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**The Ultimate Bending Moment for Plate Girders**

Moment de flexion limite des poutres à âme pleine

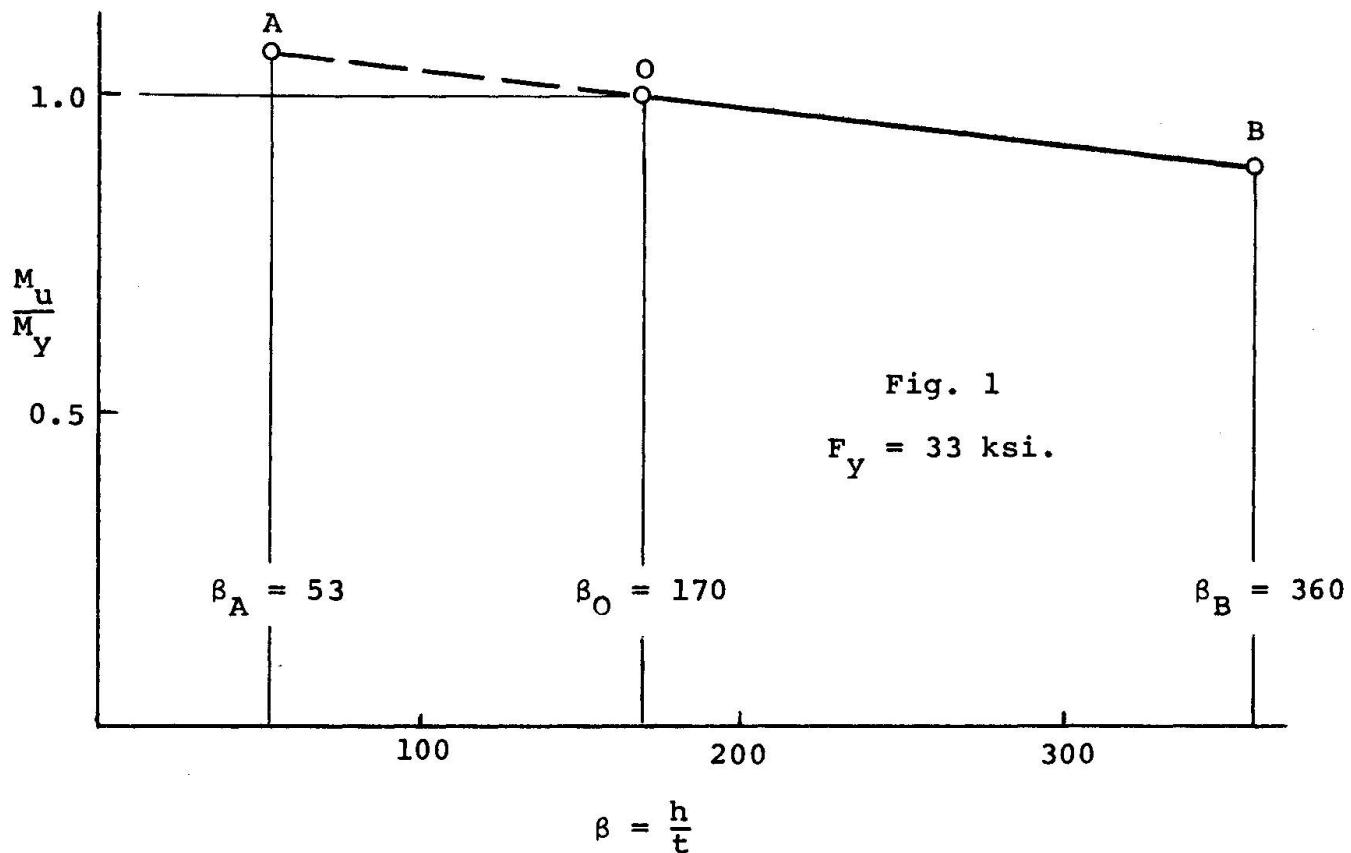
Biegetragfähigkeit von Blechträgern

**PETER B. COOPER**Department of Civil Engineering  
Kansas State University  
Manhattan, Kansas, USA**INTRODUCTION**

The purpose of this report is to review some aspects of previously developed methods for estimating the ultimate bending moment for plate girders, to examine the applicability of these methods in predicting the bending strength of test girders and to suggest some areas where additional research would be appropriate. Limited in scope to homogeneous, symmetrical girders which are statically loaded in pure bending, the report considers longitudinally stiffened as well as transversely stiffened girders.

**REVIEW OF BASLER AND THURLIMANN'S THEORY**

The basic concepts used in evaluating the bending strength of plate girders were presented by Basler and Thurlimann in 1961<sup>(1)</sup>. It was first established from ultimate load tests on plate girders subjected to pure bending that, when the applied moment exceeds the moment associated with web buckling, a redistribution of stress from the compressed portion of the web to the compression flange occurs. As a consequence of this stress redistribution, the bending strength of a girder is limited by the strength of the compression flange acting with a portion of the web as a column. Three compression flange column buckling modes were considered: torsional (local) buckling, lateral buckling and vertical buckling. As a further consequence of the stress redistribution, the ultimate bending moment must be reduced from the value calculated on the basis of a linear stress distribution.



The essential features of Basler and Thurlimann's development of an equation for the ultimate bending moment, which accounts for stress redistribution, can be described with the assistance of Fig. 1, where the ordinate is the ratio of the ultimate moment to the moment required to initiate flange yielding,  $M_u/M_y$ , and the abscissa is the web slenderness ratio,  $\beta = h/t$ .  $\beta_A$  is the highest slenderness ratio for which the full plastic moment  $M_p$  can be attained, and was taken to be 53 for  $F_y = 33 \text{ ksi}$ .  $\beta_O$  is the slenderness ratio at which web buckling would occur when the applied moment reaches  $M_y$ . Assuming partial flange restraint,  $\beta_O = 5.7/\sqrt{F_y/E}$ , which gives  $\beta_O = 170$  for  $F_y = 33 \text{ ksi}$ . At the maximum slenderness ratio permitted by a vertical buckling analysis,  $\beta_B$  ( $\beta_B = 360$  for  $F_y = 33 \text{ ksi}$ ), it was assumed that the effective section to resist bending consists of the portion on the tension side of the neutral axis plus the compression flange acting with an effective width of the web equal to  $30t$ . Values of  $M_u/M_y$  at  $\beta_A$  and  $\beta_B$  depend on the ratio of the area of the web to the area of one flange,  $A_w/A_f$ . Since a curve passing through points A, O and B is essentially a straight line, the following equation for the ultimate bending moment was adopted:

$$\frac{M_u}{M_y} = 1 - 0.0005 \frac{A_w}{A_f} \left( \beta - \frac{5.7}{\sqrt{F_y/E}} \right). \quad (1)$$

The influence of local or lateral buckling of the compression flange on the ultimate bending moment was incorporated in Eq. 1 by including the lower value of  $M_{cr}/M_y$  obtained from separate local and lateral buckling analyses,

$$\frac{M_u}{M_y} = \frac{M_{cr}}{M_y} \left\{ 1 - 0.0005 \frac{A_w}{A_f} \left( \beta - \frac{5.7}{\sqrt{\frac{M_{cr}}{M_y} \cdot \frac{F_y}{E}}} \right) \right\} \quad (2)$$

The numerical values of  $\beta_A$ ,  $\beta_o$  and  $\beta_B$  used in the development of Eq. 1 could each be modified based on more recent research findings. Research in plastic design has demonstrated that, in the absence of axial force,  $M_p$  can be attained in members with  $\beta$  - values substantially higher than  $\beta_A$ . The AISC specification<sup>(2)</sup>, for example, permits the use of members having  $\beta \leq 412/\sqrt{F_y}$  ( $\beta \leq 72$  for  $F_y = 33$  ksi.) in plastically designed structures. At least one investigator<sup>(3)</sup> has suggested that the flanges provide full fixity to the web of a welded plate girder. If full fixity is assumed, a slight increase in  $\beta_o$  to  $6.0/\sqrt{F_y/E}$  (180 for  $F_y = 33$  ksi.) would result. Experimental results cited later in this report indicate that plate girders with web slenderness ratios considerably in excess of  $\beta_B$  can be used without suffering premature vertical buckling of the compression flange.

In spite of these possible modifications to the values of  $\beta_A$ ,  $\beta_o$  and  $\beta_B$  used in developing the equation for ultimate bending moment, it has been shown that Eq. 2, when used in conjunction with appropriate local and lateral buckling equations, provides a good prediction of the observed bending strength of full size test girders.<sup>(1)</sup> Furthermore, it will be shown in the following sections that there is good reason to believe that Eq. 2 could be adopted for girders with web slenderness ratios outside the range originally proposed by Basler and Thurlimann.

#### GIRDERS WITH LOW WEB SLENDERNESS RATIOS

Basler and Thurlimann recommended that Eq. 2 be applied only when  $\beta > \beta_o$ ; thus for girders with  $\beta < \beta_o$ ,  $M_u = M_{cr}$ . For  $F_y = 36$  ksi., this corresponds

to  $\beta \leq 162$ . However, in a recently completed series of tests on eight A36 steel girders with web slenderness ratios between 61 and 123, all girders reached  $M_p$  before failure.<sup>(4)</sup> These results suggest that it would be reasonable, and probably conservative, to apply Eq. 2 in the range  $\beta_A < \beta < \beta_o$ , so that for girders with stocky webs, the calculated ultimate moment could exceed  $M_y$ . Before this extension of Eq. 5 could be permitted, it would be necessary to develop more severe local and lateral buckling requirements. The development of such requirements would be a worthwhile objective for new analytical and experimental research.

#### GIRDERS WITH HIGH WEB SLENDERSNESS RATIOS

Based on the previously mentioned vertical buckling analysis,  $\beta_B$  was originally intended to be the upper limit of the allowable web slenderness ratio for plate girders without longitudinal stiffeners. Accordingly,  $\beta_B$  also served as the upper limit of the range of applicability of Eq. 2. The results of three tests are available to investigate the possibility of using Eq. 2 to predict the bending strength of girders having  $\beta > \beta_B$ . The three specimens were fabricated from A36 steel or the equivalent, and had web slenderness ratios ranging from 388 to 751. The test results are summarized below.

Ref.	Test	$\beta$	$\frac{M_{cr}}{M_y}$	% Red.	$\frac{M_u^{ex}}{M_u^{th}}$
5	G4-T2	388	1.00	8.0	1.03
6	LBI	444	0.99	10.7	1.00
7	TTGO	751	0.98	24.4	1.01

For two of the tests,  $M_{cr}$  is slightly less than  $M_y$  according to the lateral buckling equation<sup>(1)</sup>, but in all tests, the compression flange width-thickness ratio was low enough to preclude premature local buckling. The reduction in  $M_{cr}/M_y$ , given by the second term in the brackets of Eq. 2, ranged from 8.0% to 24.4%. Each of the specimens reached the ultimate moment as a result of general yielding of the compression flange, and in tests

G4-T2 and TGO, the tests were continued beyond the ultimate load until vertical buckling of the compression flange into the web occurred. Since the experimentally observed ultimate moments  $M_u^{\text{ex}}$  were very close to the predicted values  $M_u^{\text{th}}$  based on Eq. 2, these tests indicate that Eq. 2 provides a good estimate of the bending strength of plate girders with web slenderness ratios higher than  $\beta_B$ .

#### LONGITUDINALLY STIFFENED GIRDERS

Ultimate load tests on longitudinally stiffened plate girders have indicated that longitudinal stiffeners can contribute to the bending strength by controlling lateral web deflections in the compressed portion of the web, thereby eliminating the need for the reduction in the ultimate moment represented by the second term in the brackets of Eq. 2.<sup>(8)</sup> That is, properly positioned and proportioned longitudinal stiffeners can, by controlling lateral web deflections, prevent the stress redistribution discussed previously. The increase in bending strength due to the use of longitudinal stiffeners will therefore increase with the web slenderness ratio, and will only be significant for girders with very high web slenderness ratios.

It has been suggested that longitudinal stiffeners should be located at the optimum position to increase the web buckling load.<sup>(8)</sup> For a single stiffener, a distance between the compression flange and the stiffener equal to one-fifth of the web depth has been commonly adopted. Three longitudinal stiffener proportioning requirements have been proposed for girders having a stiffener at the one-fifth depth position:

- (a) a maximum stiffener width-thickness ratio to prevent premature local buckling (for stiffeners having a rectangular cross section);
- (b) a minimum stiffener moment of inertia to force a nodal line in the deflected web up to the theoretical web buckling load; and
- (c) a minimum slenderness ratio to ensure adequate longitudinal stiffener column strength up to the ultimate moment.

For computing the stiffener moment of inertia and radius of gyration for requirements (b) and (c), a section consisting of the stiffener acting with an effective width at the web equal to  $20t$  has been suggested. Since a longitudinal stiffener, in controlling lateral web deflections, will subject the transverse stiffeners to concentrated forces, a requirement for the minimum transverse stiffener section modulus has also been formulated.

When all of the longitudinal and transverse stiffener proportioning requirements have been satisfied, the ultimate bending moment will be

$$\frac{M_u}{M_y} = \frac{M_{cr}}{M_y} \quad (3)$$

If the stiffener requirements are not satisfied, a longitudinal stiffener may still increase  $M_u/M_y$  above the value given by Eq. 2. However, no method has been developed to evaluate the magnitude of this increase; therefore, it is conservatively suggested that the influence of a longitudinal stiffener be ignored if the stiffener proportioning requirements are not satisfied.

The results of four tests, summarized below, are available to check the usefulness of Eq. 3 in estimating the bending strength of longitudinally stiffened plate girders.

Ref.	Test	$\beta$	$\frac{M_{cr}}{M_y}$	% Red.	$\frac{M_u^{ex}}{M_u^{th}}$
9	D	299	0.76	17.2 <sup>a</sup>	1.00
9	3	300	0.89	16.4 <sup>a</sup>	1.02
8	LB6	407	0.99	13.7 <sup>a</sup>	0.96
7	TG4-1	751	0.98	24.4 <sup>a</sup>	0.96

<sup>a</sup> reduction not applied in calculating  $M_u^{th}$

Constructed of structural carbon steel, each of the test specimens had a single longitudinal stiffener at the one-fifth depth position. All of the previously described stiffener requirements were satisfied in each test. Lateral buckling controlled  $M_{cr}/M_y$ , and in each case the actual length between bracing points was used in the calculations. The percent reductions shown in the table were not used in computing the predicted ultimate moments, but are listed to indicate the increase in bending strength achieved through the use of a longitudinal stiffener. The agreement between test results and predicted ultimate moments based on Eq. 3 is close enough to provide some confidence in the theory.

It would be logical to expect that, as the web slenderness ratio is increased, a point will eventually be reached where two or more longitudinal stiffeners are needed in the compression region to adequately control web deflections and prevent stress redistribution to the compression flange. Very little research on the bending strength of girders with multiple longitudinal stiffeners has been conducted to date, although some ultimate load tests have been carried out on girders having two longitudinal stiffeners.<sup>(7,9)</sup> The initial objectives of research on this topic should include the determination of the web slenderness ratio above which two longitudinal stiffeners are required to prevent stress redistribution and the development of positioning and proportioning requirements for the stiffeners.

#### LIST OF SYMBOLS

$h$	clear depth of web plate
$t$	web thickness
$A_w$	area of web
$A_f$	area of one flange
$E$	modulus of elasticity
$F_y$	yield stress
$M_{cr}$	moment at which compression flange buckling occurs
$M_p$	plastic moment
$M_u$	ultimate moment
$M_y$	yield moment
$M_u^{ex}$	experimentally measured ultimate moment
$M_u^{th}$	theoretical ultimate moment
$\beta$	web slenderness ratio ( $\beta = h/t$ )

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## SUMMARY

Basler and Thürlimann's bending strength theory for unstiffened and transversely stiffened plate girders is reviewed and discussed. Based on the available test results, the application of the theory to girders with web slenderness ratios considerably higher than the originally proposed upper limit is suggested. A bending strength theory for longitudinally stiffened plate girders is also reviewed and compared with test results. Finally, several research topics related to plate girder bending strength are suggested.

## RESUME

L'auteur discute la théorie de Basler-Thürlimann concernant la résistance à la ruine des poutres à âme pleine fléchies, non raidies ou raidies transversalement. Les résultats expérimentaux disponibles permettent de proposer une extension de la théorie à des poutres dont les âmes sont considérablement plus élancées que la limite supérieure indiquée à l'origine. De plus, on présente une théorie pour la résistance à la ruine des poutres à âme pleine fléchies, munies de raidisseurs longitudinaux, et on la compare aux résultats expérimentaux. Enfin, divers sujets apparentés sont suggérés en vue de recherches additionnelles.

## ZUSAMMENFASSUNG

Die Theorie von Basler-Thürlimann über die Tragfähigkeit von auf Biegung beanspruchten, unversteiften oder querversteiften Blechträgern wird zusammengefasst und besprochen. Auf Grund der vorhandenen Versuchsergebnisse wird eine Anwendung der Theorie auf Träger mit wesentlich über der ursprünglichen Schlankheitsgrenze liegenden Stegschlankheiten vorgeschlagen. Ferner wird eine Theorie der Tragfähigkeit von längsversteiften, auf Biegung beanspruchten Blechträgern dargestellt und mit Versuchsergebnissen verglichen. Schliesslich werden weitere Forschungsthemen auf dem gleichen Gebiet vorgeschlagen.

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