Automated Minimum Cost Design of Continuous Bridge Girders

Minimalisation automatique du coût des poutres de ponts à
plusieurs travées

Automatisierter Minimalkostenplan für durchlaufende Brückenträger

G.G. GOBLE   **P.V. DeSANTIS**
Professor of Structures   Case Western Reserve University
Cleveland, Ohio, USA

1. Introduction

As techniques of mass production find increased useage in steel bridges
greater refinement in their design becomes justified. Due to duplication, or
at least, automation, efforts to make relatively small cost savings are desirable.
In addition, a more precise knowledge of costs will become available. The above
factors point to the increased use of optimization methods in structural design.
However, it is infeasible to expect to apply mass production techniques to the
fabrication of large numbers of steel bridges through the production of standard
structures. The details of alignment geometry are much too varied to accomodate
standard designs. This may be possible for bridges having very short spans but
such structures are usually not fabricated of steel. It is much more likely that
mass production techniques may be applied to individual elements. This paper reports
the use of optimization techniques in the design of a continuous welded bridge
girder. A realistic scheme of cost optimization is used in design. A resulting
computer program, known as GAD I has been used by many design firms engaged in
bridge design. A modified version of this program, GAD II, is now going into
application and will be described.

2. Problem Formulation

The uniform depth girders designed by GAD II have a constant web height, a
constant flange width and a constant web thickness for the entire length of the
girder. The flange thickness changes along the girder with the top and bottom
flanges being identical. A set of stations is defined along the girder for analysis
and design purposes. The design forces are calculated at these points and any
changes in flange thickness will occur at these stations.

The design parameters which must always be preassigned are span lengths,
steel strength, stiffener type, number and location of stations, girder spacing
and loading type. Some of this information is internally stored and need not be

explicitly entered as data.

The design variables which the program selects to obtain the optimum design are web depth and thickness; flange width, thicknesses, points of change in flange thickness; bearing stiffener widths and thicknesses; longitudinal stiffener width and thickness; and transverse stiffener width, thickness and spacings. Some of these design variables can be changed into design parameters by apply proper constraints to fix their values.

The objective function for the uniform depth girder is made up of the following cost items:

1. Cost per pound of flange steel.
2. Cost per pound of web steel.
3. Cost per pound of installed bearing stiffeners.
4. Cost per pound of installed transverse stiffeners.
5. Cost per pound of installed longitudinal stiffeners.
6. Fixed cost of flange splice welds.
7. Cost per cubic inch of weld metal in flange splices.
8. Cost per lineal inch of flange splice.
9. Cost per cubic inch of flange to web welds.
10. Cost per lineal inch of web height penalty.
11. Cost per lineal inch of flange width penalty.

The cost of the flanges is found by multiplying the total weight of steel by the unit cost. No length, width or thickness penalties are used. The web steel cost is calculated in a similar manner with a different unit price. Because of the difference in the plate sizes used in the flanges and webs, the unit price might be different for the same grade of steel.

The three types of stiffeners have their costs based on a unit price per pound in place. The welding costs, setup costs and steel costs are all lumped into one term. This corresponds to cost estimating practices typically used by engineers and fabricators.

Flange splicing costs are made up of three terms. The first term is a fixed cost which is independent of the weld size. This accounts for setup time, inspection, and other fixed items encountered whenever a flange weld is made. The second term is a cost per unit volume of weld material deposited. This volume is calculated from the width of flange multiplied by the area of an equalateral right triangle whose leg is equal to the thickness of the smaller of the two plates being joined. The third term is a cost per inch of width of flange. This reflects the cost of X-rays and grinding of the surfaces.

The cost of the flange to web connection is based on the volume of deposited weld metal. Since the weld is required in each girder designed, the fixed cost and the cost per lineal inch would be constants and not enter into the optimization process. The volume of weld metal is calculated from four fillet welds running the length of the girder with legs equal to the thickness of the web. This is an approximation to the actual weld required. The final design gives the required welds for each subelement.

A penalty term is provided for the web height and the flange width. This term account for factors such as embankment costs, slender profiles and other terms which do not directly enter into the design of the girder. The penalty

cost is calculated from the unit penalty cost and the difference between the design being considered and a preferred design entered as data by the user. If the perferred design is not entered then the penalty is calculated for the total web depth and the flange width of the design.

Space does not permit a complete review of the limitations or constraints placed on the design. They are taken from the AASHO Bridge Design Specification (ref. 1) and include limits on bending stress, shear stress, flange and web width-thickness ratios, stiffener design criteria and many others. They are described in detail in References 2, 3, 4, and 5.

3. Design Procedure

The method used to obtain the best design for a given analysis combines several optimization techniques. The problem is decomposed to eliminate possible sources of local minima. The web plate thickness is a design variable which has a very pronounced effect on the total design space because of the discrete jumps in plate sizes. This makes the web plate cost a discontinuous function with thickness and also effects the possible web heights because of the limiting width to thickness ratios.

A one dimensional search is done on the possible range of web plate thicknesses. This search examines only discrete plate sizes available for the web. For each plate size the best girder design is found with its cost. The web thickness which yields the design with the least cost is the optimal design for a given analysis.

To find the best girder design for a particular web thickness a modified form of the steepest descent technique is used on the two design variables of flange width and web height. A point in the design space is defined by specifying a value for flange width and web height. For each point in the design space a complete girder is designed and its cost is calculated. The modified steepest descent technique finds the point with the least cost by a systematic search of the design space. This point yields the best girder design for the given web thickness.

In order to find the best flange thicknesses and splice locations for a given design point, the dynamic programming technique is used. There are two methods by which the flange can be designed. The first method, which is preferred, is to minimize the total cost of the flange. This dynamic programming routine balances the cost of excess plate thickness against the cost of making flange splices. The second method requires the number of splices in each span to be specified in the data set. The flange is designed such that the material used is minimized. Both of these methods design the girder one span at a time starting from the left end. Continuity at the supports is insured by forcing the plates on either side to be identical.

4. Dynamic Programming Application

The dynamic programming technique is applicable to problems where a sequence of decisions is made. The sequence of decisions which gives the best design is called the optimal policy. The girder design problem is well suited for the dynamic programming technique. The variables are discrete values of plate sizes and the system (i.e. the girder) is partitioned into subelements or stages in the nomenclature of the process. The dynamic programming concept insures that the global optimum and not a local optimum will be found.

Since the dynamic programming technique is used only as a tool to obtain the

best practical design, a detailed discussion of the theory behind the method is not warrented. Rather a sample formulation is given below. A complete numerical example for the girder problem is contained in reference 4. A complete discussion of the techniques are found in reference 5.

The example formulation will be set up for the flange of a span with uniform depth. The flange width, web height and web thickness are known at this point in the optimization process. The determination of the flange thicknesses along the span length will be done using the dynamic programming computational technique. An attempt will be made to use both the nomenclature of the optimization method and the actual engineering equivalent name. Let r_1, r_2,..., r_n be a preassigned sequence of demands, where r_k is the demand at the kth stage. These are the required minimum plate thicknesses in each of the N subelements.

Let x_k = the capability of the system at the kth stage,

$$k = 1,2,...,N$$

This represents the best plate thickness for the kth subelement. It is required that the provided plate thickness must always be greater than or equal to the required plate thickness. Therefore the following relationship must hold.

$$x_k \geq r_k \qquad k = 1,2,...,N \tag{1}$$

Two cost functions can now be set up. The first is the cost of material used in the kth stage.

$$M_k(x_k) \tag{2}$$

The second is the cost of the splice between the k and the k-1 stages. Obviously the cost is zero if x_k and x_{k-1} are the same or if k is one.

$$S_k(x_k, x_{k-1}) \tag{3}$$

Total cost of the flange is then

$$C(x_1, x_2,..., x_n) = \sum_{k=1}^{N} M_k(x_k) + S_k(x_k, x_{k-1}) \tag{4}$$

The objective is to choose the x_k, k = 1,2,...,N subject to the minimum plate conditions so as to obtain the least cost flange.

It is now necessary to transform this problem to the form required by the minimization technique. A family of problems is constructured which requires the minimization of the function

$$C_R = \sum_{k=R}^{N} [M_k(x_k) + S_k(x_k, x_{k-1})] \tag{5}$$

over the region defined by $x_k \geq r_k$, k = R,R+1,...,N with x_{R-1} = t, for R = 1,2,...,N. This requires the minimization of the girder from subelement R to the end if the plate size at subelement R-1 is set at thickness t. A new function can be defined

$$f_R(t) = \min_{x_k} C_R, \quad R = 1, 2, \ldots, N \tag{6}$$

where the minimum is taken over the x_k region defined above. This is the minimum cost of the girder from subelement R to N if subelement R-1 is set at thickness t. Then

$$f_N(t) = \min_{x_N \geq r_N} [M_N(x_N) + S_N(x_N, t)] \tag{7}$$

is the least cost of the N or end subelement if subelement N-1 is thickness t. This is easily found by direction enumeration.

The Principle of Optimality upon which the dynamic programming method is based is given below.

> An optimal policy has the property that whatever the
> initial state and initial decision are, the remaining
> decisions must constitute an optimal polich with regard
> to the state resulting from the first decision (ref. 6).

This principle states that having chosen some initial thickness x_N at the end of the span, then all policies involving that particular choice of x_N need not be examined, but rather only those policites which are optimal for a N-1 stage process.

Using the above argument the following relation can be formulated.

$$f_R(t) = \min_{x_R \geq r_R} [M_R(x_R) + S_R(x_R, t) + f_{R+1}(x_R)] \tag{8}$$
$$\text{for } R = 1, 2, \ldots, N-1$$

This is the algorithm for obtaining the computational solution of the flange selection problem.

5. Cost Parameter Study

The capabilities of this program can be illustrated with an example problem. In this case a number of designs will be obtained for a particular girder using different cost coefficients. The behavior of the design can be examined. The girder considered has three continuous spans having lengths of 80.0, 120.0 and 75.0 feet. It is designed to carry one HS20 loading including both the equivalent lane and truck load in addition to a superimposed dead load of 1.5 kilopounds per foot. The material used has a yield point of 36.0 ksi and the requirements of the Bridge Specification of the American Association of State Highway Officials, including fatigue will be satisfied. A design using transverse stiffeners will be used.

A number of cost constants were used as shown in Table 1. The cost of flange splices is increased while the material cost is held constant. It is observed that with increasing flange splicing cost the number of flange splices is reduced. Total girder weight increases since it becomes advantageous to reduce fabrication cost by using more material. Of course, since unit costs are changing it is meaningless to compare optimum costs.

TABLE I

COST PARAMETER STUDY

Mat. Cost $	Flange Splice Fixed Cost $	Flange Splice Depth Weld Metal $/in³	Web Depth in.	Web Thickness in.	Flange Width in.	Weight lbs.	No. Flange Splices
.15	10.	1.	59	3/8	12.	50,540	10
.15	10.	2.	71	7/16	10.	51,123	7
.15	10.	3.	71	7/16	10.	52,222	6
.15	10.	4.	71	7/16	10.	52,282	6
.15	20.	1.	59	3/8	12.	50,825	9
.15	20.	2.	71	7/16	10.	52,282	6
.15	20.	3.	71	7/16	10.	52,282	6
.15	20.	4.	71	7/16	10.	52,282	6

6. Conclusions

The GAD programs have been used extensively in the automated optimum design of bridge girders. These techniques are ideally suited for use in mass produced structures. In cases where mass production techniques are applied it is reasonable to expect that considerable detailed knowledge of fabrication costs will be developed. This information can be fruitfully used in designing more economical structures.

7. References

1. The American Association of State Highway Officials, "Standard Specifications for Highway Bridges", Ninth Edition, AASHO, Washington D. C., 1965.

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SUMMARY

As technique of mass production find increased usage in steel bridges greater refinement in their design becomes justified. Due to duplication, or at least, automation, efforts to make relatively small cost savings are desirable. In addition, a more precise knowledge of costs will become available. The above factors point to the increased use of optimization methods in structural design. The GAD (Girder Automated Design) programs have been used extensively in the automated optimum design of bridge girders. These techniques are ideally suited for use in mass produced structures.

RESUME

Comme la technique de la production en série trouve toujours plus d'applications dans la construction des ponts, une optimisation dans les calculs est tout indiquée Même des économies relativement faibles sont justifiées, grâce à la répétition des pièces ou à l'automation de leur fabrication. On obtient en outre un calcul plus exact des prix de revient. Ces différents avantages nous encouragent à employer des méthodes d'optimisation dans le calcul des structures. Les programmes du GAD (Girder Automated Design) ont été largement utilisés pour l'optimisation des âmes de poutres. Cette technique est particulièrement bien adaptée pour le calcul des structures produits en série.

ZUSAMMENFASSUNG

Da die Technik der Seriefertigung vermehrte Anwendung für Stahlbrücken findet, ist eine erhöhte Verfeinerung im Entwurf gerechtfertigt. Infolge der Vervielfachung oder zumindest Automation sind Bestrebungen, relativ geringe Ersparnisse zu erzielen, wünschenswert. Ausserdem wird eine genauere Kenntnis der Kosten zugänglich. Die obigen Faktoren weisen auf vermehrte Anwendung von Optimierungsmethoden im Entwurf von Bauwerken hin. Die GAD- (Girder Automated Design) Programme wurden im automatisierten optimalen Entwurf von Brückenträgern ausgiebig verwertet. Diese Technik eignet sich ideal für die Anwendung auf massengefertigte Bauten.