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A Computer Approach to the Analysis of a Free Cantilever Prestressed Concrete Bridge

Calcul électronique d'un pont en béton précontraint avec un cantilever sans support

Ein Computerverfahren zur Berechnung von vorgespannten Freivorbaubrücken

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1. INTRODUCTION

The construction of large pre-stressed concrete bridges built by the free cantilever technique is normally carried out in a series of repetative operations. The travelling carriages move the shutters forward at each end of the cantilever, the formwork positioned, and the concrete poured. As soon as the concrete has gained sufficient strength these units are pre-stressed, the cycle completed, and the carriages moved forward again. This cycle is repeated for most of the bridge, and is controlled in time by the construction program. The repetative nature of the construction readily lends itself to analysis by computer methods.

The extent of the bridge constructed, and the nature of its support will change during building - a temporary prop being used in various positions to take the out of balance moment. During construction the internal forces are statically determinate, while for final conditions they may well be indeterminate.

The behaviour of concrete is affected by shrinkage, creep and Young's Modulus, and is therefore age and time dependent as well as being affected by the previous stress history. These effects are taken into account.

The program is designed in such a way as to evaluate loss of pre-stress at each program stage and print the cumulated stresses and vertical deflections at any section and any stage, both during construction and after completion.

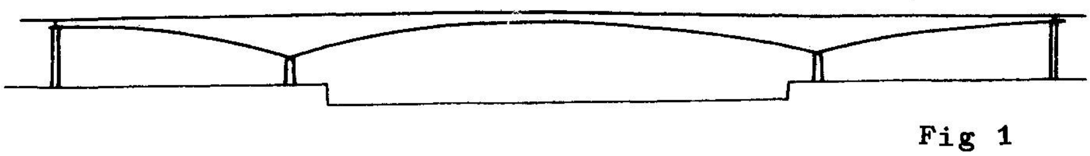


Fig 1

The computer program described was developed, for symmetrical three span bridges as shown in Fig. 1, and has been used on the Aire Bridge[1] - England, and the Kingston Bridge[2] - Scotland. Because of symmetry only half the bridge is analysed.

2. GENERAL PRINCIPLES

The program is built up from a series of routines. This is helpful not only for the purpose of the organisation of the program itself but also enables the program to be easily modified to take into account the peculiarities of a particular bridge.

In dealing with the problem of creep the Principle of Superposition of Strains[3] is adopted, creep being assumed to be linearly proportional to stress. All forces and deflexions are therefore dealt with on an incremental basis.

The cycle of construction is split into two parts:

(a) Pre-stressing and moving the carriages - always the even program stage.

(b) Pouring of the two extreme concrete units, diaphragms or ballast - normally the odd program stage.

There are several ways of solving this problem. In this program it was decided to operate on each bridge section (i) through all its stages (n) for 1<n<nt, rather than all sections through each stage. The method is perfectly valid while the structure remains statically determinate, however when this is no longer so, the pre-stress losses together with the creep deflexions have to be modified to conform with the new indeterminate boundary conditions.

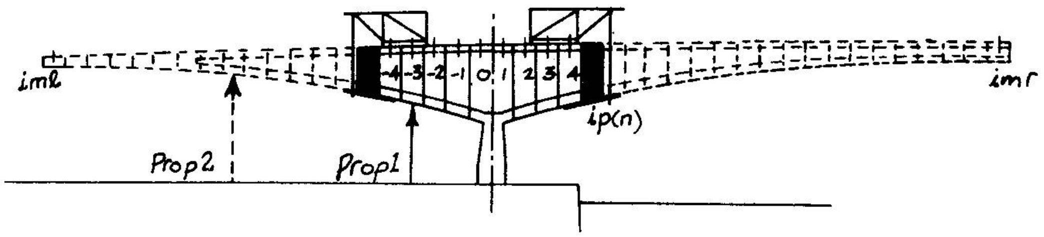


Fig 2

3. PROGRAM CONTROLS

3.1 <u>Sections</u> The bridge is split up into a series of concrete pours, the mid point of which is called a section (i) at which bending moments and stresses are evaluated - see Fig. 2, the extent of the completed half structure being defined in the data by section limits iml and imr.

3.2 <u>Extent of construction</u> The extent of construction at stage (n) is defined by routine LIMCON, ipl(n) and ipr(n) define the section limits poured, and conversely np(i) defines the stage at which section (i) is poured. The routine LIMCON is important as it defines CYCLE LIMITS for other parts of the program.

3.3 <u>Boundary conditions</u> The stage and place of boundary condition changes due to prop movement or the indeterminate nature of the completed bridge, are defined early in the program, as they affect the distribution of moments.

3.4 <u>Concrete age</u> Young's Modulus, shrinkage and creep are all time dependent properties. Time is defined by the age of the concrete-age (i,n). Routine AGECON generates this information from data read in for age (0,n) and age (i,nt). An example of part of the generated data is shown in Table 1.

3.5 <u>Section properties</u> Values of area, section modulii, second moment of area, position of centroid and eccentricity are generated by routine SECPR from the vertical profile and cross section data of the bridge.

4. LOADING

External loading is in two forms: (a) that due to the self weight of the concrete, ballast or surfacing, and (b) due to

	Age of Concrete .(days)												
	Section (i) No												
stage(n)	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	82	82	82	2	0	0	0	0	0	0	0	0	0
2	84	84	84	4	0	0	0	0	0	0	0	0	0
3	112	112	112	32	2	0	0	0	0	0	0	0	0
4	114	114	114	34	4	0	0	0	0	0	0	0	0
5	129	129	129	49	19	2	0	0	0	0	0	0	0
6	131	131	131	51	21	4	0	0	0	0	0	0	0
7	141	141	141	61	31	14	2	0	0	0	0	0	0
8	143	143	143	63	33	16	4	0	0	0	0	0	0
9	151	151	151	71	41	24	12	2	0	0	0	0	0
10	153	153	153	73	43	26	14	4	0	0	0	0	0
11	162	162	162	82	52	35	23	13	2	0	0	0	0
12	164	164	164	84	54	37	25	15	4	0	0	0	0
13	173	173	173	93	63	46	34	24	13	2	0	0	0
14	175	175	175	95	65	48	36	26	15	4	0	0	0

Table I

the movement of the carriages. In case (a) the loads are applied once in a given position while for (b) the same load is moved along the superstructure. The bending moments in the statically determinate condition is evaluated by routine CONCMTS, typical input for which is shown in Table II.

	Concrete self weight loading data			
stage(n)	W_r(tonf)	x_r(ft)	W_l(tonf)	x_l(ft)
1	275.1	15	261.1	-25
1	265.5	25	251.7	-35
3	256.9	35	240.7	-45

Table II

The bending moments due to loads applied in the indeterminate state e.g. for surfacing and finishes, are dealt with as above but modified for continuity by routine CONMTS, using the flexibility method of analysis. The moments due to the carriage movements could have been evaluated using alternate loading and unloading in one position followed by loading in the next position - tedious. A special routine CARRMTS was therefore written. The incremental bending moment due to external loading dmc(i,n) is derived from either one or a combination of the above routines.

5. PRE-STRESSING

There are various ways of inputting this information depending on the layout of the pre-stress. In Kingston Bridge most of the tendons are in the top flange of the box section, with rather smaller numbers in the soffit slab at midspan and near the abutments, none changing position from top to bottom or vice versa. The cables lie close together and so an average eccentricity could be assumed for evaluating the losses. Two routines are used, MAINBAR for the top and SOFBAR for the soffit bars. The data is input for MAINBAR as shown in Table III.

	Prestress bar data			
stage(n)	Jacking Section	No. of Bars	Anchorage Section	Position of Bars (in)
4	4	5	-4	10
6	5	4	-5	10
6	5	2	4	10

Table III

The data is first used to generate and accumulate the number of bars dnpt(i,n). The pre-stress moment dmpt(i,n) and dmps(i,n) are then evaluated from a centroid distance generated by SECPR and its position within the slab given by Column 5 in Table III. The midspan soffit tendons are-stressed while the structure is statically indeterminate, the continuity moments being then obtained by CONMTS. It is assumed in all cases that the tendons are grouted immediately after pre-stressing, so that there is no need to average the losses over the length of the tendon.

6. MATERIAL PROPERTIES

As for most bridges of this kind the time dependent properties of the concrete were examined at the early stages of the contract. Uniaxial compression tests on cylinders were carried out at various stress levels and concrete ages, under controlled temperature and humidity conditions. The results were used as a guide to form the theoretical expressions used in the program.

6.1 Young's Modulus

$$ym(t)=mk(1)./(\log_{10}(1+\log_{10}(t+1))) \quad \text{where } t=age(i,n)$$

6.2 Shrinkage For shrinkage and creep two relationships were used: (a) a logrithmic expression, giving rather high long term values -

$$\text{shrinkage strain}(t)=(-mk(2)+mk(3).\log_{10}(t+1)) \quad \text{where } t=age(i,n)$$

(b) an expression after Ross[4]

$$\text{shrinkage strain}(t)=mk(4).t./(mk(5)+mk(6).t)$$

The increment of shrinkage strain in the interval (n-1) to (n) is given by

$$dshs=\frac{mk(4).age(i,n)}{mk(5)+mk(6).age(i,n)} - \frac{mk(4).age(i,n-1)}{mk(6)+mk(6).age(i,n-1)}$$

The corresponding loss in steel stress = est.dshs. The loss of pre-stress moment for top tendons = mpt(n-1).est.dshs/po. - since the increment of shrinkage effects all previously stressed tendons by the same amount.

6.3 <u>Creep of concrete</u> The creep strain is assumed to be linearly proportional to stress. At each stage the principle of superposition of strains is used in conjunction with the stress history to evaluate the incremental creep strain. Two separate functions are used to define the creep behaviour.
(a) The ultimate specific creep, based on the age (t=age(i,m)) of the concrete when stressed, and shown in Fig. 3a.
(b) The proportion of this creep which has taken place since stressing, and shown in Fig. 3b.
The increment of specific creep in interval (n-1) to (n) is therefore given by

$$\mathrm{ddc} = \left\{ \frac{\mathrm{age}(i,n)-\mathrm{age}(i,m)}{\mathrm{mk}(8)+\mathrm{age}(i,n)-\mathrm{age}(i,m)} - \frac{\mathrm{age}(i,n-1)-\mathrm{age}(i,m)}{\mathrm{mk}(8)+\mathrm{age}(i,n-1)-\mathrm{age}(i,m)} \right\} \mathrm{eult}$$

and the increment in creep strain at the level of the top tendons due to the stress applied at stage (m) is ddc.dftt(m). The total increment of creep strain due to the previous loading history is

$$\mathrm{dctt} = \sum_{m=np(i)+1}^{n-1} \mathrm{ddc.dftt}(m) \quad ; \quad \mathrm{dcts} = \sum_{m=np(i)+1}^{n-1} \mathrm{ddc.dfts}(m)$$

for top and soffit tendons respectively. The incremental loss in pre-stress moment for top tendons at stage (n) due to creep is given by

$$\mathrm{dmlcrt} = \frac{\mathrm{est}}{\mathrm{po}}.\mathrm{dctt.mpt}(n-1)$$

Apart from its effect on loss of pre-stress, creep also produces a change of curvature which is independent of the direct stress. This is taken into account by postulating an equivalent elastic creep moment decm(i,n).

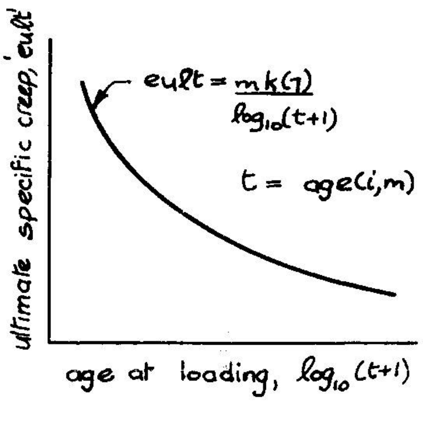


Fig 3 a

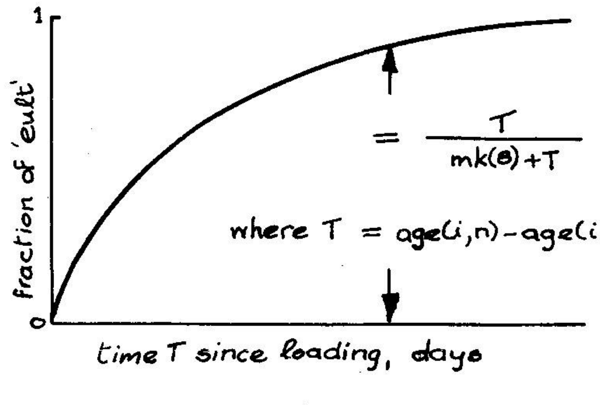


Fig 3b

6.4 <u>Relaxation in steel tendons</u> An expression such as

$$\mathrm{dpr}(n,m) = .01.\log_{10}(\mathrm{mk}(9).t+1)$$

is used to define fractional loss of pre-stress due to relaxation, where t = age(i,n)-age(i,m). The fractional loss in the interval (n-1) to (n) is dpr(n,m)-dpr(n-1,m), due to pre-stressing carried out at stage (m). The incremental loss at stage (n) due to all previous top pre-stressing bar is given by

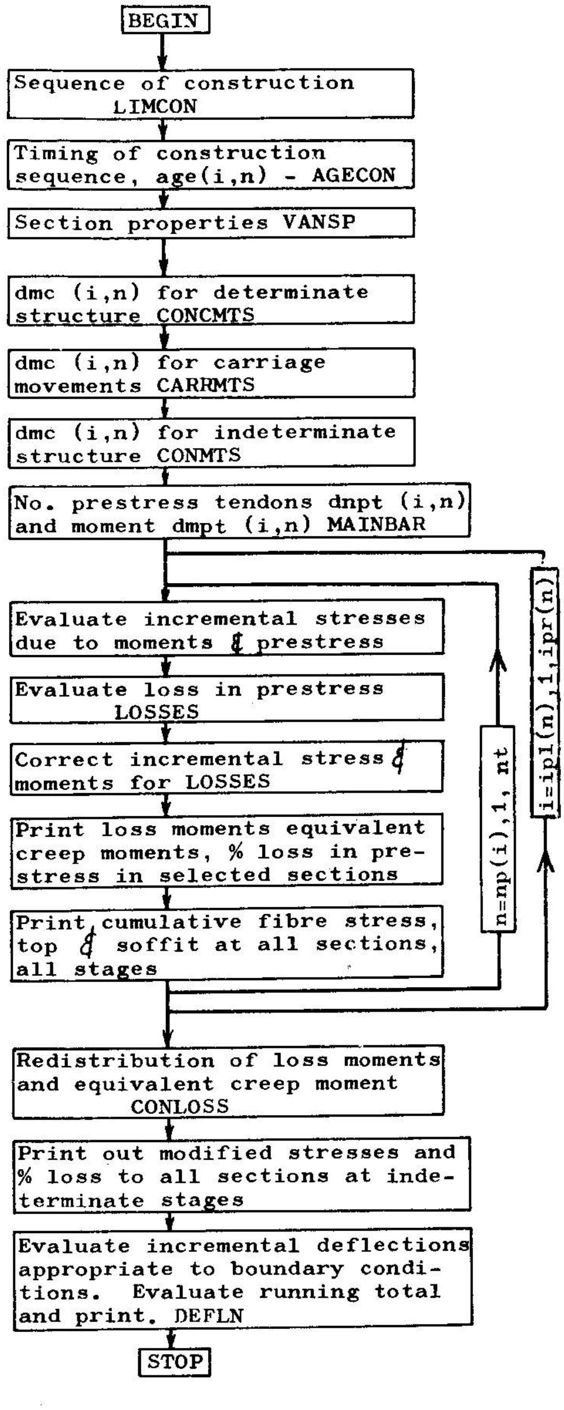
$$dmlrt = \sum_{m=np(i)+1}^{m=n-1} dmpt(i,m).(dpr(n,m)-dpr(n-1,m))$$

6.5 <u>Friction losses</u> The majority of tendons in the two bridges examined followed the curvature of the top and bottom slabs, consequently the main source of friction was wobble of the ducts. The expression for the tendon stress is

$$px = po.e^{-(mk(10).x)}$$

The loss of pre-stress moment due to friction is given by

$$dmlft = dmpt(i,m).(1-e^{-mk(10).x})$$



FLOW CHART OF COMPUTER PROGRAM

Fig 4

6.6 <u>Losses</u> The losses described above are evaluated in routine LOSSES, and individual loss moments and cumulative values for top and soffit tendons may be printed on call if required. The effect of losses taking place during the interval (n-1) to (n) are applied to the structure with the loading at stage (n).

7. PROGRAMMING

A flow chart of the program is shown in Fig. 4. It is split into four major sections.
Part 1 deals with the preparation of program controls, incremental bending moments, pre-stressing forces and moments as previously described.
Part 2 contains the routine LOSSES and deals with the incremental stresses due to moments and forces in Part 1, at a given section for each advancing stage. Prestress losses are evaluated, and stresses corrected and accumulated. The equivalent creep moment is stored. Losses and equivalent creep moments are evaluated assuming no interaction between secions. A bending moment dm(i,n), is calculated which is the sum of all the moments, losses and equivalent creep moment to serve Part 4 in evaluating the deflexions. This bending moment will also be modified in Part 3.

Where the structure finally becomes statically indeterminate the results of Part 2 are not valid. Part 3, which contains the routine CONLOSS resolves this problem, dealing with the re-distribution of loss moments and equivalent creep moments. A flow diagram of this

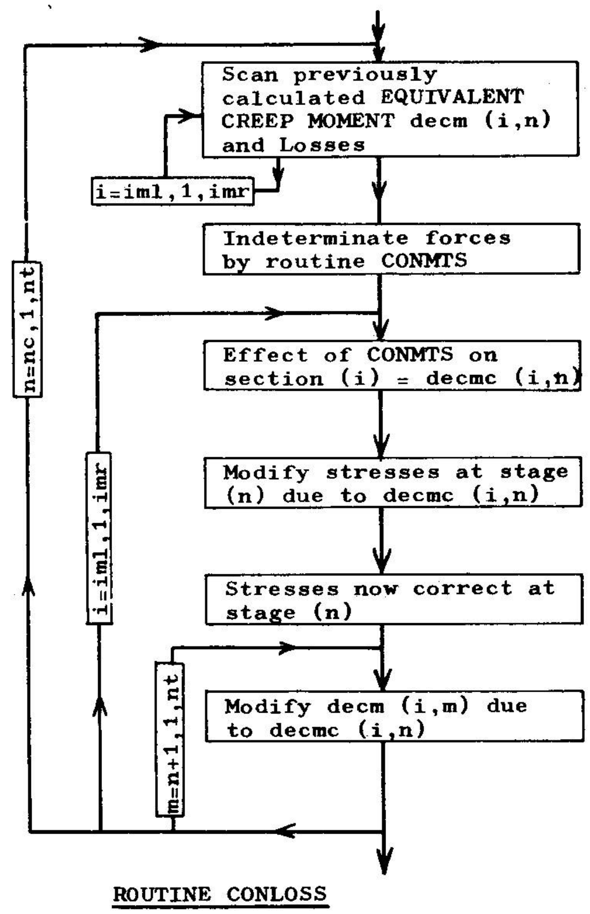


Fig 5

part of the program is shown in Fig. 5. It becomes effective at stage (nc) when the structure becomes continuous. At stage (n) the loss at each section is scanned and routine CONMTS is applied to obtain the corresponding continuity forces. The consequential stresses are evaluated at each section and added to those previously obtained. However the presence of these consequential stresses will affect creep strains at subsequent stages (m>n), so that the loss of pre-stress due to creep and the equivalent creep moments previously calculated will be modified, assuming again that there is no interaction between section.

It should be clearly understood that the previously calculated equivalent creeep moments decm (i,n) themselves are only to be used to determine creep deflection. However the equivalent creep moment distribution along the length of the bridge will (using CONMTS) produce real continuity forces decmc (i,n) at each stage (n), which will affect the stresses at that stage. When these stresses are added to those previously obtained - the stresses are then correct at stage (n). These additional stresses will also affect the creep strains at later stages. The process described in this paragraph is known as creep re-distribution.

The program will have stored and will now have available a total incremental moment dm (i,n) from which Part 4,(which includes the routine DEFLN) evaluates the incremental deflections at all sections, and prints out the accumulated deflections at each stage. The deflections are available as absolute vertical values, and also as relative to the 'attitude' of the superstructure.

The program was written in Atlas Autocode, and for the Kingston bridge 44 sections and 47 stages were considered. The program took some 9 minutes to run on the KDF 9 computer. The program now is being rewritten in Fortran.

8. CONCLUSIONS

The program described has been successfully used on two major bridges. It provides a useful tool for the designer who wishes to examine the effect of various shapes of superstructure and different layouts and stages of pre-stress. For the contractor precambering details are easily obtained from the calculated deflection. Since the deflections are printed at each section and each computer stage the deformation of the real structure may be compared with that of the computer program model at each construction stage.

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NOTATION

age(i,n)	age of concrete at section (i) and stage (n)
ddc	increment of specific creep due to stress at given age of concrete
dctt,dcts	increment of creep strain at level of top and soffit bars due to previous stress history
dshs	increment of shrinkage strain
dpr	fraction of tendon stress due to relaxation
dftt(m)	increment of concrete stress at level of top tendon at stage (m) due to direct and bending action
dmlcrt,dmlrt,dmlft,	incremental loss of top pre-stress moment due to creep, relaxation and friction respectively
decm(i,n)	equivalent creep moment
decmc(i,n)	the continuity moment resulting from a distribution of decm(i,n) to conform with new boundary conditions
dmc(i,n)	incremental bending moment due to external loading
dnpt(i,n),dmpt(i,n)	incremental number of top bars and corresponding pre-stress moment
dm(i,n)	an incremental moment taking account of all factors producing deflexion
mpt(n)	total of top pre-stress moment up to stage (n)
est	Young's Modulus for steel bars
i	section number, iml and imr defining the extreme sections of completed bridge
ipl(n),ipr(n)	section members defining extent of bridge constructed at stage (n)
mk(1)...(10)	constants defining material behaviour
m,n,nc,nt	construction stages, including continuity and final stages
np(i)	the stage at which the concrete for section (i) is poured
po	initial pre-stress on jacking
t	time in days
ym(t)	Young's Modulus of concrete at age (t) days
x	distance along tendon from jacking end

SUMMARY

The paper describes a method of calculating the prestress losses, stresses, and deflexions of large concrete bridges subject to creep re-distribution.

RESUME

La communication décrit une méthode pour calculer les pertes de précontrainte, ainsi que les contraintes et les déformations de grands ponts en béton sujets à la rédistribution par le fluage des moments fléchissants.

ZUSAMMENFASSUNG

Diese Arbeit behandelt ein Berechnungsverfahren zur Bestimmung der Vorspannverluste, Spannungen und Biegungen von grossen Betonbrücken, die der Kriechumteilung ausgesetzt sind.