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### III

#### **A Computer Approach to the Analysis of a Free Cantilever Prestressed Concrete Bridge**

Calcul électronique d'un pont en béton précontraint avec un cantilever sans support

Ein Computerverfahren zur Berechnung von vorgespannten Freivorbaubrücken

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#### **1. INTRODUCTION**

The construction of large pre-stressed concrete bridges built by the free cantilever technique is normally carried out in a series of repetitive operations. The travelling carriages move the shutters forward at each end of the cantilever, the formwork positioned, and the concrete poured. As soon as the concrete has gained sufficient strength these units are pre-stressed, the cycle completed, and the carriages moved forward again. This cycle is repeated for most of the bridge, and is controlled in time by the construction program. The repetitive nature of the construction readily lends itself to analysis by computer methods.

The extent of the bridge constructed, and the nature of its support will change during building - a temporary prop being used in various positions to take the out of balance moment. During construction the internal forces are statically determinate, while for final conditions they may well be indeterminate.

The behaviour of concrete is affected by shrinkage, creep and Young's Modulus, and is therefore age and time dependent as well as being affected by the previous stress history. These effects are taken into account.

The program is designed in such a way as to evaluate loss of pre-stress at each program stage and print the cumulated stresses and vertical deflections at any section and any stage, both during construction and after completion.



**Fig 1**

The computer program described was developed, for symmetrical three span bridges as shown in Fig. 1, and has been used on the Aire Bridge<sup>1</sup> - England, and the Kingston Bridge<sup>2</sup> - Scotland. Because of symmetry only half the bridge is analysed.

## 2. GENERAL PRINCIPLES

The program is built up from a series of routines. This is helpful not only for the purpose of the organisation of the program itself but also enables the program to be easily modified to take into account the peculiarities of a particular bridge.

In dealing with the problem of creep the Principle of Superposition of Strains<sup>3</sup> is adopted, creep being assumed to be linearly proportional to stress. All forces and deflexions are therefore dealt with on an incremental basis.

The cycle of construction is split into two parts:

- (a) Pre-stressing and moving the carriages - always the even program stage.
- (b) Pouring of the two extreme concrete units, diaphragms or bal-last - normally the odd program stage.

There are several ways of solving this problem. In this program it was decided to operate on each bridge section (i) through all its stages (n) for  $1 \leq n \leq nt$ , rather than all sections through each stage. The method is perfectly valid while the structure remains statically determinate, however when this is no longer so, the pre-stress losses together with the creep deflexions have to be modified to conform with the new indeterminate boundary conditions.

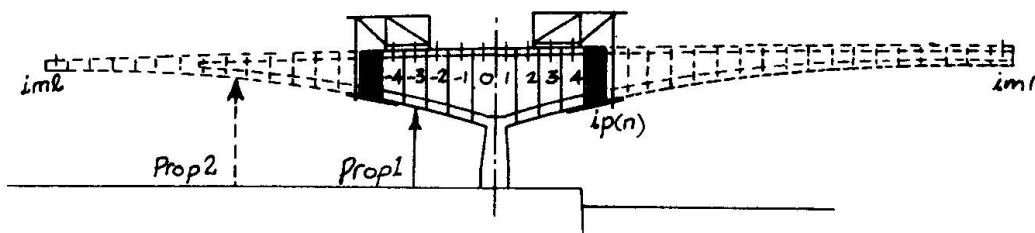


Fig 2.

## 3. PROGRAM CONTROLS

**3.1 Sections** The bridge is split up into a series of concrete pours, the mid point of which is called a section (i) at which bending moments and stresses are evaluated - see Fig. 2, the extent of the completed half structure being defined in the data by section limits *iml* and *imr*.

**3.2 Extent of construction** The extent of construction at stage (n) is defined by routine LIMCON, *ipl(n)* and *ipr(n)* define the section limits poured, and conversely *np(i)* defines the stage at which section (i) is poured. The routine LIMCON is important as it defines CYCLE LIMITS for other parts of the program.

**3.3 Boundary conditions** The stage and place of boundary condition changes due to prop movement or the indeterminate nature of the completed bridge, are defined early in the program, as they affect the distribution of moments.

**3.4 Concrete age** Young's Modulus, shrinkage and creep are all time dependent properties. Time is defined by the age of the concrete-age (i,n). Routine AGECON generates this information from data read in for age (0,n) and age (i,nt). An example of part of the generated data is shown in Table 1.

**3.5 Section properties** Values of area, section moduli, second moment of area, position of centroid and eccentricity are generated by routine SECPR from the vertical profile and cross section data of the bridge.

#### 4. LOADING

External loading is in two forms: (a) that due to the self weight of the concrete, ballast or surfacing, and (b) due to

| stage(n) | Age of Concrete (days) |     |     |    |    |    |    |    |    |   |    |    |    |
|----------|------------------------|-----|-----|----|----|----|----|----|----|---|----|----|----|
|          | Section (i) No         |     |     |    |    |    |    |    |    |   |    |    |    |
|          | 0                      | 1   | 2   | 3  | 4  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 |
| 0        | 0                      | 0   | 0   | 0  | 0  | 0  | 0  | 0  | 0  | 0 | 0  | 0  | 0  |
| 1        | 82                     | 82  | 82  | 2  | 0  | 0  | 0  | 0  | 0  | 0 | 0  | 0  | 0  |
| 2        | 84                     | 84  | 84  | 4  | 0  | 0  | 0  | 0  | 0  | 0 | 0  | 0  | 0  |
| 3        | 112                    | 112 | 112 | 32 | 2  | 0  | 0  | 0  | 0  | 0 | 0  | 0  | 0  |
| 4        | 114                    | 114 | 114 | 34 | 4  | 0  | 0  | 0  | 0  | 0 | 0  | 0  | 0  |
| 5        | 129                    | 129 | 129 | 49 | 19 | 2  | 0  | 0  | 0  | 0 | 0  | 0  | 0  |
| 6        | 131                    | 131 | 131 | 51 | 21 | 4  | 0  | 0  | 0  | 0 | 0  | 0  | 0  |
| 7        | 141                    | 141 | 141 | 61 | 31 | 14 | 2  | 0  | 0  | 0 | 0  | 0  | 0  |
| 8        | 143                    | 143 | 143 | 63 | 33 | 16 | 4  | 0  | 0  | 0 | 0  | 0  | 0  |
| 9        | 151                    | 151 | 151 | 71 | 41 | 24 | 12 | 2  | 0  | 0 | 0  | 0  | 0  |
| 10       | 153                    | 153 | 153 | 73 | 43 | 26 | 14 | 4  | 0  | 0 | 0  | 0  | 0  |
| 11       | 162                    | 162 | 162 | 82 | 52 | 35 | 23 | 13 | 2  | 0 | 0  | 0  | 0  |
| 12       | 164                    | 164 | 164 | 84 | 54 | 37 | 25 | 15 | 4  | 0 | 0  | 0  | 0  |
| 13       | 173                    | 173 | 173 | 93 | 63 | 46 | 34 | 24 | 13 | 2 | 0  | 0  | 0  |
| 14       | 175                    | 175 | 175 | 95 | 65 | 48 | 36 | 26 | 15 | 4 | 0  | 0  | 0  |

Table I

the movement of the carriages. In case (a) the loads are applied once in a given position while for (b) the same load is moved along the superstructure. The bending moments in the statically determinate condition is evaluated by routine CONCMTS, typical input for which is shown in Table II.

| stage(n) | Concrete self weight loading data |            |              |            |
|----------|-----------------------------------|------------|--------------|------------|
|          | $W_r$ (tonf)                      | $x_r$ (ft) | $W_l$ (tonf) | $x_l$ (ft) |
| 1        | 275.1                             | 15         | 261.1        | -25        |
| 1        | 265.5                             | 25         | 251.7        | -35        |
| 3        | 256.9                             | 35         | 240.7        | -45        |

Table II

The bending moments due to loads applied in the indeterminate state e.g. for surfacing and finishes, are dealt with as above but modified for continuity by routine CONMTS, using the flexibility method of analysis. The moments due to the carriage movements could have been evaluated using alternate loading and unloading in one position followed by loading in the next position - tedious. A special routine CARRMTS was therefore written. The incremental bending moment due to external loading  $dmc(i,n)$  is derived from either one or a combination of the above routines.

## 5. PRE-STRESSING

There are various ways of inputting this information depending on the layout of the pre-stress. In Kingston Bridge most of the tendons are in the top flange of the box section, with rather smaller numbers in the soffit slab at midspan and near the abutments, none changing position from top to bottom or vice versa. The cables lie close together and so an average eccentricity could be assumed for evaluating the losses. Two routines are used, MAINBAR for the top and SOFBAR for the soffit bars. The data is input for MAINBAR as shown in Table III.

| Prestress bar data |                 |             |                   |                       |
|--------------------|-----------------|-------------|-------------------|-----------------------|
| stage(n)           | Jacking Section | No. of Bars | Anchorage Section | Position of Bars (in) |
| 4                  | 4               | 5           | -4                | 10                    |
| 6                  | 5               | 4           | -5                | 10                    |
| 6                  | 5               | 2           | 4                 | 10                    |

Table III

The data is first used to generate and accumulate the number of bars  $dnpt(i,n)$ . The pre-stress moment  $dmpt(i,n)$  and  $dmps(i,n)$  are then evaluated from a centroid distance generated by SECPR and its position within the slab given by Column 5 in Table III. The midspan soffit tendons are stressed while the structure is statically indeterminate, the continuity moments being then obtained by CONMTS. It is assumed in all cases that the tendons are grouted immediately after pre-stressing, so that there is no need to average the losses over the length of the tendon.

## 6. MATERIAL PROPERTIES

As for most bridges of this kind the time dependent properties of the concrete were examined at the early stages of the contract. Uniaxial compression tests on cylinders were carried out at various stress levels and concrete ages, under controlled temperature and humidity conditions. The results were used as a guide to form the theoretical expressions used in the program.

### 6.1 Young's Modulus

$$ym(t) = mk(1) / (\log_{10}(1 + \log_{10}(t+1))) \text{ where } t = \text{age}(i,n)$$

6.2 Shrinkage For shrinkage and creep two relationships were used: (a) a logarithmic expression, giving rather high long term values -

$$\text{shrinkage strain}(t) = (-mk(2) + mk(3) \cdot \log_{10}(t+1)) \text{ where } t = \text{age}(i,n)$$

(b) an expression after Ross<sup>4</sup>

$$\text{shrinkage strain}(t) = mk(4) \cdot t / (mk(5) + mk(6) \cdot t)$$

The increment of shrinkage strain in the interval (n-1) to (n) is given by

$$dshs = \frac{mk(4) \cdot \text{age}(i,n)}{mk(5) + mk(6) \cdot \text{age}(i,n)} - \frac{mk(4) \cdot \text{age}(i,n-1)}{mk(5) + mk(6) \cdot \text{age}(i,n-1)}$$

The corresponding loss in steel stress =  $est \cdot dshs$ . The loss of

pre-stress moment for top tendons =  $mpt(n-1).est.dshts/po.$  - since the increment of shrinkage effects all previously stressed tendons by the same amount.

**6.3 Creep of concrete** The creep strain is assumed to be linearly proportional to stress. At each stage the principle of superposition of strains is used in conjunction with the stress history to evaluate the incremental creep strain. Two separate functions are used to define the creep behaviour.

(a) The ultimate specific creep, based on the age ( $t=age(i,m)$ ) of the concrete when stressed, and shown in Fig. 3a.

(b) The proportion of this creep which has taken place since stressing, and shown in Fig. 3b.

The increment of specific creep in interval (n-1) to (n) is therefore given by

$$ddc = \left\{ \frac{age(i,n)-age(i,m)}{mk(8)+age(i,n)-age(i,m)} - \frac{age(i,n-1)-age(i,m)}{mk(8)+age(i,n-1)-age(i,m)} \right\} eult$$

and the increment in creep strain at the level of the top tendons due to the stress applied at stage (m) is  $ddc.dftt(m)$ . The total increment of creep strain due to the previous loading history is

$$dctt = \sum_{m=np(i)+1}^{n-1} ddc.dftt(m) \quad ; \quad dcts = \sum_{m=np(i)+1}^{n-1} ddc.dfts(m)$$

for top and soffit tendons respectively. The incremental loss in pre-stress moment for top tendons at stage (n) due to creep is given by

$$dmlcrt = \frac{est.dctt.mpt(n-1)}{po}$$

Apart from its effect on loss of pre-stress, creep also produces a change of curvature which is independent of the direct stress. This is taken into account by postulating an equivalent elastic creep moment  $decm(i,n)$ .

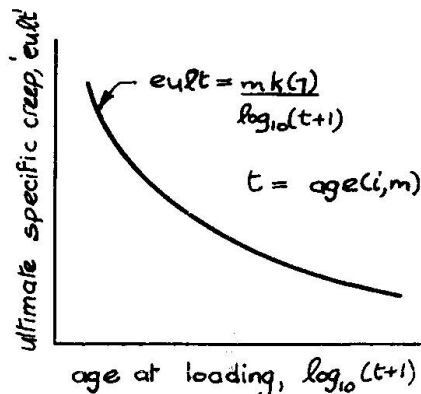


Fig 3a

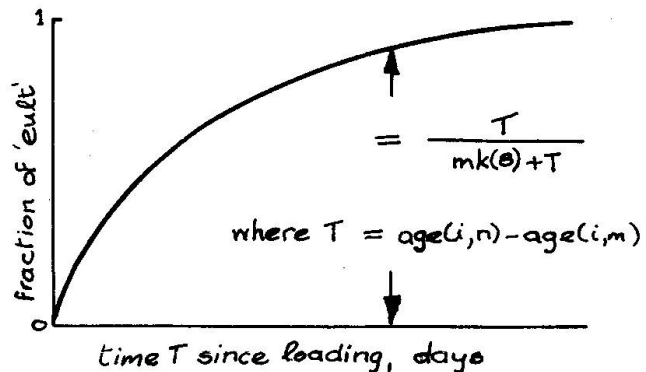


Fig 3b

**6.4 Relaxation in steel tendons** An expression such as

$$dpr(n,m) = .01.log_{10}(mk(9).t+1)$$

is used to define fractional loss of pre-stress due to relaxation, where  $t = age(i,n)-age(i,m)$ . The fractional loss in the interval (n-1) to (n) is  $dpr(n,m)-dpr(n-1,m)$ , due to pre-stressing carried out at stage (m). The incremental loss at stage (n) due to all previous top pre-stressing bar is given by

$$dmlrt = \sum_{m=np(i)+1}^{m=n-1} dmpt(i,m) \cdot (dpr(n,m) - dpr(n-1,m))$$

**6.5 Friction losses** The majority of tendons in the two bridges examined followed the curvature of the top and bottom slabs, consequently the main source of friction was wobble of the ducts. The expression for the tendon stress is

$$px = po \cdot e^{-(mk(10) \cdot x)}$$

The loss of pre-stress moment due to friction is given by

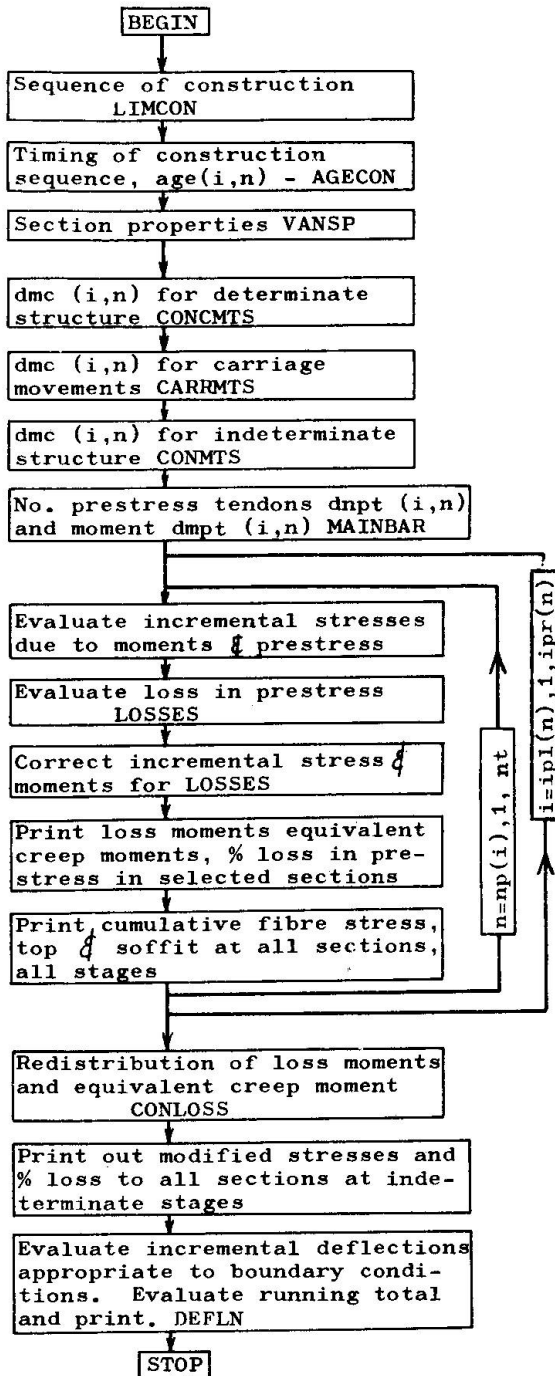
$$dmlft = dmpt(i,m) \cdot (1 - e^{-mk(10) \cdot x})$$

**6.6 Losses** The losses described above are evaluated in routine LOSSES, and individual loss moments and cumulative values for top and soffit tendons may be printed on call if required. The effect of losses taking place during the interval (n-1) to (n) are applied to the structure with the loading at stage (n).

## 7. PROGRAMMING

A flow chart of the program is shown in Fig. 4. It is split into four major sections. Part 1 deals with the preparation of program controls, incremental bending moments, pre-stressing forces and moments as previously described. Part 2 contains the routine LOSSES and deals with the incremental stresses due to moments and forces in Part 1, at a given section for each advancing stage. Pre-stress losses are evaluated, and stresses corrected and accumulated. The equivalent creep moment is stored. Losses and equivalent creep moments are evaluated assuming no interaction between sections. A bending moment  $dm(i,n)$ , is calculated which is the sum of all the moments, losses and equivalent creep moment to serve Part 4 in evaluating the deflexions. This bending moment will also be modified in Part 3.

Where the structure finally becomes statically indeterminate the results of Part 2 are not valid. Part 3, which contains the routine CONLOSS resolves this problem, dealing with the re-distribution of loss moments and equivalent creep moments. A flow diagram of this



FLOW CHART OF COMPUTER PROGRAM

Fig 4 moments. A flow diagram of this







2. Davies, G., 'Kingston Bridge, Glasgow, Report on Deflexion of the Superstructure'. Report to Messrs Logan-Marples Ridgeway Ltd. 1968.
3. McHenry, D., 'A New Aspect of Creep in Concrete and its Application to Design', Proceedings, A.S.T.M, V.5., No. 43, 1943
4. Ross, A.D., Concrete creep data. The Struct.E., V.15 pp314-26 (London 1937).

## NOTATION

|                     |  |
|---------------------|--|
| age(i,n)            | age of concrete at section (i) and stage (n)   |
| ddc                 | increment of specific creep due to stress at given age of concrete                                       |
| dctt,dcts           | increment of creep strain at level of top and soffit bars due to previous stress history                 |
| dshs                | increment of shrinkage strain  |
| dpr                 | fraction of tendon stress due to relaxation  |
| dftt(m)             | increment of concrete stress at level of top tendon at stage (m) due to direct and bending action        |
| dmlcrt,dmlrt,dmlft, | incremental loss of top pre-stress moment due to creep, relaxation and friction respectively             |
| decn(i,n)           | equivalent creep moment  |
| decnc(i,n)          | the continuity moment resulting from a distribution of decn(i,n) to conform with new boundary conditions |
| dmc(i,n)            | incremental bending moment due to external loading   |
| dnpt(i,n),dmpt(i,n) | incremental number of top bars and corresponding pre-stress moment                                       |
| dm(i,n)             | an incremental moment taking account of all factors producing deflexion                                  |
| mpt(n)              | total of top pre-stress moment up to stage (n)   |
| est                 | Young's Modulus for steel bars   |
| i                   | section number, iml and imr defining the extreme sections of completed bridge                            |
| ipl(n),ipr(n)       | section members defining extent of bridge constructed at stage (n)                                       |
| mk(1)...(10)        | constants defining material behaviour  |
| m,n,nc,nt           | construction stages, including continuity and final stages   |
| np(i)               | the stage at which the concrete for section (i) is poured  |
| po                  | initial pre-stress on jacking  |
| t                   | time in days   |
| ym(t)               | Young's Modulus of concrete at age (t) days  |
| x                   | distance along tendon from jacking end   |

## SUMMARY

The paper describes a method of calculating the prestress losses, stresses, and deflexions of large concrete bridges subject to creep re-distribution.

## RESUME

La communication décrit une méthode pour calculer les pertes de précontrainte, ainsi que les contraintes et les déformations de grands ponts en béton sujets à la redistribution par le fluage des moments fléchissants.

## ZUSAMMENFASSUNG

Diese Arbeit behandelt ein Berechnungsverfahren zur Bestimmung der Vorspannverluste, Spannungen und Biegungen von grossen Betonbrücken, die der Kriechumteilung ausgesetzt sind.