I

Limit Design of Aluminium Shear Webs

Calcul à la ruine des âmes cisaillées en aluminium

Bemessung von Aluminiumstehblechen auf Traglast

J.W. CLARK
Chief, Engineering
Design Division

M.L. SHARP
Senior Research
Engineer

Alcoa Research Laboratories
New Kensington, Pennsylvania, USA

Introduction

The object of this investigation was to develop equations for analyzing thin-web aluminum girders loaded in shear, taking into account the effect of elastic flexural rigidity of the flanges on the ultimate strength of the web, stiffeners and fasteners. Flange stiffnesses varying from zero to complete rigidity are included. Consideration is also given to the effect of web buckles on the appearance of the girder. Results are compared with experimental data for thin-web aluminum girders.

The authors believe that the factors considered in this investigation are the areas of primary importance relative to the shear strength of thin-web aluminum girders in current applications, probably the most common of which are highway van trailers and shipping containers. However, it should be noted that a number of potentially interesting areas for investigation are not included, such as combined shear and bending, longitudinal stiffeners, and plastic hinge formation in the flanges.

Analysis of Girder Web

Figure 1 illustrates three types of idealized stress distribution that can act in the web of a girder under shear loading [1, 11]. It is assumed in this analysis that at loads above the shear buckling stress, the girder continues to carry a component of pure shear equal to the shear buckling stress. Superimposed on the shear buckling stress is a component of diagonal tension of the kind shown

in Figure 1B and an additional component like that shown in Figure 1C, the ratio between the latter two components depending on the flange flexibility. It is assumed that the stresses introduced in the flanges by the tension field action are roughly of the same magnitude as stresses introduced in the stiffeners, with the result that the component of the tension field stress corresponding to rigid flanges is at an angle of $\pi/4$. The stress distributions assumed are considerable oversimplifications of the actual distribution [2, 8, 13], but are believed to be satisfactory for the purposes of this analysis.

The total shear capacity of a girder, V_T, is assumed to be the sum of the contributions from the three stress components illustrated in Fig. 1.

$$V_T = V_{cr} + V_1 + V_2 \tag{1}$$

where V_{cr} is the total shear corresponding to the shear buckling stress, τ_{cr}, and V_1 and V_2 are the shearing forces contributed by the tensions σ_1 and σ_2, respectively. These three components are [1, 11]

$$V_{cr} = \tau_{cr} ht \tag{2}$$

$$V_1 = \frac{\sigma_1 ht}{2\sqrt{1 + \alpha^2}} \tag{3}$$

$$V_2 = \frac{\sigma_2 ht}{2} \tag{4}$$

in which h is the depth of web (distance between centroids of flange fasteners), t the web thickness, and α the aspect ratio (ratio of length of shear panel s to depth of panel, h).

The total shear carried by the web is thus

$$V_T = \tau_{cr} ht + \frac{\sigma_1 ht}{2\sqrt{1 + \alpha^2}} + \frac{\sigma_2 ht}{2} \tag{5}$$

All quantities in Eq. 5 are known for a given case except σ_1 and σ_2. Reference [19] shows how σ_1 and σ_2 can be evaluated, based on a consideration of the deformations of the web and assuming that the total shear is limited by the combination of stresses which causes general yielding in the web. The following relationships result:

$$\sigma_2 = K\sigma_1 \tag{6}$$

$$K = \frac{770 \sqrt{1 + \alpha^2}}{\alpha} \left(\frac{I}{s^3 t}\right) \tag{7}$$

A. SHEAR STRESSES ; $T \leq T_{cr}$

B. ADDITIONAL STRESSES IN GIRDER WITH FLEXIBLE FLANGES.

$\phi = TAN^{-1}(\sqrt{1+\alpha^2} - \alpha)$

C. ADDITIONAL STRESSES IN GIRDER WITH RIGID FLANGES

ASSUMED STRESSES IN BUCKLED GIRDER WEB
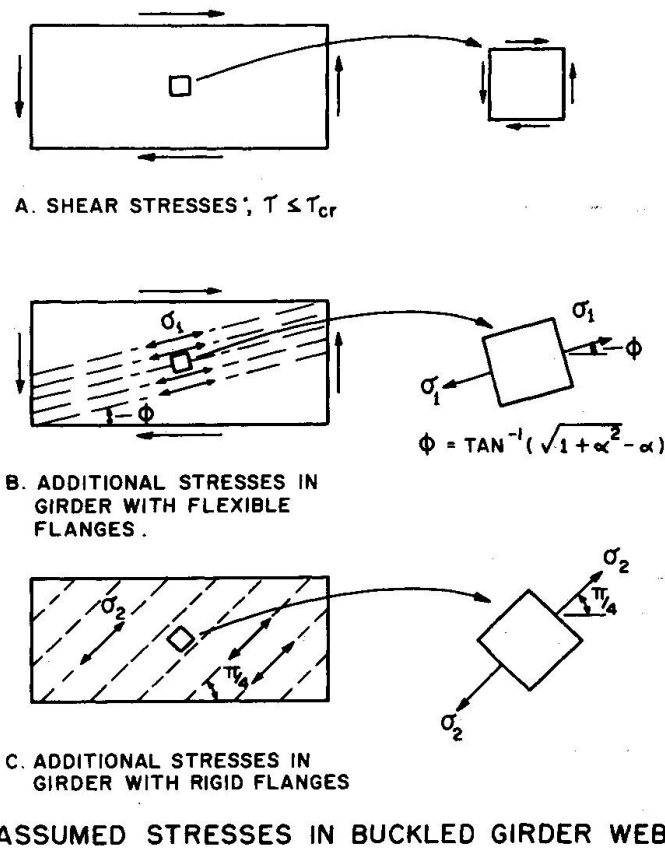
FIGURE I

in which I is the flange moment of inertia.

As shown in Ref. [19], the following equations can be derived for the average shear stress in the web at yielding, τ_T, the corresponding compressive force in the stiffeners, F_s, and the increment in flange force, F_f, due to diagonal tension:

$$\tau_T = \tau_{cr} + C_1 \frac{\sqrt{3}}{2} (\tau_y - \tau_{cr}) \qquad (8)$$

$$F_s = C_2 (s - s_e) t (\tau_T - \tau_{cr}) \qquad (9)$$

$$F_f = \frac{C_3 h t}{2} (\tau_T - \tau_{cr}) \qquad (10)$$

The coefficients in these equations are (see Fig. 2)

$$C_1 = (\frac{1}{K+1}) (\frac{1}{\sqrt{1 + \alpha^2}} + K) \qquad (11)$$

$$C_2 = \frac{\sqrt{1 + \alpha^2} - \alpha + K \sqrt{1 + \alpha^2}}{1 + K \sqrt{1 + \alpha^2}} \qquad (12)$$

$$C_3 = \frac{(K+1)\sqrt{1 + \alpha^2}}{1 + K\sqrt{1 + \alpha^2}} \qquad\qquad (13)$$

FIGURE 2

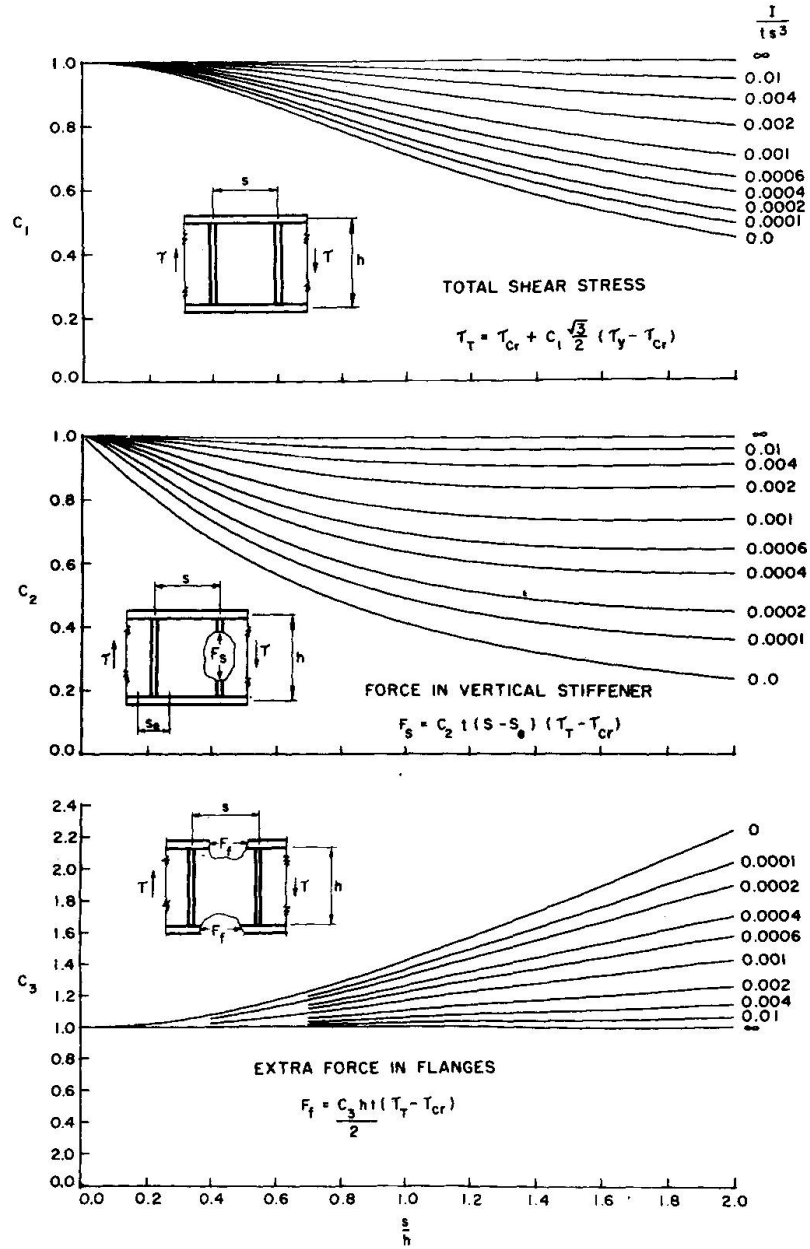In Equation 9, the effective width of web acting with the stiffener, s_e, is given by

If $\frac{s}{h} \leq 0.3$, $s_e = \frac{s}{2}$ $\qquad\qquad$ (14a)

If $\frac{s}{h} > 0.3$, $s_e = 0.15h$ $\qquad\qquad$ (14b)

The experimental basis for Eq. 14a and 14b is discussed in Ref. [19].

If the ratio I/ts^3 exceeds 0.01, little accuracy is lost by considering the flanges to be completely rigid and letting $C_1 = C_2 = C_3 = 1.0$. This is true for many applications of aluminum girders.

Shear Buckling of Web

Although practical webs have some initial out-of-flatness and thus do not undergo true buckling, tests on aluminum girders [14, 15] have shown that the theoretical buckling stress gives an approximate indication of the load at which the web deflections become large and appreciable diagonal tension develops. For design purposes, the shear buckling stress can be expressed [3]:

In the elastic range: $\lambda > C_s$

$$\tau_{cr} = \frac{\pi^2 E}{\lambda^2} \tag{15}$$

In the inelastic range: $\lambda \leq C_s$

$$\tau_{cr} = B_s - D_s \lambda \tag{16}$$

where E is the modulus of elasticity, λ the equivalent slenderness ratio for shear buckling, and B_s and D_s are coefficients that depend on the yield strength and type of alloy [3].

Cook and Rockey have discussed the numerous factors that affect buckling behavior [4, 5, 6, 17]. The value of λ which is numerically about halfway between values corresponding to fixed and simply supported edges is [3]:

$$\lambda = \frac{a}{t} \sqrt{\frac{1.6}{1 + 0.7(a/b)^2}} \tag{17}$$

where a = smaller dimension of shear panel
      b = larger dimension of shear panel

When the web plate is sandwiched between flange members or between stiffener members, a and b are equal to the clear distances between flanges and between stiffeners. For one-sided flanges or stiffeners, a and b are equal to the distance between fastener centerlines.

Intermediate Stiffeners

Stiffeners on thin-web girders perform two functions. They divide the web into panels, thereby increasing the buckling strength of the web; and when the shear stress is above the buckling stress, the stiffeners act as compression struts.

The following formulas for the stiffener moment of inertia, I_s, required to develop the shear buckling strength of the panels correspond to the design equations used in current United States specifications for aluminum structures [10, 20]:

$$\text{For } \frac{s}{h_c} \leq 0.4, \quad I_s = \frac{V_{cr} h_c^2}{0.154} \left(\frac{s}{h_c}\right) \tag{18a}$$

$$\text{For } \frac{s}{h_c} > 0.4, \quad I_s = \frac{V_{cr} h_c^2}{0.96} \left(\frac{h_c}{s}\right) \tag{18b}$$

where I_s in mm^4 is measured about the face of the sheet for stiffeners on one side and h_c is the clear height of the web in mm. V_{cr} is the shear buckling load in MN.

For many girders, stiffeners designed in accordance with Eqs. 18a and 18b will have adequate column strength in the post-buckling range. For example, if s/h is 0.4 or less and the average web stress, τ_m, does not exceed about 10 τ_{cr}, there is no need to check column strength of the stiffeners, provided that Eq. 18a is satisfied. For more severe loading the stiffener should be checked to insure that it can carry the force F_s given by Eq. 9, acting as a column with a length h_c. The use of the depth of web as the effective length of stiffener is conservative because the lateral support provided by the web is neglected [11]. In one-sided stiffeners the combined axial plus bending stress should be less than the yield stress of the material divided by a suitable factor of safety.

Kuhn [11] has noted that thin-walled stiffeners may be deformed by the buckle waves in the web and subsequently fail locally by "forced crippling" under the compression in the stiffener. Figure 3 shows that forced crippling can be avoided if the thickness of the stiffener, t_s, is:

$$t_s \geq t \left(0.65 + 0.35 \sqrt{\frac{\tau}{\tau_{cr}}}\right) \tag{19}$$

where τ is the average shear stress on the web.
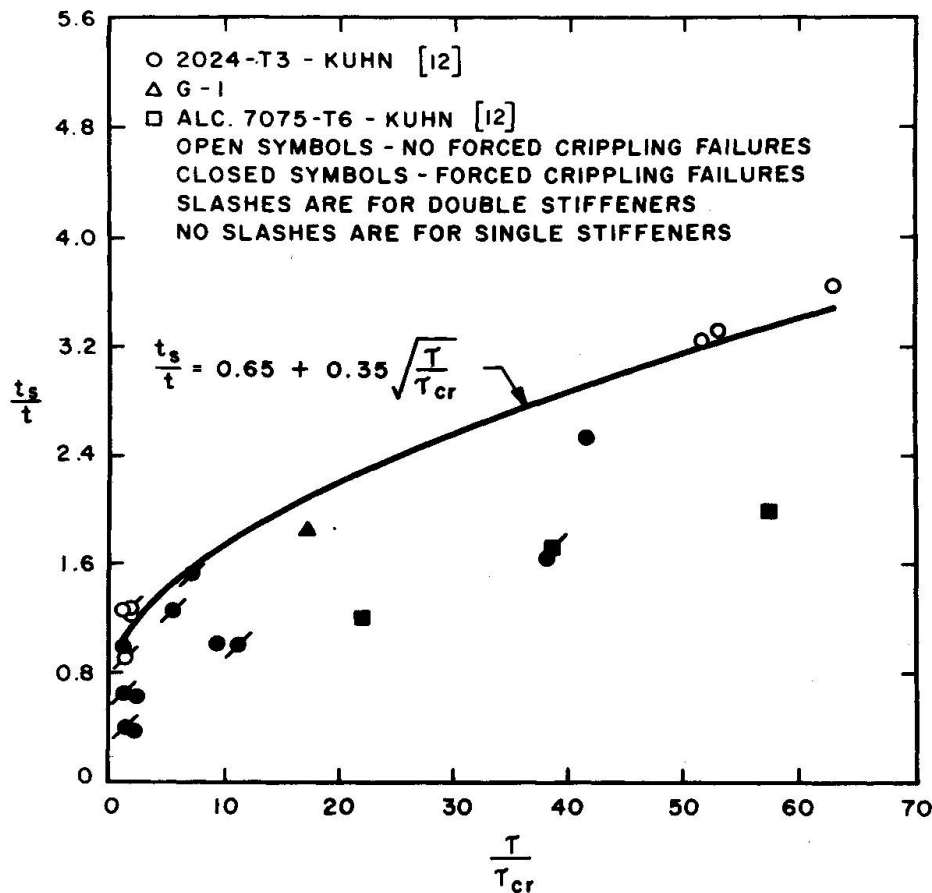
**THICKNESS OF STIFFENER TO PROHIBIT
FORCED CRIPPLING FAILURE OF STIFFENER**

FIGURE 3

Flanges

The total axial force on the flange is that from Eq. 10 plus that from beam action. The flange should be checked for local buckling of the components, torsional buckling, lateral buckling and in the case of very thin webs, flexural buckling in the plane of the web between intermediate stiffeners. In the absence of a precise analysis, it is conservative to treat the flange as a column with an unsupported length equal to the stiffener spacing.

In the analysis in Ref. [19], an expression is developed for the lateral force on the flange exerted by the web. This force causes bending of the flange in the plane of the web. In practice this bending is generally ignored, as indicated by steel design rules [1, 21] and also aircraft experience [11]. The analysis of Fujii [7] and the tests of Rockey et al [18], however, tend to show that these forces are significant for design.

Connection of Web to Flanges and Stiffeners

An extensive survey of average stresses in the web near boundary members in steel girders [22] showed that these stresses were adequately given by calculations based on shear resistant behavior. Other recent tests [9] also indicate that membrane stresses near the edges of the panel are between values calculated for tension field and shear resistant behavior but nearer to the latter. This has been corroborated by strain measurements on an aluminum girder discussed later in this paper. Thus, it should be satisfactory to calculate the strength of the connection of web to flange members as though the web were shear resistant.

The load per unit length, R_f, is:

$$R_f = \tau_T t \qquad\qquad (20)$$

The fastening between web and stiffener must at least be sufficient to develop the total load in the stiffener over one-half the length of the stiffener. To allow for some concentration of load, it has been proposed [1] to develop the load in one-third the length of the stiffener. The load per unit length, R_s, is thus:

$$R_s = \frac{3F_s}{h} \qquad\qquad (21)$$

The framing at the ends of the girder must provide anchorage for the horizontal component of the web tension force at the end. This problem may be handled by making the end panel a shear resistant panel [1].

Appearance of Girder Web

In some cases it is desirable to proportion the girder so that the buckle waves present at working loads are small and not readily noticeable [16]. Reference [19] shows that this can be accomplished by limiting the average shear stresses on the web to a value equal to the shear buckling stress plus 10 MN/m^2.

While fatigue strength is not treated in this paper, it should be borne in mind that if the shear buckling stress is exceeded at working loads, the girder will be more susceptible to fatigue cracking than would a shear resistant web [22].

Test Results

Figure 4 shows details of a riveted aluminum girder having an h/t ratio of 532. The value of I/ts^3 for this girder was 0.027, so that its flanges were relatively rigid. The web material had a tensile strength of 450 MN/m^2. Figure 5 shows that the principal stresses at the centers of
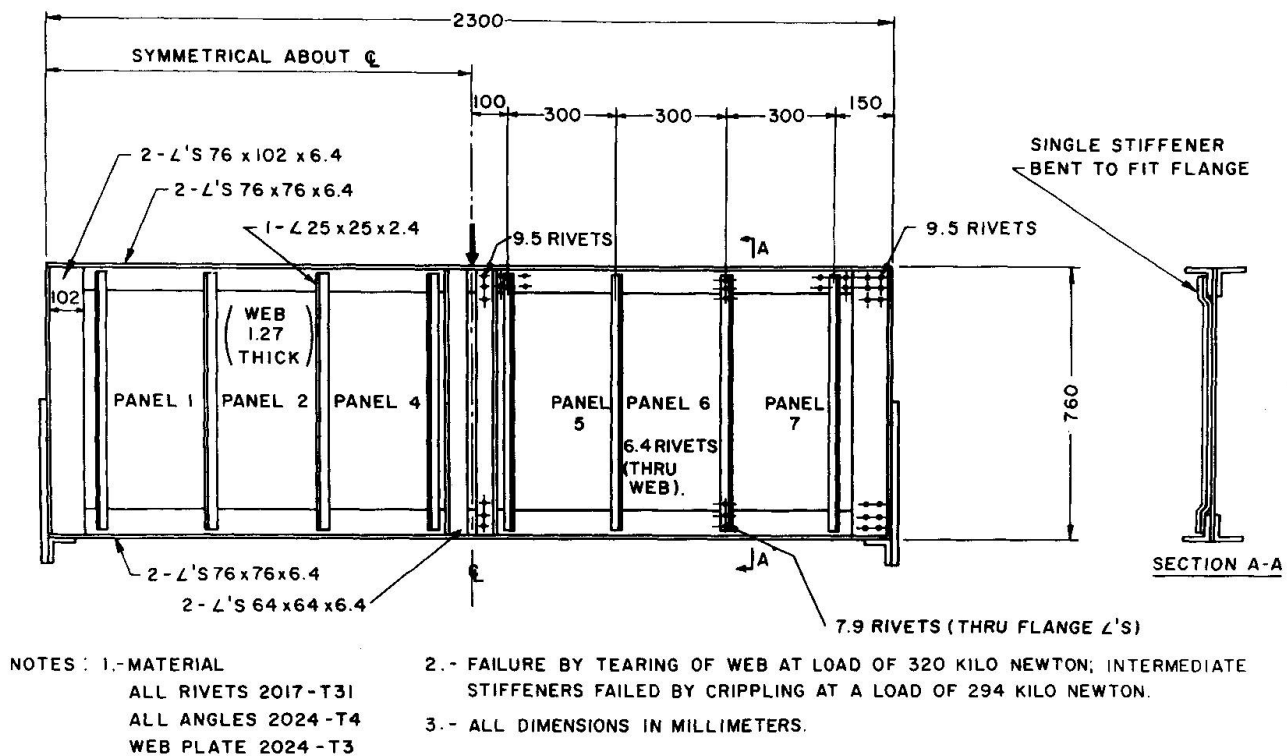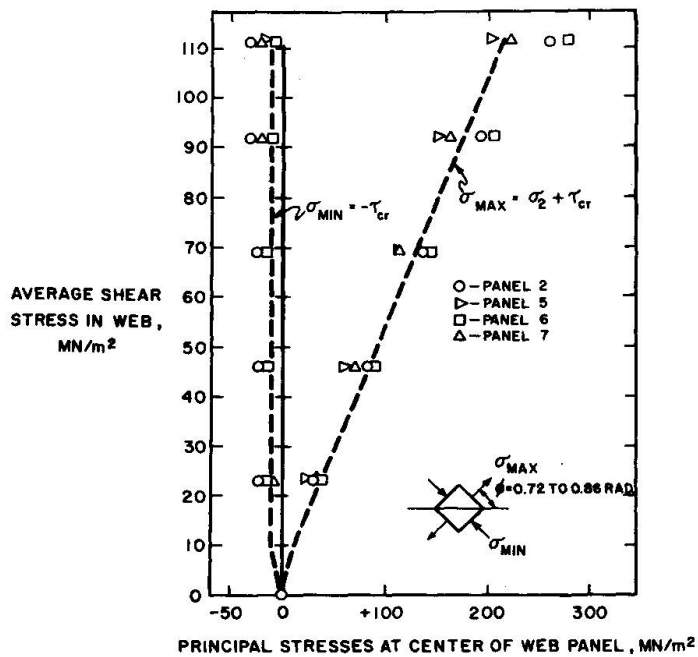
GIRDER G-I

FIGURE 4

the panels calculated using Eq. 5 are in agreement with the values from test. The stresses at the edge of the panel for this girder were intermediate to the calculated values for tension field and shear resistant webs but nearer to the values corresponding to shear resistant behavior. Figure 6 presents the portion of flange stress due to diagonal tension in the web. Equation 10 is seen to give reasonably conservative values of flange stress.

In most of the tests on aluminum girders or shear panels, the ultimate strength has been measured but not the load to cause general yielding. Shear stress values given by Eq. 8, which is based on yielding, would be expected to be conservative in comparison to the ultimate strengths, and this is generally found to be the case, as illustrated by data [12] for riveted panels given in Fig. 7.

Figure 8 compares test strengths of aluminum girders reported by Moore [14, 15] and Girder G1 with those calculated by the use of Eq. 8. In some cases the ultimate load for the girders was limited by torsional or local buckling of the compression flange so that the test values are a "lower bound" to web strength. The test data, however, tend to confirm that Eq. 8 provides a reasonable estimate of the variation of strength with flange rigidity for these girders.

CALCULATED AND MEASURED STRESSES AT THE CENTER
OF A PANEL – GIRDER G-I
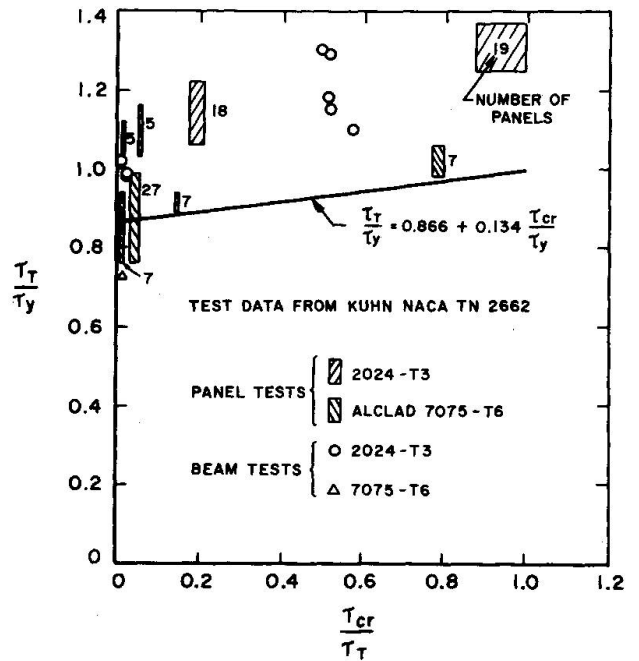
FIGURE 5

SHEAR STRENGTH OF PANELS AND
BEAMS WITH RIGID FLANGES

FIGURE 7
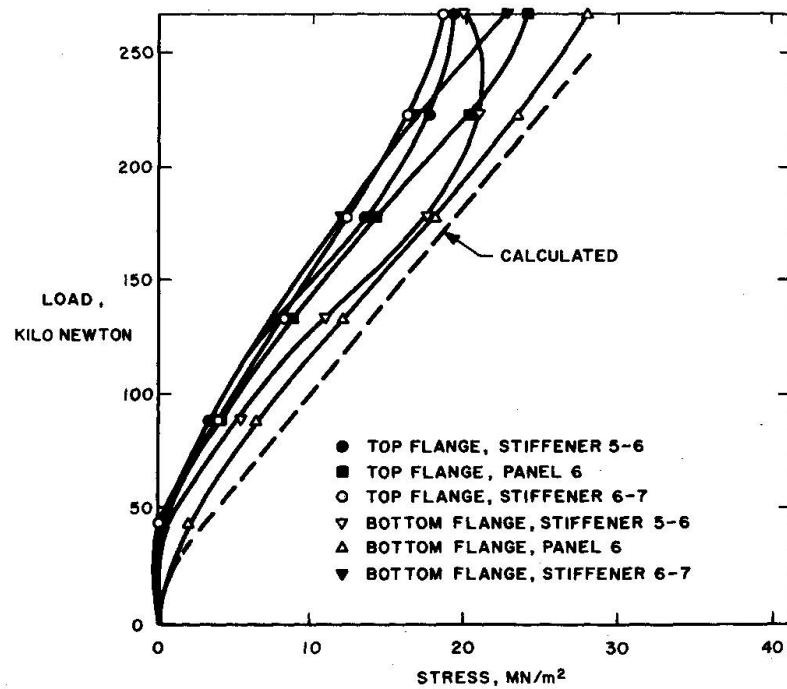
COMPRESSION STRESS IN FLANGE DUE TO DIAGONAL TENSION IN WEB

FIGURE 6
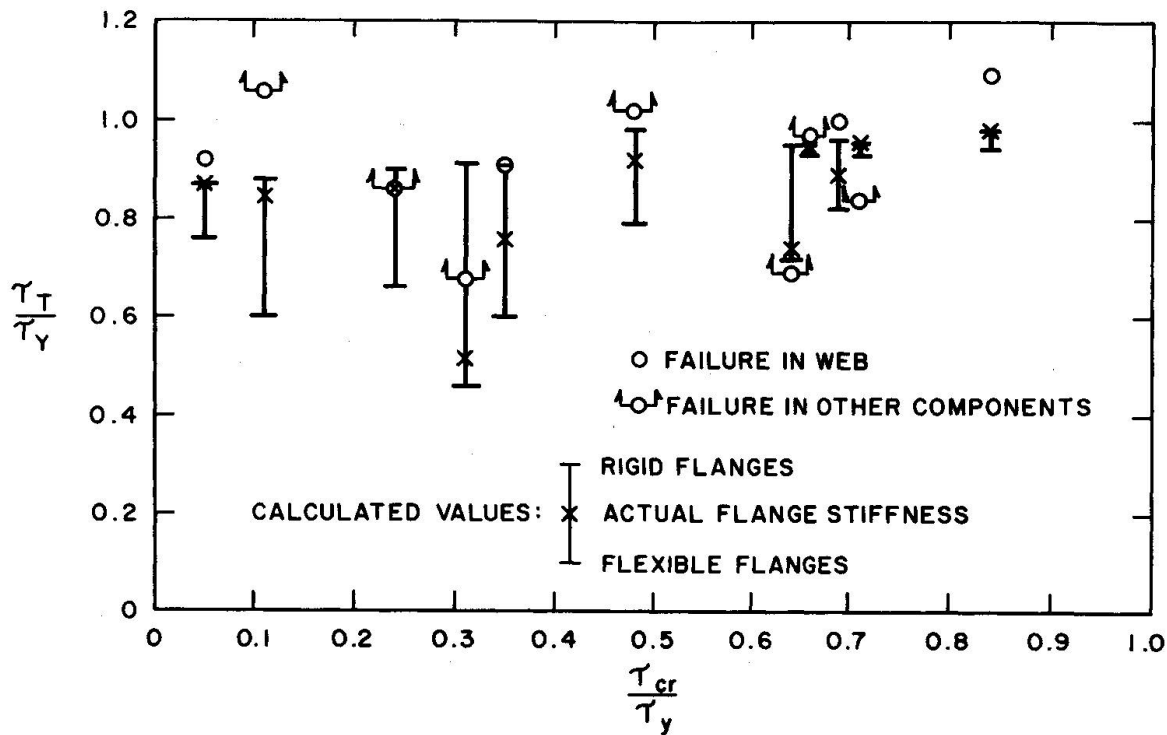
COMPARISON OF MEASURED AND CALCULATED SHEAR STRENGTHS OF ALUMINUM
GIRDERS

FIGURE 8

Conclusions

Methods have been developed in this paper for analyzing the strength of thin aluminum shear webs in girders with flanges that are either flexible, rigid, or of intermediate stiffness. The steps to be taken in the analysis are summarized in Table 1.

Table 1

Steps in Analyzing Thin Web Aluminum Girders

1. Calculate τ_{cr} (Eq. 15-17). If fatigue is a major consideration, the web shear stress should not exceed τ_{cr}. If the web must not appear too wavy at design loads, the allowable shear stress in MN/m^2 should not exceed (τ_{cr} + 10), divided by a suitable factor of safety such as 1.2.

2. Calculate I/ts^3. If it exceeds 0.01, the flange is relatively rigid, and the coefficients C_1, C_2, and C_3 in Eqs. 8, 9, and 10 can be considered as unity. If I/ts^3 < 0.01, C_1, C_2, and C_3 can be determined from Fig. 2.

3. Calculate τ_T from Eq. 8 and Fig. 2. The average shear stress in the web should not exceed τ_T divided by a suitable factor of safety, such as 1.65.

4. Check the stiffener moment of inertia I_s to make sure that it meets the requirements of Eqs. 18 a and 18b. Check the stiffener thickness by Eq. 19 to avoid forced crippling of the stiffener due to web buckling.

5. If τ_T/τ_{cr} >10 or if s/h>0.4, calculate the stiffener stress from Eq. 9 and Fig. 2. Compare with allowable column stresses.

6. Calculate the increment in flange stress from Eq. 10 and Fig. 2. Add the beam stress (Mc/I). The total flange stress must be within allowable limits for compression flanges of beams.

7. Check the spacing of flange and stiffener fasteners, using Eq. 20 and 21.

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