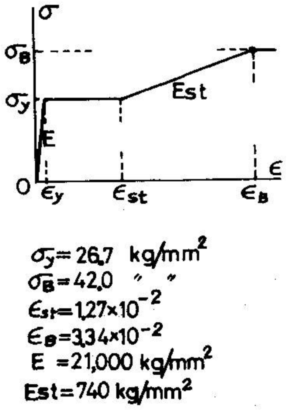

Fig.4 $\sigma-\epsilon$ Relation

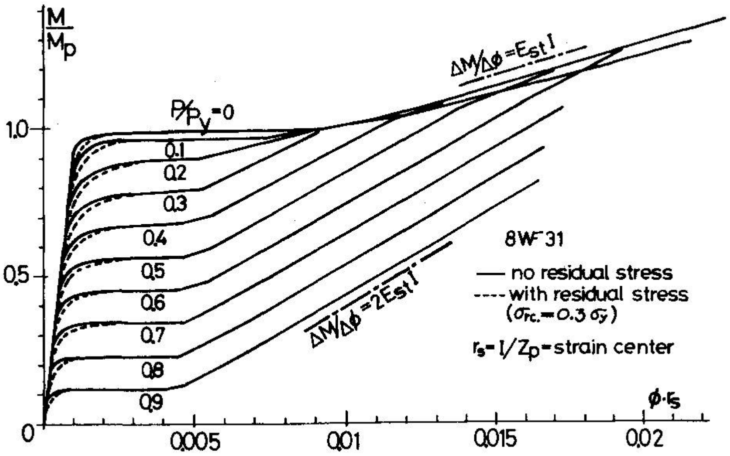

Fig.5 $M-\phi$ Relation

show the effect of residual stress. It can be seen the effect of residual stress is negligible in the region of large curvature. For a member with substantial axial thrust($P \geq 0.3P_y$),flexural rigidity in strain-hardening region is nearly twice the value of that for a member with no axial thrust(beam).

4.LOAD-DEFLEXION RELATIONSHIPS FOR MEMBERS AND FRAMES

4.1. Beams

When a member does not carry the axial compression,load-deflexion curve for monotonic loading can be readily obtained by the integration of the curvature obtained in chapter 3.

$$y=\iint \phi(M)\ dx\ dx \tag{1}$$

Since the calculation of $M-\phi$ relation means a kind of geometrical transformation of $\sigma-\epsilon$ relationship,mechanical characteristics of $\sigma-\epsilon$ relation will remain unaltered in the expression of $M-\phi$ relationship. Calculation of deflexion according to eq.(1) concerns the moment distribution along the member length only,and the relation between bending moment and applied force is linear,then the mechanical characteristics of $\sigma-\epsilon$ relation will remain unaltered in the load-deflexion relation again. From above observations,it can be assessed that load-deflexion curves for cyclic loading can be constructed from those for monotonic loading by the same technique as described for material behavior in chapter 2. Let OAU be the load-deflexion curve for monotonic loading in Fig.6,where U represents the maximum load attributed to local buckling or lateral torsional buckling or some other causes. The curve for repeated and reversed loading can be constructed from OAU as follows; The beam is first subjected to an increasing transverse load up to point B. The load is then removed at point C and re-applied in the opposite direction. For further reversed loading,the response of the beam is represented by O'A'D' which is identical to the virgin curve OAD. If the beam is unloaded again at point D',the moment will be removed at point E'. The reloading curve C'B'F' will be identical to CBF. This process is continued until point U' is reached which corresponds to the maximum load point U of the monotonic curve.

4.2.Beam-Column

Though calculation of the deflexion becomes rather complicated due to the existence of $P-\Delta$ effect, the inelastic behavior of beam-columns subject to monotonic lateral loads is fairly well understood. $P-\Delta$ effect may cause the instability of beam-columns.

When a beam-column is subjected to repeated and reversed lateral force H under constant axial compressive force P,this $P-\Delta$ moment induces the additional effect(3),which can be explained with reference to Fig.7. For simplicity of discussion,elastic component of the deformation is ignored. There is no lateral deflexion along the column until the bending moment at the base reaches the plastic moment M_{pc}. The corresponding lateral load is equal to M_{pc}/L. The response of the column for monotonic loading condition is given by OABDFU in Fig.8. If the lateral load is removed after the column has reached a deflexion Δ ,there will be a residual moment equal to $P\times\Delta$ at the base of the column. The horizontal force that is required to remove this moment is

$$H = -\frac{P\Delta}{L} \tag{2}$$

This equation is represented by line a-a' in Fig.8. Any point on this line defines a residual moment free condition for a column. Instead of horizontal axis, this line should be used as the base line in the analysis for the subsequent cycles of load application. In the construction of cyclic curve for beam-columns, this is the only difference from the case of beams.

The process of constructing the cyclic curve is illustrated in Fig.8. Note that the expansion of the loop is more pronounced than that for the case of beam. For both beams and beam-columns,it has been shown that the collupse point for cyclic loading can be predicted if the monotonic load-deflexion curves and their terminal points(U) are known.

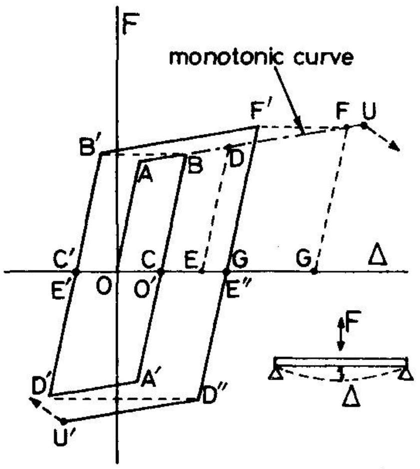

Fig.6 Construction of Cyclic Curve from Monotonic Curve of a Beam

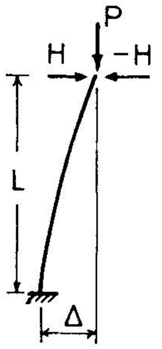

Fig.7 Cantilever Beam-Column

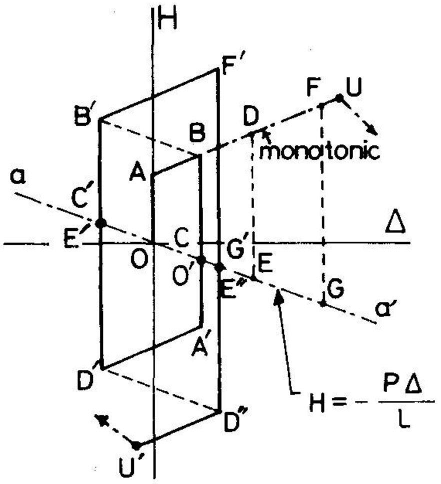

Fig.8 Construction of Cyclic Curve from Monotonic Curve of a Beam-Column

4.3. Frame

Examples of typical frame behavior subject to cyclic lateral loading are shown schematically in Fig.9*.

The behavior of frames which carry substantial vertical load is basically similar to that of beam-column, but the shape of loop varies with material property and loading condition. After attainment of full plastic moment,capacity of the member section increases further due to the strain-hardening of the material,so is the load carrying capacity of the member if it does not carry the axial compression. When this increasing rate overcomes the reducing rate due to P-Δ effect,slope in inelastic region is positive and the loop expands for every additional cycle(a1). When the increasing rate is smaller than the reducing rate due to P-Δ effect,slope in inelastic region is negative but the loop expands(a2). When there is no strain-hardening of the material,member shows elastic perfectly plastic behavior. In this case,slope is negative and the loop closes(a3). Finally when the moment capacity decreases in plastic region by local buckling or by some other causes,the slope is negative and loop diminishes(a4). The behavior of frames which carry very small vertical loads or no vertical loads is similar to that of beam,which is shown in (b) of this figure.

5.COMPARISON WITH TEST RESULTS

Available test results are compared with the prediction by the method described in the foregoing chapters in Fig.10. Except for the elastic-plastic transitive parts,for which Baushinger effect plays prominent role,theoretical predictions show satisfactory agreement with test results. Introduction of Baushinger effect and thus the modification of the curve must be made reflecting the knowledge obtained from the experimental investigation.

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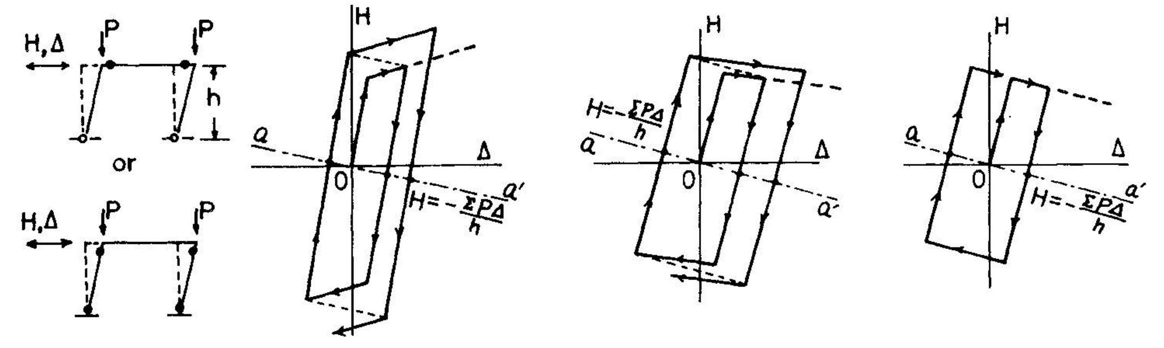
* · compare with Fig.21 of "Experimental Studies concerning Steel Structure,their Elements and Connections", Introductory Report of this Symposium (p.56),which seems to be incorrect.



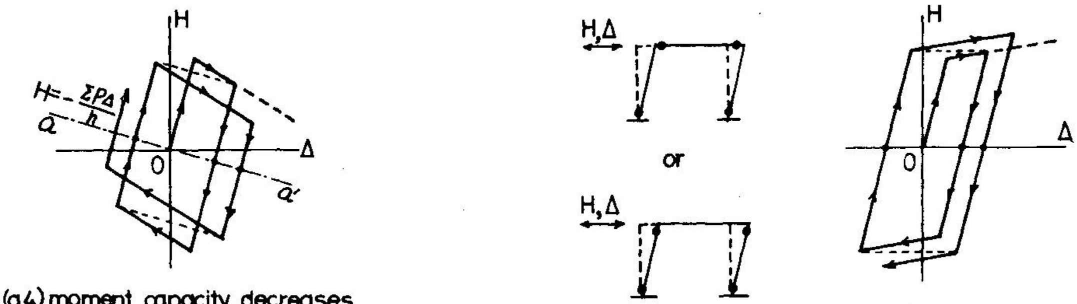
(a) with axial load

moment capacity increases over full plastic value

(a1) increasing rate overcomes the P-Δ effect

(a2) P-Δ effect overcomes the increasing rate

(a3) moment capacity keeps M_p



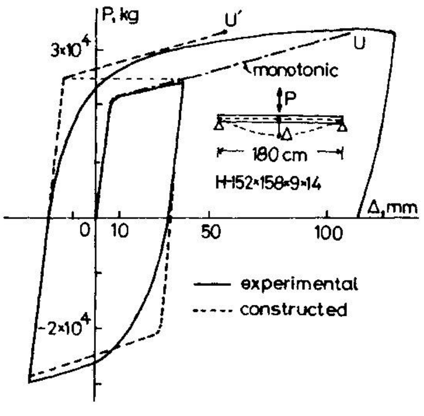
(a4) moment capacity decreases from M_p

(b) no axial loads

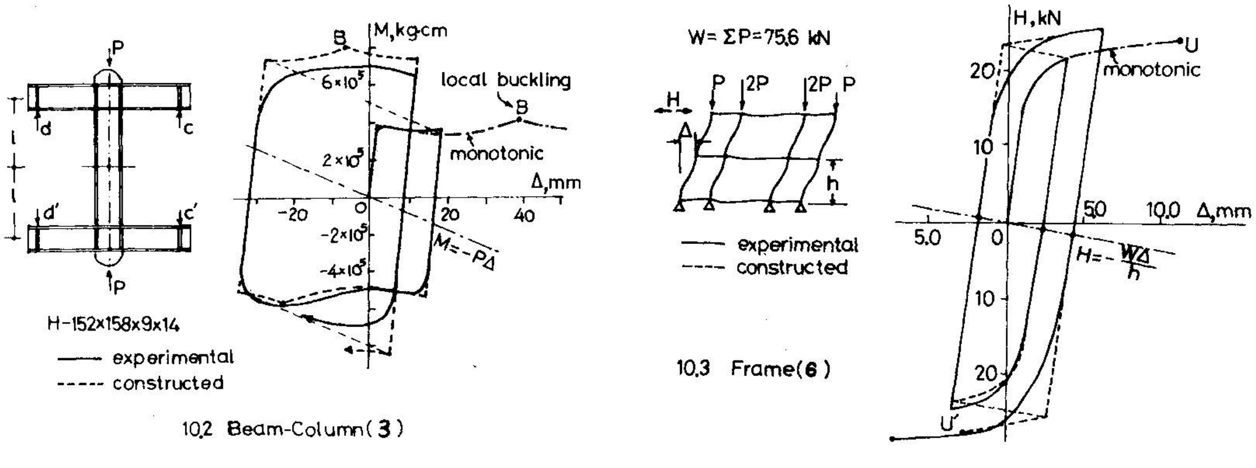
Fig.9 Various Types of Cyclic Behavior of Steel Frames

Fig.10 Correlation between Test Results and Theoretical Predictions

	Beam
Beam-Column	Frame



10.1 Beam(5)



10.2 Beam-Column(3)

10.3 Frame(6)

6. CONCLUSION

1. Load-deflexion curves of steel members and frames subject to repeated and reversed loading have the property that the loop expands for every additional cycle provided that all kinds of buckling are properly prevented.
2. This phenomenon of expansion is associated with the property of strain-hardening of steel material and/or P-Δ effect of the structural system.
3. The cyclic behavior can be predicted theoretically from the monotonic loading curve if the loading conditions are well defined.
4. Baushinger effect is not taken into account in this theory. Introduction of Baushinger effect and thus the modification of the curve must be made reflecting the knowledge obtained from the experimental investigation.

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SUMMARY

Inelastic behavior of steel members and frames subject to repeated and reversed loading was analysed, and a technique to construct the load-deflexion relationship for cyclic loading directly from that for the monotonic loading condition was presented. The correlation between this theoretical prediction and the test results were shown to be satisfactory.

RESUME

On analyse dans ce rapport le comportement inélastique d'éléments et de cadres en acier soumis à des charges répétées et alternées et on présente une méthode permettant de déterminer la relation charge-déformation pour des charges cycliques directement à partir de celle obtenue pour des charges monotones. La concordance entre les résultats théoriques et les résultats des essais est satisfaisante.

ZUSAMMENFASSUNG

Das inelastische Verhalten von Stahlstäben und Rahmen unter wiederholter und wechselseitiger Belastung wurde untersucht und eine Methode zum Aufzeichnen der Last-Ausbiegungs-Abhängigkeit für zyklische Belastung, direkt ausgehend von der einseitigen, konstanten Belastung, angegeben. Es wurde gezeigt, dass der Unterschied zwischen dieser theoretischen Voraussage und den Versuchsresultaten innerhalb befriedigender Grenzen liegt.