

<b>Zeitschrift:</b>	IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen
<b>Band:</b>	6 (1970)
<b>Rubrik:</b>	Free discussion

#### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

#### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

#### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 20.08.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## DISCUSSION LIBRE / FREIE DISKUSSION / FREE DISCUSSION

### Diskussionsbeitrag zum Einfluß des Kriechens und Schwindens in Stahlbetonkonstruktionen: Rückkriechen

HELMUT HÖLZENBEIN

Dipl. Ing.  
Wien, Oesterreich

Das Rückkriechen (Erholkriechen) wird in der Regel als kleiner angegeben als das Kriechen selbst. Es sei hier aber ein Beispiel wiedergegeben, das scheinbar eine andere Aussage macht.

Vor einigen Jahren wurden von Dr. SORETZ Langzeitversuche mit Stahlbetonplatten durchgeführt, bei denen sich u.a. folgendes bei der Durchbiegung ergab.

Stahlbetonplatten ( $b = 115 \text{ cm}$ ,  $d = 17 \text{ cm}$ ,  $h = 15,7 \text{ cm}$ ,  $F_e = 12 \phi 8 = 5,86 \text{ cm}^2$ ) mit Bewehrung aus TOR-Stahl 60 und aus Beton mit  $\beta_{W28} = 273 \text{ kg/cm}^2$  wurden bei konstanter Temperatur mehrmals bis zu einer Stahlspannung von  $4200 \text{ kg/cm}^2$  belastet und anschliessend wieder entlastet bis sich jeweils bei der Verformung ein stabiler Endzustand einstellte. Die Belastung erfolgte mit zwei Linienlasten in den Drittelpunkten; die Stützweite betrug 5,00 m.

Fig. 1 zeigt die Durchbiegung in Plattenmitte in Abhängigkeit von der Zeit in Tagen. Man erkennt deutlich die Belastungs- und Entlastungsperioden. Bei letzteren wirkte nur das Eigengewicht. Fig. 2 veranschaulicht die Durchbiegungen in Abhängigkeit von der Beanspruchung, ausgedrückt durch die Stahlspannung. Ab der 3. Belastung scheint dem Kriechen infolge der Last ein etwa gleich grosses Rückkriechen bei der Entlastung zu folgen. Im Diagramm entstehen Schleifen, die ein quasi-elastisches Verhalten andeuten. Dies wäre jedoch nur möglich, wenn das Rückkriechen gleich gross wie das Kriechen wäre.

Nach der 4. Belastungsperiode wurde bis zum Bruch zügig belastet (Pfeil).

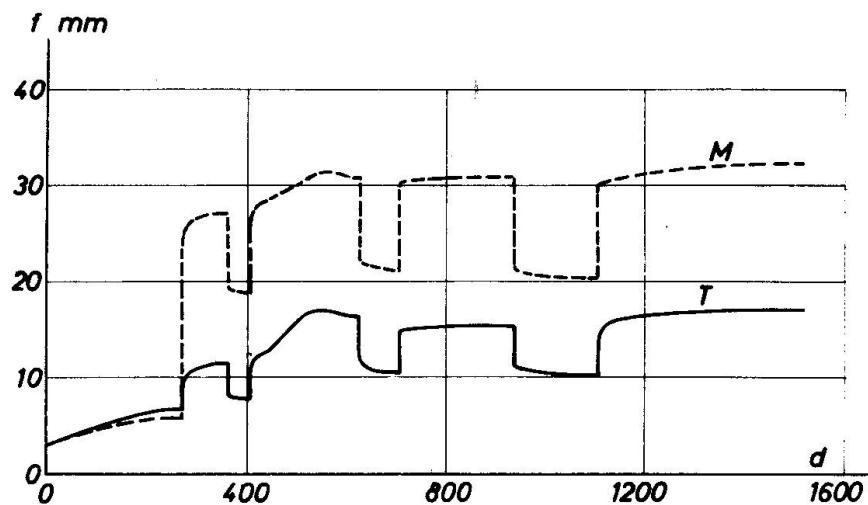


Fig. 1

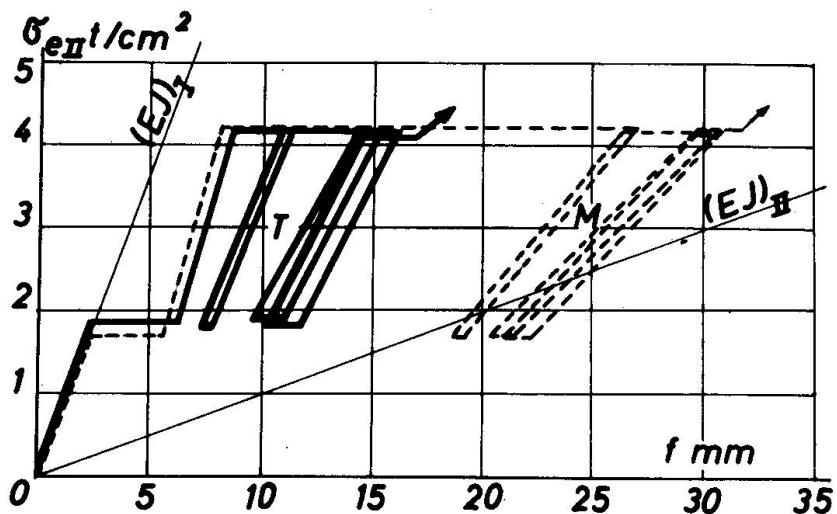
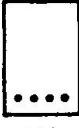
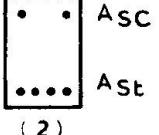
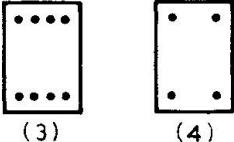


Fig. 2

**Free Discussion / Discussion libre / Freie Diskussion**

**M.R. HOLLINGTON**  
 Dr.  
 Cement and Concrete Association  
 Stough, England

The 1963 edition of the American Building Code Requirements for Reinforced Concrete (ACI 318-63) states that the additional long-term deflexion is a multiple of the instantaneous deflexion as shown in the following table :

REINFORCEMENT	SECTION	RATIO OF ADDITIONAL LONG-TERM DEFLEXION TO INSTANTANEOUS DEFLEXION
$A_{SC} = 0$	 (1)	2.0
$A_{SC} = 0.5 A_{St}$	 (2) $A_{SC}$ $A_{St}$	1.2
$A_{SC} = A_{St}$	 (3)      (4)	0.8

The results given in the Cement and Concrete Association Technical Report TRA.442 - "A series of long-term tests to investigate the deflexion of a representative precast concrete floor component" showed that the provision of compression steel in a lightly reinforced member did not reduce the long-term deflexions by the amounts predicted by the ACI Code. This investigation indicated that it was incorrect to assume that the long-term behaviour of the lightly reinforced section (4) would be the same as the heavily reinforced section (3) although the ratio of compression to tension steel areas is the same in both cases. The tests showed that section (4) behaved in a similar manner to section (2), and that the amount of restraint to long-term deformation was dependent on the arrangement of reinforcement in the compression zone, which need not be related to the area of tension steel determined by a collapse method of design as suggested by the American Code.

## II

### Behavior of Concrete under Variable Temperature and Loading

MUHAMMAD MUKADDAM

Assistant Professor  
American University of Beirut  
Lebanon

For designing concrete structures subjected to thermal gradients, the structural engineer requires the complex constitutive time and temperature dependent relations of concrete. Below, a mathematical model is developed which generates the specific compliance surface of concrete for temperatures ranging between 68°F - 200°F, age at loading varying between 28-400 days, and observation time up to 1000 days after loading. The mathematical model is developed for sealed specimen with stress/strength ratio of 35-45% or less.

#### Terminology:

specific creep = time dependent deformation/unit stress  
specific compliance = instantaneous + time dependent deformation/unit stress  
thermal strain =  $\alpha \Delta T$ , where

$\alpha$  = coefficient of thermal expansion

$\Delta T$  = change in temperature

T.S. material = thermorheologically simple material.

\* \* \*

In order to develop the mathematical model the effects of the temperature to which the concrete is subjected to are first studied. Then the effects of age of concrete at loading are discussed separately. Finally the effects of varying the two above parameters simultaneously are determined.

#### Temperature Effects:

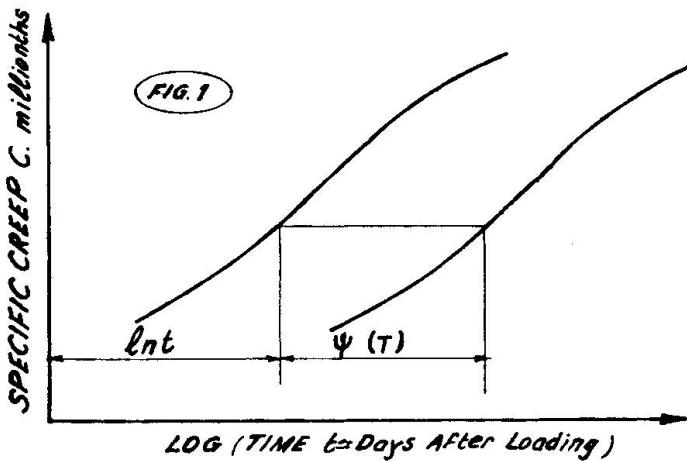
It is demonstrated that concrete under some conditions behaves as a thermorheologically simple material. In order to decide whether a material is T.S. or not one has to check whether all experimental curves of creep at different temperatures fit together by shifting them along the axis of logarithmic time.

Mathematically, the principle of shift function is simple. Let  $T_0$  be a known fixed reference temperature, and  $T$  a constant temperature  $T > T_0$ .

Then as seen from Fig. 1,

$$c_T(\ln t) = c_{T_0}(\ln t + \psi(T))$$

or  $\ln(t)$  for  $T$  curve corresponds to  $(\ln(t) + \psi(T))$  for the  $T_0$  curve. Thus  $t$  for  $T$  curve corresponds to  $\text{Exp}(\ln t + \psi(T))$  for  $T_0$  curve, but  $\text{Exp}(\ln t + \psi(T)) = t \text{Exp}(\psi(T))$  call  $\text{Exp}(\psi(T)) = \phi(T) \equiv$  shift function for temperature  $T$  having  $T_0$  as reference temperature.



Hence knowing the specific creep curve at one temperature for T.S material one can determine the specific creep curves for that material at any other constant temperature  $T$ , by simply replacing the argument  $t$  in reference curve by  $t \psi(T)$ .

The number of reliable published experimental results on creep characteristics of concrete at high temperatures is limited. Using the available data it is demonstrated that concrete behaves as a T.S material for temperatures between  $60 - 200^{\circ}\text{F}$  and age at loading of 10 days or more.

A.D. Ross (Ref 1) presented data for total strain (elastic + creep)/ksi for uniaxial cylinders loaded for 80 days at different temperatures ranging from  $68^{\circ}\text{F}$  to  $284^{\circ}\text{F}$ .

All specimens were sealed and loaded at an age of 10 days. Also included in the data is the instantaneous response on loading for different temperatures. Fig. (2) shows how good the fit is for different temperatures, ranging from  $68^{\circ}\text{F} - 176^{\circ}\text{F}$ .

Another independent set of data is obtained from Ref.(2) by D. Hannant. In this case concrete was cured for 24 hours under wet sacks before demolding, followed by five months under water at  $20^{\circ}\text{C}$  ( $68^{\circ}\text{F}$ ) and one month in the sealed saturated condition. The temperature range that the concrete was subjected to, is  $27^{\circ}\text{C} - 77^{\circ}\text{C}$  ( $80.6-170.6^{\circ}\text{F}$ ). Figure (3) shows that the shift principle leads to accurate results in this case.

#### Effect of Age at Loading:

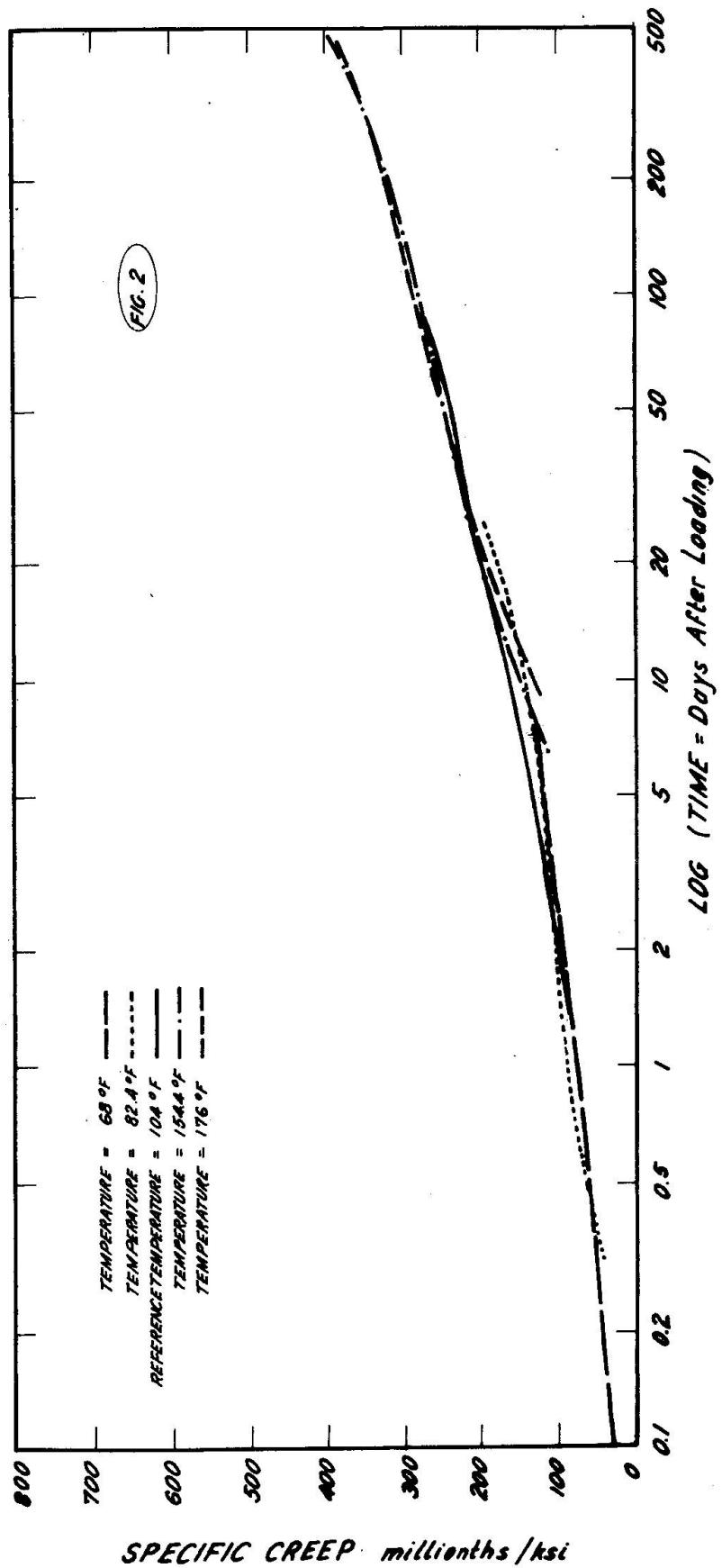
It is known that specific creep is a function of age at loading. Recently, research has been done on the mathematical form of the function representing the variation in the specific creep with a change in the age at loading of concrete.

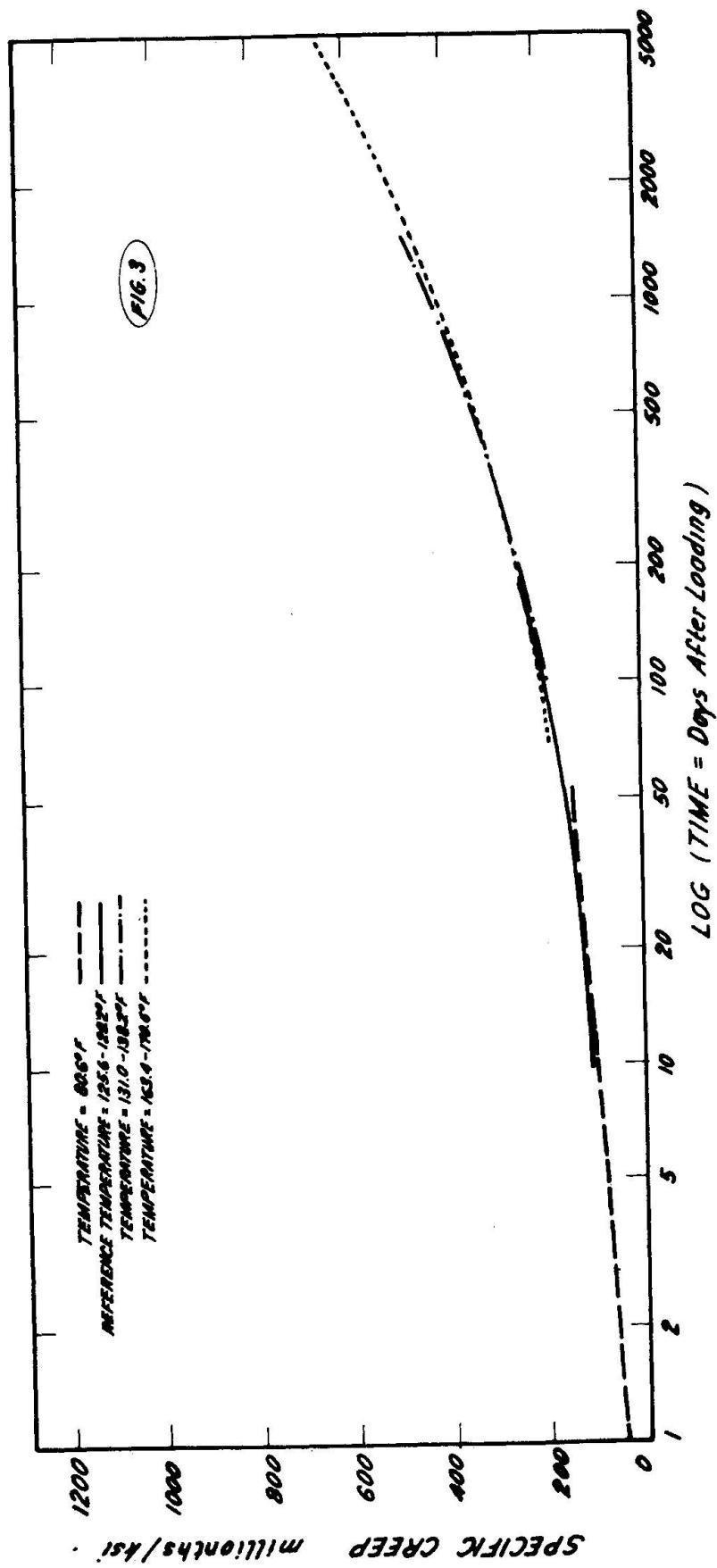
After close examination of the shapes of the specific creep curves at different ages of loading, and keeping in mind that the rate of chemical reaction in concrete is affected by temperature as well as by time, the following is proposed:

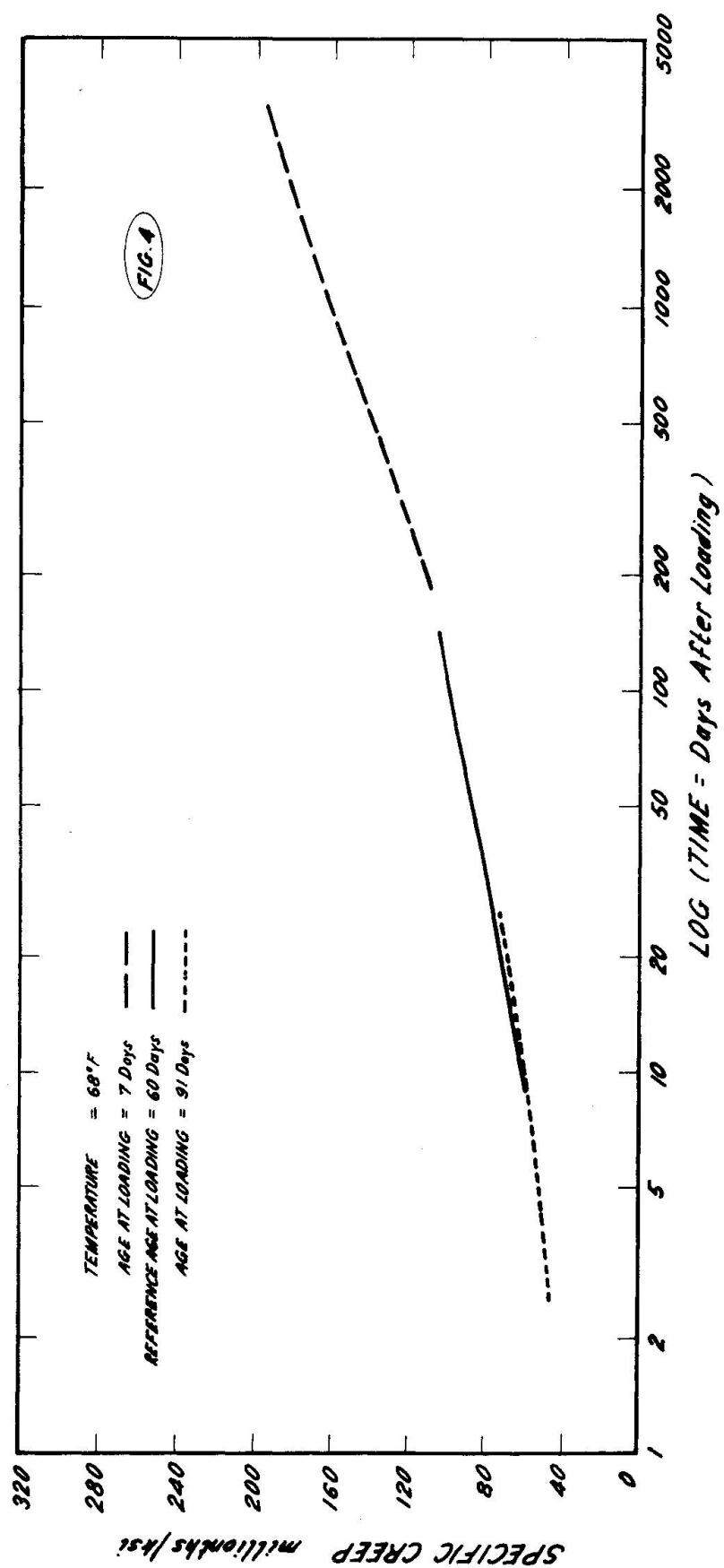
$$\text{If } C(K_0) = \sum_{i=1}^n C_i e^{-\lambda_i t}$$

represents the specific creep at a certain reference age at loading  $K_0$ ,

where  $C_i$ ,  $\lambda_i$  are constant coefficients, and  $t$  is time in days after loading.







$$\text{then } C_{(K)} = \sum_{i=1}^n c_i e^{-h_i t} \psi_i(K)$$

represents the specific creep of the concrete loaded at a different age K. This is similar to temperature shift function principle.

$\psi_i(K)$  is the shift function for ageing of the concrete.

To check the validity of this proposal, the same data used by Selna Ref. (3) and presented by A. Ross (4) is used. Figures (4,5) show how good the proposed model is. In addition to giving good results, this method is simple to apply, gives better interpolation values than Selna's Model, and much better extrapolation values for specific creep. This principle leads to a compatible formulation for the simultaneous consideration of temperature and age at loading on the creep characteristics of concrete.

#### Combined Effects of Temperature and Ageing on Creep Characteristics of Concrete:

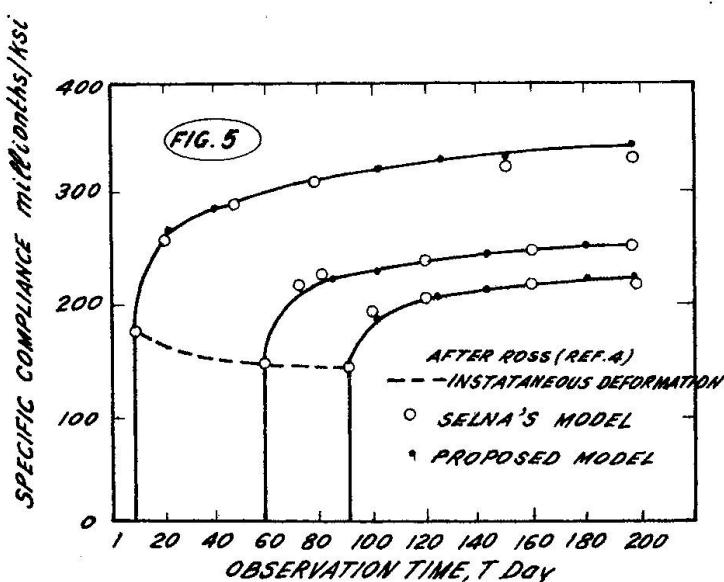
If temperature and age at loading of concrete were independent then the principle of shift function could be applied simply as:

If  $C = \sum_{i=1}^n c_i e^{h_i t}$  represents the specific creep of concrete at reference temperature  $T_0$  and reference age at loading  $K_0$ , then

$C = \sum_{i=1}^n c_i e^{h_i t} \phi(T) \psi_i(K)$  gives the specific creep for same concrete loaded at an age of K and temperature T.

However, since it is known that age of concrete (maturity) is a function of temperature, the following form of shift function is proposed.

$$TS(K, T) = \phi(T) \psi(K, T)$$



Where  $TS(K,T)$  is the total amount the specific creep curve loaded at an age  $K$  and temperature  $T$  needs to be shifted to fit the reference creep curve.

$\phi(T)$  is the temperature shift function as discussed before.

It is proposed that

$$\psi(K,T) = \psi(COAGE(K,T))$$

$$\text{where } COAGE(K,T) = K_0 + (K-K_0) e^{-A(T-T_0)}$$

where  $K_0, T_0$  are the reference age at loading and temperature respectively.

$A$  - is a constant obtained in such a way that the curve  $\psi(K,T)$  gives a continuous monotonic curve so that interpolation from such curve is possible.

#### General Procedure for Finding Shift Functions:

- 1- Calculate the specific creep for each case where  $C(t,\tau) = \text{specific compliance}$   $(t,\tau)$ . Instantaneous strain  $(\tau)$ , where  $t$  is observation time.  $\tau$  is age at loading.
- 2- Plot all specific creep curves on semi-log papers.
- 3- Select the specific creep curve corresponding to reference age at loading  $K_0$  and reference temperature  $T_0$  as the reference curve.
- 4- By shifting the specific creep curves corresponding to age at loading  $K_0$  and different temperatures  $T_1, T_2, T_3$  etc... determine the temperature shift function  $\phi(T)$ .
- 5- By shifting the rest of the specific creep curves corresponding to different temperatures and ages at loading so that they fit the reference specific creep curve, determine  $TS(K,T)$ .
- 6- Determine  $\psi(K,T) = TS(K,T) / \phi(T)$  for each specific creep curve.
- 7- By trial and error between steps 7 and 8, determine  $A$  such that  $\psi_1(COAGE(K,T))$  will be a smooth monotonic curve. Then determine  $COAGE(K,T)$  for each case as  $COAGE(K,T) = K_0 + (K-K_0) \text{Exp}(A(T-T_0))$ .
- 8-  $\psi(COAGE(K,T))$  is determined and plotted.

Thus with the above procedure the specific creep for any temperature and any age at loading can be determined.

#### Verification of Above Procedure:

There is extremely limited data on the response of concrete loaded at different ages and different temperatures.

R. Browne Ref. (5) presented data for concrete loaded at ages of 28, 60, 180, 400 days and for each age some specimens were subjected to 68, 104, 149 and 200°F.

For this concrete, the specific creep curve corresponding to age at loading 28 days and temperature 68°F is chosen as reference. The equation which represents the reference curve is found.

$$C(28,68) = 919.01 - 46.27 e^{-0.2t} - 95.08 e^{-0.02t} - 156.32 e^{-0.002t} -$$

$$197.60 e^{-0.0002t} - 370.22 e^{-0.00002t} \quad (1.a)$$

where  $t$  is time in days after loading.

The shift function  $\phi(T)$  is found in this case to be

$$\phi(T) = -3.55 \left(\frac{T}{68}\right)^3 + 21.57 \left(\frac{T}{68}\right)^2 - 33.83 \left(\frac{T}{68}\right) + 16.79 \quad (1.b)$$

where  $T$  is temperature in  $^{\circ}$ F.

$\phi(T)$  is temperature shift function.

Also it is found that in this case

$$\text{COAGE } (K, T) = K_0 + \sum_{i=1}^n \Delta K(i) e^{-A(T(i) - T_{ref})}$$

$$\text{or COAGE } (K, T) = 28. + \sum_{i=1}^n \Delta K(i) e^{-0.6(T(i) - 68)} \quad (1.c)$$

where  $(K_0 + \sum_i \Delta K(i))$  gives actual age of concrete in days if no corrections for temperature, or temperature history are applied.

$$\psi_1 (\text{COAGE } (K, T)) = .056 + .686 e^{-0.01(\text{COAGE } (K, T))}$$

for COAGE  $> 28.3$  days (1.d)

The instantaneous deformation / ksi is found as a function of age at loading and temperature.

$$\text{Instantaneous def. } (\gamma) / \text{ksi} = 210. \left\{ -.086 \left(\frac{T}{68}\right)^3 + .352 \left(\frac{T}{68}\right)^2 - .019 \left(\frac{T}{68}\right) + .753 \right\} + \frac{1600}{T} \quad (1.e)$$

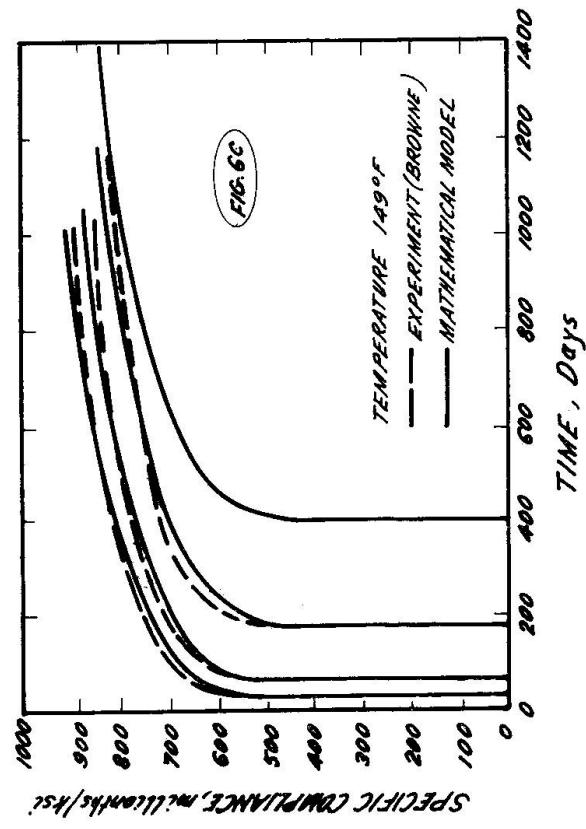
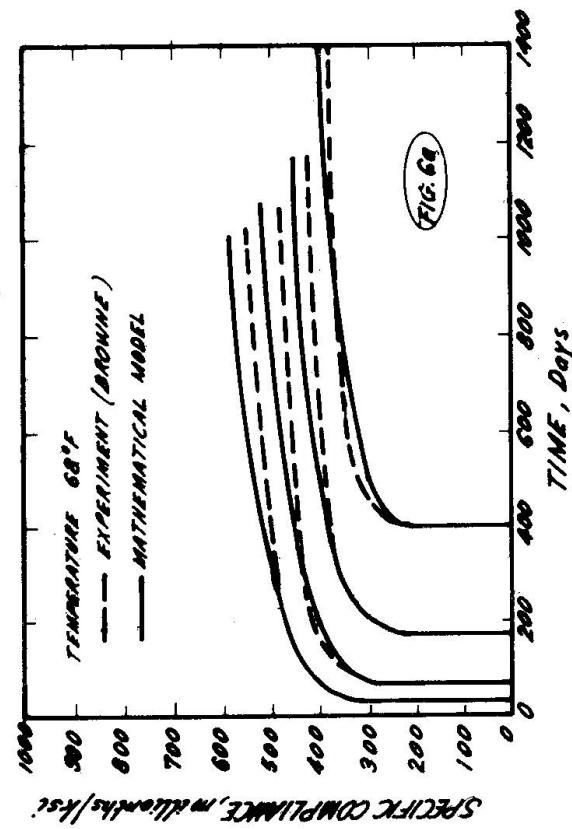
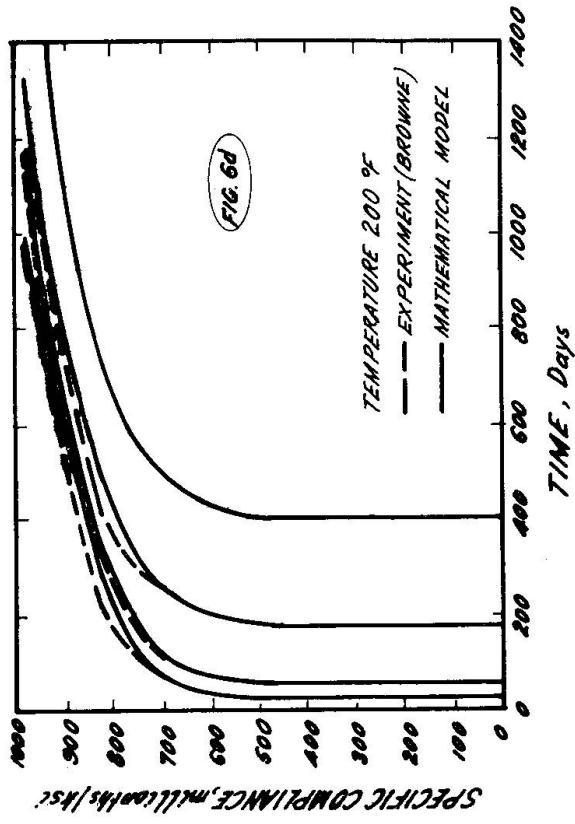
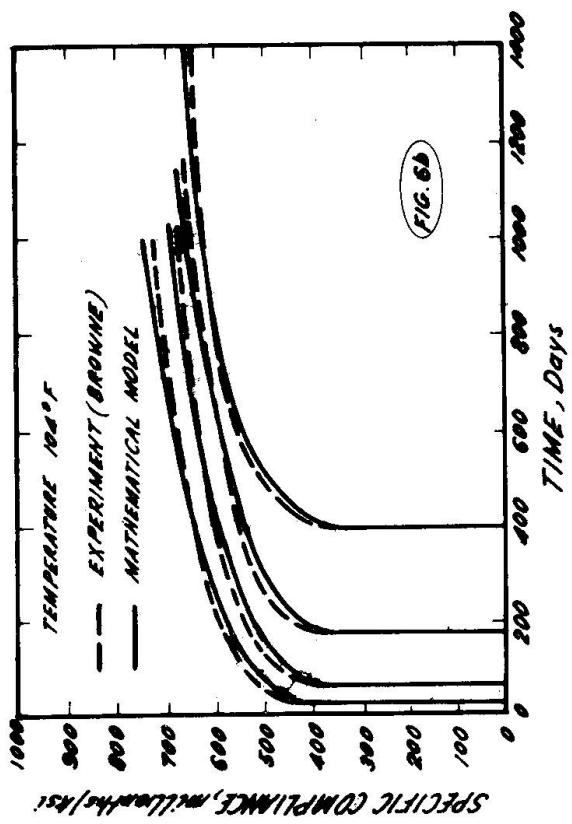
where  $\gamma$  is the times at loading in days  
 $T$  is temperature in  $^{\circ}$ F.

Using the above mathematical Model equations (1.a - 1.e) the total deformation/ksi for all the cases are computed and plotted as shown in Figs. (6.a - 6.d).

The results fit well with the experimental results if one considers the accuracy of results of experimental work Ref.(5).

To illustrate the capability of the mathematical model two examples in which simple uniaxial concrete cylinders are subjected to different loads and temperatures are solved. Also, a study of the behavior of a concentrically prestressed concrete beam subjected to thermal gradients is done. Finally, in order to simulate the nuclear reactor pressure vessel, a thick-wall ring structure subjected to internal thermal gradients, internal pressure and external pressure (Prestressing) is analyzed.

Incremental approach using finite element method is used in the solution of these structural problems. It is only for the lack of space that the results of these examples shall not be included here.



References:

- 1- A. Ross, "The Influence of Sustained Temperature on the Shrinkage and Creep of Concrete and the Effects in Some Structures", Colloquium on the Nature of Inelastic Behavior of Concrete and its Structural Effects, Cornell University, June 1965.
- 2- D. Hannant, "Strain Behavior of Concrete Up to 95°C Under Compressive Stresses", Proc. of Conferences on Prestressed Concrete Pressure Vessel, Group C, Paper 17, London 1967.
- 3- L. Selna, "Time - Dependent Behavior of Reinforced Concrete Structures", SESM Report No. 67, 19 August 1967, University of California, Berkeley.
- 4- A. Ross, "Creep of Concrete Under Variable Stress", Proc. ACI, Vol. 54, 1958, pp. 739-757.
- 5- R. Browne, "Properties of Concrete in Reactor Vessels", Proc. of Conference on Prestressed Concrete Pressure Vessel, Group C, Paper 13, London 1967.

\* \* \*

## Control of Shrinkage by Laboratory Tests

B. BRESLER  
USA

There is general agreement that shrinkage, creep and temperature changes can no longer be neglected in design of reinforced concrete structures. Consideration of strength alone is not sufficient as a measure of serviceability. Various methods for taking into account time-dependent volume changes in concrete have been proposed and availability of large digital computers greatly simplifies use of these methods by the designer.

Reliance on such design calculations requires a prescription of shrinkage and creep characteristics as part of the general specification for the structure and a subsequent verification of compliance with this prescription. The prescription and verification must be based on standard laboratory tests, just as prescription and verification of compressive strength is based on such tests.

The Structural Engineers Association of California in 1963 appointed a special committee to study the problems of shrinkage in concrete structures and to recommend methods of control from a practical point of view. A report of this committee was issued in 1965<sup>1</sup>. One part of this report deals with control of shrinkage of concrete by proper specifications and laboratory tests. It is based on the idea that shrinkage of a small laboratory specimen observed at a reasonably early age is an index of potential shrinkage of concrete used on the job. Troxell<sup>2</sup>, based on test results, concluded that actual long-time shrinkage may be predicted from short-time shrinkage provided a proper conversion factor for the special test conditions has been determined.

The SEAOC report proposes to use a 4 x 4 x 11 in. prism as a standard specimen, moist cured for 7 days, and then dried under 50 percent relative humidity and 73.4° F (23° C). Measurements of shrinkage shall be made on a 10 inch gage length at 7, 14, 21 and 28 days of drying. From experimental studies it was found that shrinkage of such a specimen approaches its ultimate value in about 64 weeks, that at 28 days the observed shrinkage is about 0.4 and at 21 days the observed shrinkage is about 0.33

of ultimate value. It was then assumed that due to variation in humidity, temperature, size of members, etc. the "effective" shrinkage of concrete member in the structure is about 0.5 that of a 4 x 4 in. laboratory specimen. Effective shrinkage is defined as that observed in unreinforced member. Thus, should the design of the structural element be based on effective ultimate shrinkage  $S$ , the laboratory specimen after 28 days of drying should exhibit a shrinkage of  $S_{28} = (S/0.5) (0.4) = 0.8S$ , or after 21 days of drying it should exhibit a shrinkage of  $S_{21} = (S/0.5) (0.33) = 0.66S$ .

Three classes of concrete are proposed, as shown below:

Class	A	B	C
Value of $S$ , millionths	400	600	800

Due to variability in test results and conditions on which these relationships are based, a variation of 15 percent in the observed values can be allowed. It is also recognized that laboratory tests on small specimens cannot predict the shrinkage of concrete members of all sizes under widely varying conditions. Future tests on correlation between laboratory and field shrinkage may show that the above values should be increased or decreased.

The general specifications can be written to require control tests in order to verify shrinkage characteristics of the particular concrete. Two types of test should be required:

1. For the concrete mix proposed for the project, a "trial batch" mix shall be prepared and at least three shrinkage specimens and six compression test specimens should be tested to verify shrinkage and strength characteristics prior to placing any concrete.
2. During construction shrinkage specimens for each class of concrete should be taken at the same time as compression strength test specimens. At least one set of three specimens should be taken for each 150 cubic yards of concrete placed, and in no case less than one set of specimens should be taken for each day's concreting.

Above remarks focused attention on shrinkage characteristics of concrete. Other properties, such as creep and thermal coefficient of expansion, also require attention. Appropriate short term tests to predict these characteristics have not yet been developed, and it is hoped that this discussion will stimulate efforts in this direction.

#### References

1. Control of Shrinkage of Concrete, Report of Committee on Shrinkage of Concrete, Structural Engineers Association of California, 1965.
2. Troxell, G. E., "Short-Time Tests for the Effect of Type of Cement on Concrete Shrinkage," Proc. ACI, Vol. 35, 1939.

**Comparaison des résultats des essais de flambement sous charges soutenues  
de MM. Thürlimann, Baumann, Grenacher et Ramu avec les charges critiques calculées**

PIERRE FAESSEL  
Ingénieur  
Professeur au C.H.E.C.  
France

REMARQUES PRELIMINAIRES

1<sup>o</sup>) - Une méthode de calcul est absolument nécessaire dans l'évaluation des charges critiques de flambement, particulièrement en longue durée.

En effet, trop de paramètres interviennent pour que les laboratoires puissent étudier toutes les combinaisons de ceux-ci; citons seulement l'élancement, l'excentricité initiale de la charge, le pourcentage des aciers et leur position, la qualité du béton et son coefficient de fluage, la limite élastique des aciers, enfin, la forme géométrique de la section qui peut varier à l'infini.

2<sup>o</sup>) - Cette méthode doit cependant être contrôlée par des essais rigoureusement conduits, c'est-à-dire où ces divers paramètres sont aussi bien connus que possible. Remarquons en particulier que les excentricités initiales nulles n'existent pas, et doivent être considérées dans le calcul comme identiques à une très faible excentricité initiale.

3<sup>o</sup>) - Réciproquement, cette méthode permet de guider dans le choix des charges soutenues. En effet, en longue durée, on risque d'obtenir une rupture trop rapide, ou de se placer trop en dessous de la charge critique.

METHODE PROPOSEE

Le fait que les parties tendues, ou entrant en traction au cours du temps sous l'influence de l'augmentation de la flèche, peuvent être fissurées et doivent être considérées comme telles dans le calcul (par sécurité) permet de ne pas se préoccuper des problèmes difficiles du fluage par déchargement.

Nous proposons donc d'utiliser la méthode qui a déjà été parfaitement vérifiée par l'analyse de tous les essais connus sous charges instantanées (1). Mais dans le cas de charges soutenues, on considérera que les déformations du béton sont multipliées par le coefficient  $(1 + \Psi)$ ,  $\Psi$  étant le coefficient de fluage. Ce coefficient ayant été mesuré pour la plupart des poteaux essayés par les auteurs cités (2), il nous a été possible de calculer les charges critiques soutenues correspondantes.

## POTEAUX N'AYANT PAS CASSE

Tableau I

N°	$\frac{e}{H}$	$\mu = \frac{Fe}{BH}$ (%)	$\mu' = \frac{Fe}{BH}$ (%)	$t_0$	$\lambda$	DURÉE DU CHARGEMENT	CHARGE APPLIQUÉE	CHARGE CRITIQUE CALCULÉE
51	0.033	.84	.84	28	100	104	43.8	43.9
11	0.10	.84	.84	28	100	93	15.1	23.
12	0.10	.84	.84	28	100	321	23.4	27.2
21	0.10	.84	.84	28	100	199	26.6	25.6
23	0.25	.84	.84	28	100	186	14.	17.5
25	0.25	.84	.84	28	100	188	16.4	18.9
52	0.25	.84	.84	28	100	105	18.9	21.5
32	1.	.84	.84	28	100	138	7.	7.7
33	1.	.84	.84	70	100	140	7.3	8.
63	0.25	2.15	2.15	56	100	477	35.1	37.3
82	0.05	.85	0.85	28	150	265	10.3	12.3
74	0.033	.84	.84	28	50	69	65.1	82.4

POTEAUX AYANT CASSE  
EN PLUSIEURS JOURS

Tableau II

N°	$\frac{e}{H}$	$\mu = \frac{Fe}{BH}$ %	$\mu' = \frac{Fe}{BH}$	$t_0$	$\lambda$	DURÉE DU CHARGEMENT	CHARGE CRITIQUE DE COURTE DUREE CALCULÉE	CHARGE APPLIQUÉE	CHARGE CRITIQUE DE LONGUE DUREE CALCULÉE	FLÈCHE CRITIQUE MESURÉE (cm)	FLÈCHE CRITIQUE CALCULÉE (cm)
42	.033	.84	.84	33	100	10	60.5	41.6	35.5	2.64	2.5
13	.1	.84	.84	28	100	14	48	31.1	26.4	3.33	3.8
43	.033	.84	.84	28	100	18	53.5	43.5	29.8	2.5	2.5
54	.033	.84	.84	16	100	26	71.5	44.	42.2	2.62	2.7
72	.375	.85	.85	28	150	26	7.5	6.2	5.8	6.09	7.7
71	.05	.85	.85	28	150	27	21.3	14.	10.5	2.53	1.9
61	0.	.84	.84	28	100	27	80.1	65.9	47.7	1.63	1.8
83	0.	2.15	2.15	28	100	34	95.8	77.5	68.2	1.57	2.5
56	.25	.84	.15	28	100	39	24.7	18.7	17.7	7.	6.8
22	.25	.84	.84	28	100	68	23.8	18.9	17.	6.14	6.4
64	.033	2.15	2.15	27	100	80	77.2	64.2	53.2	2.85	3.2
62	.033	.84	.84	90	100	153	72.5	49.6	44.2	2.79	2.5

RESULTATS

Ceux-ci sont donnés dans deux tableaux.

Dans le premier figurent les poteaux qui sont restés stables.

On peut remarquer qu'ils ont tous une charge inférieure à la charge critique calculée (ou supérieure de 4 % pour le poteau 21). Il aurait été intéressant de charger plus fortement certains poteaux qui sont trop nettement en dessous de la charge calculée.

Réciprocement, tous les poteaux qui se sont rompus avaient une charge supérieure à la charge critique calculée. Dans le 2ème tableau, nous avons groupé ceux qui se sont ainsi rompus et qui ont été chargés assez longtemps pour que les auteurs aient pu mesurer une flèche critique.

On pourra vérifier que cette flèche est correctement prévue pour le calcul, ce qui est une preuve supplémentaire de la bonne conformité du mode de calcul à la réalité.

REFERENCES

- (1) J.R. ROBINSON et S.S. MODJABI : La prévision des charges de flambement des poteaux en béton armé par la méthode de M.P. FAESSEL (Supplément aux Annales de l'I.T.B.T.P. septembre 1968)
- (2) P. RAMU, M. GRENAKER, M. BAUMANN, B. THÜRLIMANN : Versuche an gelenkig gelagerten Stahlbetonstützen unter Dauerlast (Institut für Baustatik, Zurich, mai 1969)

Zur Abschätzung der Kräfteumlagerungen bei abschnittsweise hergestellten Spannbetontragwerken

H.-J. WOLFF

Dipl.-Ing.

Technische Universität Hannover  
Deutschland

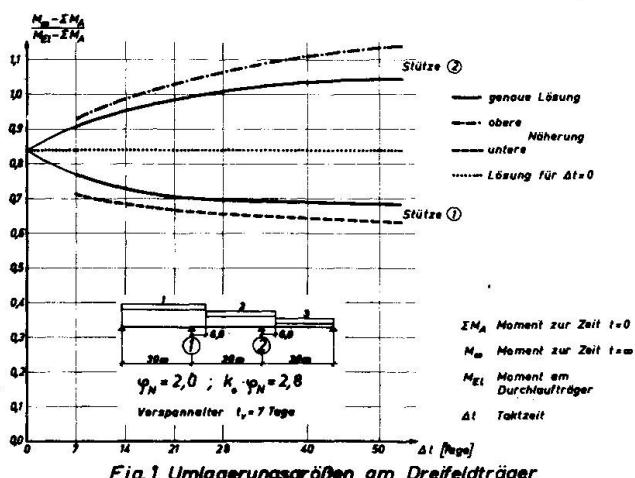
Durch das zeitabhängige Verhalten des Betons entstehen bei abschnittweise erstellten Tragwerken Kräfteumlagerungen für die ständigen Beanspruchungen. Diese Umlagerungen müssen zusätzlich zu der Summe der Ausrüstungszustände zur Zeit  $t = 0$  berücksichtigt werden.

Die genauere rechnerische Ermittlung der Kräfteumlagerungen führt nach der Kriechdifferentialgleichung von Dischinger auf umfangreiche Differentialgleichungssysteme und auch bei vereinfachten Annahmen zu einem erheblichen numerischen Aufwand.

Auf der Grundlage einer linearen zeitabhängigen Spannungsdehnungsbeziehung, die auf den in den CEB-Empfehlungen normierten Kriecheigenschaften des Betons beruht, wurde in [1] ein Verfahren zur Ermittlung der Kräfteumlagerungen angegeben.

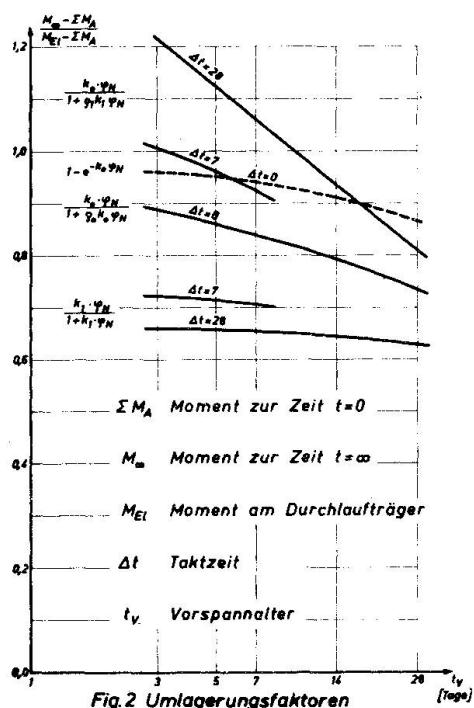
Die auf diese Weise aus linearen Gleichungen gewonnene Momentbeanspruchung ist vom zeitlichen Verlauf des Kriechens, dem jeweiligen Alter des Betons beim Vorspannen und der Art der Systemänderung abhängig.

Fig. 1 zeigt die Umlagerungsgrößen an den Mittelstützen eines Dreifeldträgers, der in drei Abschnitten nacheinander hergestellt wird. Als Abzisse ist die Zeitdifferenz zwischen den einzelnen Bauabschnitten oder kurz die Taktzeit  $\Delta t$  gewählt worden. Als Ordinate erscheint das Verhältnis der reinen Umlagerungsgröße  $M_{\text{u}} - \Sigma M_A$  zu der Differenz  $M_{\text{E1}} - \Sigma M_A$ . Hierin bedeutet  $M_{\text{E1}}$  den Momentenzustand



des in einem Zuge errichteten Trägers und  $\Sigma M_A$  die Summe der Ausrüstungszustände.

Die genaue Lösung für die Kräfteumlagerung entwickelt sich für die beiden Innenstützen sehr unterschiedlich. Während für die jüngere Stütze (2) das Moment  $M_{E1}$  am Durchlaufträger sogar überschritten werden kann, wird es an der Stütze (1) für größere Taktzeiten  $\Delta t$  nur zu etwa 70% erreicht. Die gepunktete Gerade gibt die Größe der Umlagerungen an, wenn die Kriechfähigkeiten in allen Bereichen gleich sind oder wenn die Taktzeit  $\Delta t$  vernachlässigt wird. Für kurze Taktzeiten  $\Delta t$  können damit die Kräfteumlagerungen einfach abgeschätzt werden. Für größere Taktzeiten  $\Delta t$  sollen mit oberen und unteren Umlagerungsfaktoren Näherungen vorgeschlagen werden. Diese obere und untere Näherung bilden eine Grenzabschätzung der entstehenden Umlagerungen.



In Fig. 2 sind diese Umlagerungsfaktoren in Abhängigkeit von dem Vorspannalter  $t_v$  und der Taktzeit  $\Delta t$  für ein Normalkriechmaß  $\psi_N = 2,0$  dargestellt. Als Ordinate erscheint wie im vorherigen Bild das Verhältnis der Umlagerungsgröße zur Differenz  $M_{E1} - \Sigma M_A$ . Für  $\Delta t = 7$  oder 28 Tage und mit den angegebenen Verhältnissen erhält man mit den beiden oberen Kurven den oberen Umlagerungsfaktor und mit den unteren Kurven den unteren Umlagerungsfaktor zur Grenzabschätzung der Kräfteumlagerungen. Mit dem mittleren Kurvenzug für  $\Delta t = 0$  ergibt sich die Umlagerung bei Systemänderungen in Tragwerken mit gleichen Kriecheigenschaften in allen Bereichen wie beim Zusammenbau von gleichalten Fertigteilen. Für diesen Fall stellt die gestrichelte Kurve die Lösung von Dischinger dar.

In den angegebenen Verhältniswerten berücksichtigt der Wert  $k$  den Einfluß des Betonalters bei der Belastung und der Relaxationskennwert  $\rho$  berücksichtigt das abgeminderte Kriechvermögen, auf das die erzwungenen Spannungsänderungen treffen. Die Werte  $k_0$  und  $\rho_0$  ergeben sich für das Betonalter im Zeitpunkt des Vorspannens und  $k_1$  und  $\rho_1$  für das Betonalter nach  $t_v + \Delta t$  Tagen.

Da die Werkstoffkennwerte des Betons, insbesondere die zeitabhängigen Eigenschaften, gewissen Schwankungen unterworfen sind, sollte man mögliche Abweichungen der ermittelten Beanspruchungen berücksichtigen. Deshalb wird dem Konstrukteur mit den oberen und unteren Grenzumlagerungsfaktoren eine Möglichkeit gegeben, unter Benutzung der elastisch ermittelten Momentendifferenz  $M_{E1} - \Sigma M_A$  die Einflüsse der Taktzeit und des Belastungsalters auf die Kräfteumlagerung abzuschätzen.

[1] H.Trost und H.-J.Wolff: Zur wirklichkeitsnahen Ermittlung der Beanspruchungen in abschnittweise hergestellten Spannbetontragwerken. Der Bauingenieur 48 (1970), H. 5, S. 155 - 169.

**Leere Seite**  
**Blank page**  
**Page vide**