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Effect of Creep on the Flexural Strength and Deformation of Concrete Beams

Influence du fluage sur la résistance et déformation en flexion des poutres en béton armé et précontraint

Einfluß des Kriechens auf die Biegefestigkeit und die Verformung von Stahlbetonträgern

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INTRODUCTION:

Some years ago the writers were engaged in an investigation of the effect of creep on the deformations and strength of statically determinate beams of reinforced concrete (1) and arrived at the conclusion that a previous creep history has little effect on the flexural strength. From limit analysis it can be inferred that this will also be true for continuous beams. The same conclusion has been reported by Messrs. Ghosh and Cohn in the Preliminary Publication of this Symposium (2).

A non-linear analysis program was used for solving the stresses in a rectangular concrete member by successive approximations. The effect of time dependant deformation was introduced by both the reduced modulus concept and by numerical integration of specific creep curves. The experimental program was limited in scope and included tests of aluminum reinforced resin models and simply reinforced concrete beams.

More recently the second author extended the method of solution to unsymmetrical double-T sections, typical of pre-stressed concrete (3) and had the opportunity to compare the theoretical solutions of long-time behaviour with the experimental data of the investigation under way at the Institut du Genie Civil of the University of Liege.

SECTION ANALYSIS:

The usual assumption of a linear strain distribution across the section is made while compressive and eventual tensile stresses in the viscoelastic material as well as in the reinforcement may follow arbitrary stress-strain laws (Fig. 1). Defining the

Stress by their secant moduli and corresponding strains and combining equilibrium and compatibility equations a quadratic expression for the neutral axis position is obtained.

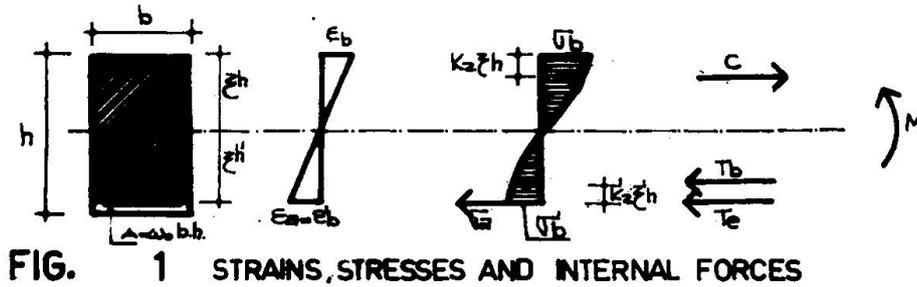


FIG. 1 STRAINS, STRESSES AND INTERNAL FORCES

The equations have general validity for any state of stress up to failure. The solution is obtained by a process of successive approximations. Moment-curvature relationships are also obtained. Special cases, as f. i. linear stress-strain laws, absence of tensile stresses and combinations thereof are readily obtained. The failure moment is a limiting condition for which the strain-dependant coefficients take known values.

TIME DEPENDANT DEFORMATION:

For non-aging materials with linear creep and linear behaviour under instantaneous loading the stress-strain relation can be expressed by :

$$\epsilon_b(t) = \sigma_b(t) \left[\frac{1}{E_b(t_0)} + \bar{\epsilon}_o(t, \tau_0) \right] \quad (1)$$

where $\bar{\epsilon}_o(t, \tau)$ is the specific creep. The term between parenthesis can be interpreted as the inverse of a reduced modulus. Eq.(1) represents the first term of the exact solution of the problem by power series and constitutes in many cases of practical importance

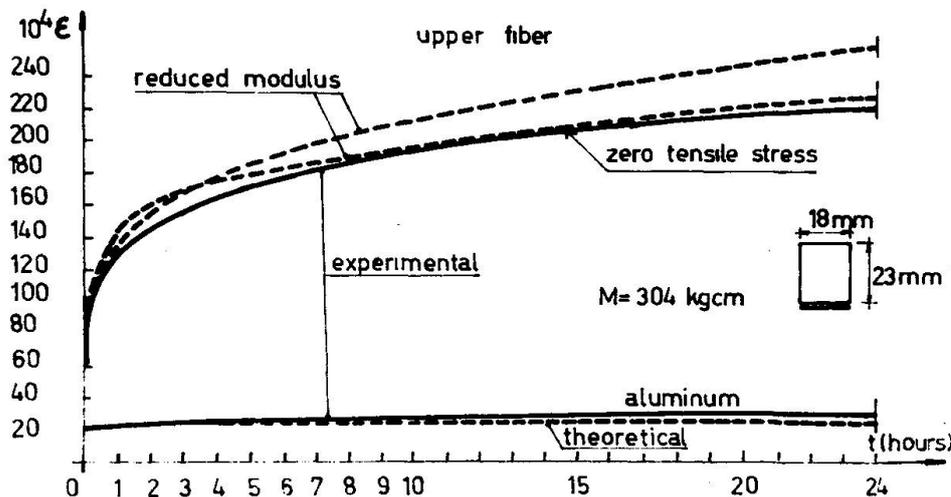


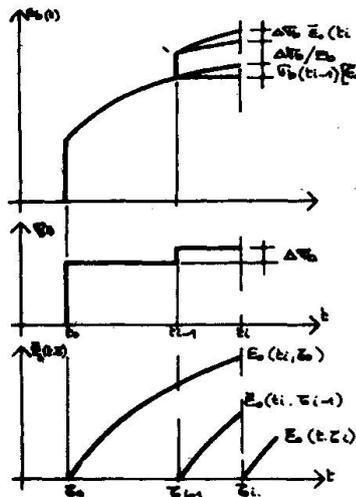
FIG.2 DEFORMATION-TIME RELATIONSHIP FOR ALUMINUM-REINFORCED EPOXY MODEL

a very good approximate solution (4). Introducing this reduced modulus in the equations of section analysis an approximate solution for the redistribution of stresses is obtained.

Fig. 2 shows the results of redistribution obtained with an aluminum reinforced resin model maintained under a constant moment and the comparison with reduced modulus solutions.

NUMERICAL SOLUTION:

For linear creep the strain increment can be expressed in function of the corresponding specific creep curves. From Fig. 3 the following expression is obtained:



$$\begin{aligned} \Delta \epsilon_b &= \bar{\sigma}_b(t_{i-1}) \left[\bar{\epsilon}_o(t_i, \tau_0) - \bar{\epsilon}_o(t_{i-1}, \tau_0) - \bar{\epsilon}_o(t_i, \tau_{i-1}) - \right. \\ &\quad \left. - \frac{1}{E_b(t_{i-1})} \right] + \bar{\sigma}_b(t_i) \left[\bar{\epsilon}_o(t_i, \tau_{i-1}) + \frac{1}{E_b(t_i)} \right] = \\ &= \bar{\sigma}_b(t_{i-1}) f_{i-1} + \bar{\sigma}_b(t_i) f_i \quad (2) \end{aligned}$$

FIG. 3 FINITE VARIATION OF CREEP STRAIN

Substituting Eq. (2) into the equilibrium equations it is possible by iteration to obtain the unknown value of $\bar{\sigma}_b(t_i)$ for each

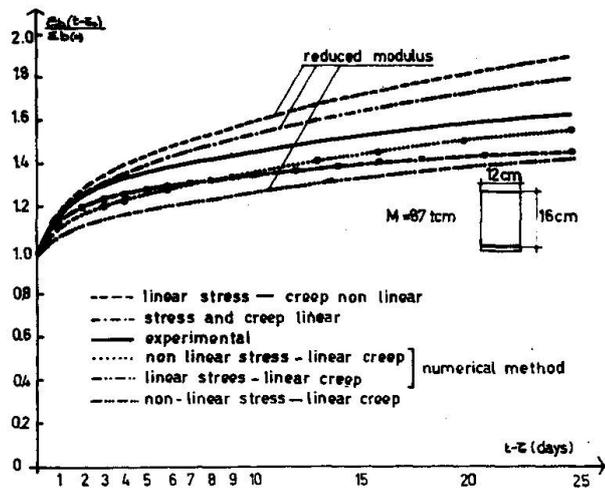


FIG. 4 COMPARISON OF THEORETICAL AND EXPERIMENTAL DEFORMATION RATIOS OF REINFORCED CONCRETE BEAM

time increment. This calculation is best performed by a computer. Fig. 4 shows the experimental and analytical results for a rectangular reinforced concrete beam, loaded at the age of 7 days just below its failure moment which was maintained constant for a period of 25 days when the beam was tested to failure.

ULTIMATE MOMENT OF A BEAM WITH PREVIOUS CREEP HISTORY:

In order to determine the failure moment of a reinforced concrete section with a previous creep history it can be assumed that at a certain instant, t , the deformations in both concrete and steel are known and that the applied moment is increased up to failure in a short period of time such that further creep deformation is excluded. Maintaining the hypothesis of linear strain distribution, failure will occur by limiting states of either concrete or steel reinforcement.

Designating the corresponding values of the limiting state by a horizontal bar, the following ratios of moments with and without a previous creep history are obtained:

$$\begin{aligned} \text{Failure by steel: } \frac{\bar{M}_r}{M_r} &= \frac{1 - k_{2c} \bar{\xi}}{1 - k_{2c} \xi} && \text{(underreinforced)} \\ \text{Failure by concrete: } \frac{\bar{M}_r}{M_r} &= \frac{\bar{\sigma}_s}{\sigma_s} \frac{1 - k_{2c} \bar{\xi}}{1 - k_{2c} \xi} && \text{(overreinforced)} \end{aligned} \quad (3)$$

where $\bar{\sigma}_s$ is the steel stress, inferior to its yield stress, but affected by the redistribution process. In order to evaluate the effect, comparative calculations were made for the test beam section with different assumptions for the form of the stress-strain relationship (linear, parabolic, rectangular) and for the ultimate strength of concrete with (146 and 195 Kg/cm²) and without creep history (174 Kg/cm²). It was found that even for extreme combinations the influence was only of the order of $\pm 10\%$.

The results are not surprising since it is well known that the failure moment of underreinforced concrete sections is not influenced very much by even rather big concrete strength variations. It is also evident that an experimental determination would be virtually impossible because of inherent variations of material properties.

T-SECTIONS OF REINFORCED AND PRESTRESSED CONCRETE:

The calculations of creep deformations by the reduced modulus concept have been extended to double-T sections typical of prestressed concrete (Fig. 5) subjected to flexural moments applied in two stages.

The results of calculated deformations for zero prestress are shown in Fig. 6 and 7 by dashed lines for the maximum compressive

and tensile strain respectively. There is quite a good agreement between these calculated values and the experimental results (solid lines) of an investigation under way at the Institut du Genie Civil of the University of Liege.

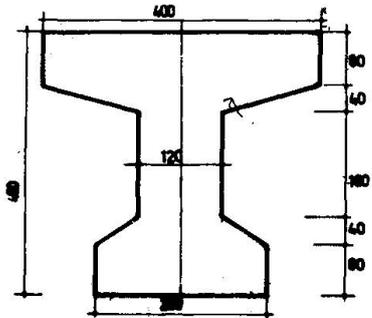


FIG. 5 CROSS SECTION CHARACTERISTICS OF TESTED BEAM (IN MM.)

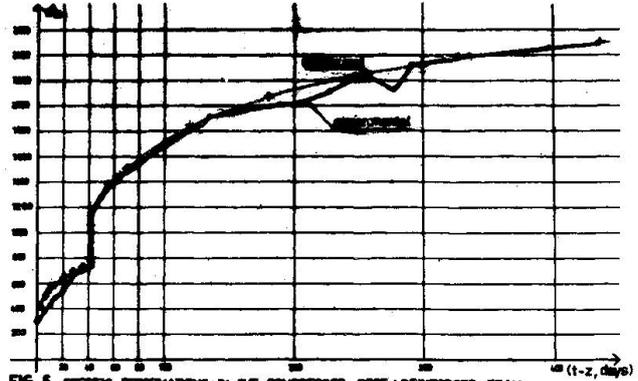


FIG. 6 SPECIFIC DEFORMATIONS IN THE COMPRESSED EDGE / REINFORCED BEAM

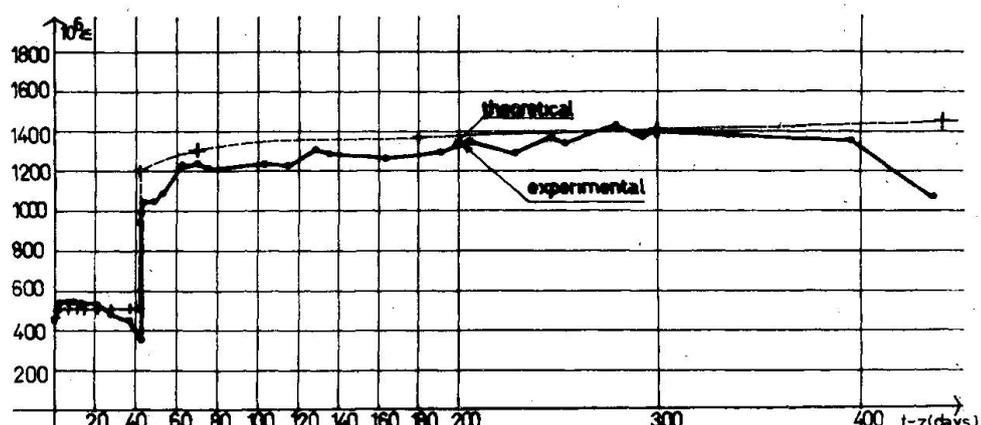
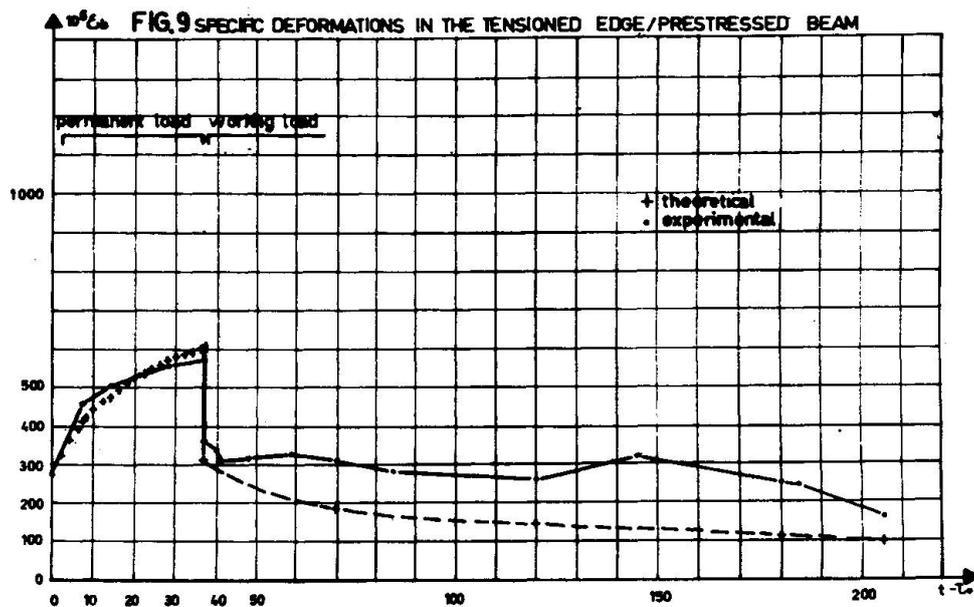
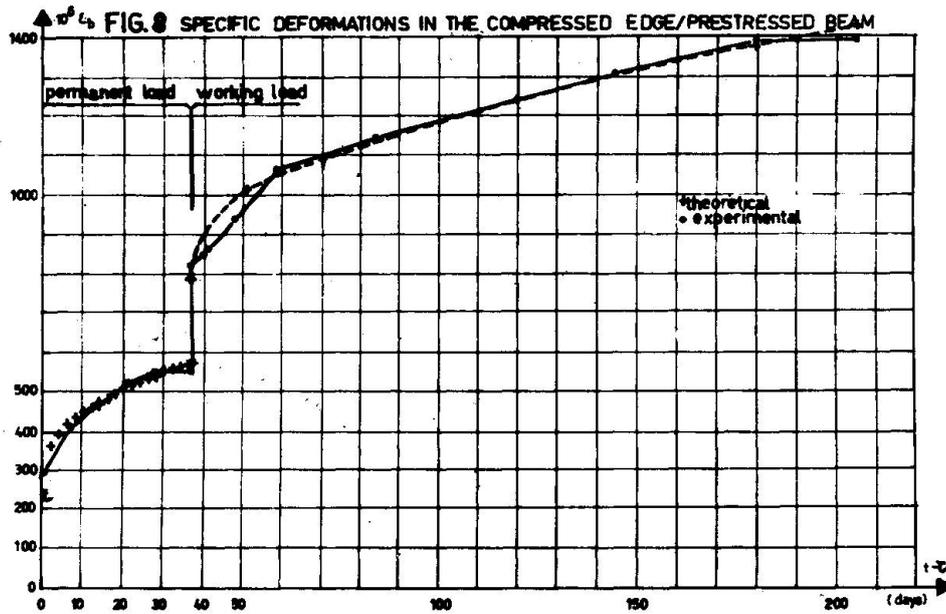


FIG. 7 SPECIFIC DEFORMATIONS IN THE TENSIONED EDGE / REINFORCED BEAM

In the case of fully prestressed concrete, the substitution of the reduced modulus in the conventional equations for concrete and cable stresses gives, together with the perfect bond condition between the cable and the surrounding concrete at all times, the wanted solution. Unfortunately, the cable eccentricity has also become a function of time so that a procedure of successive approximations has to be used. A computer program was written for these calculations. The same section shown in Fig. 5 was analysed and compared with the experimental results under way at the Institut du Genie Civil already mentioned earlier. Again a very good agreement was obtained between theoretical and experimental results (Figs. 8 and 9).



CONCLUSIONS:

The time dependant deformations of reinforced and prestressed concrete beams can be predicted with a good approximation by the reduced modulus concept or numerical integration of specific creep curves.

A previous creep history has a very small effect upon the flexural strength of reinforced concrete beams and the inherent variations of material properties make an experimental verification very difficult.

NOTATION:

| | |
|--------------------|--|
| E_b | secant modulus of concrete in compression |
| M | applied moment |
| M_r | failure moment |
| k_1 | compressed area coefficient |
| k_2 | coefficient for c. g. of compressed area |
| $\bar{\epsilon}_b$ | concrete compressive strain in extreme fiber |
| $\bar{\epsilon}_o$ | specific creep strain |
| $\bar{\sigma}_b$ | compressive concrete stress in extreme fiber |
| ξ | neutral axis coefficient |
| $\bar{\sigma}_a$ | stress in steel reinforcement |
| τ | age |
| t | time |

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SUMMARY

The effect of creep on the deformation and strength of statically determinate beams of rectangular and T-section of reinforced and prestressed concrete was studied both experimentally and theoretically. The effect of time-dependant deformation was introduced by both the reduced modulus concept and by numerical integration of specific creep curves.

The conclusion was made that a previous creep history has a very small effect on the flexural strength of reinforced concrete.

RESUME

On étudie de façon théorique et expérimentale l'influence du fluage sur la déformation et la résistance des poutres statiquement déterminées de section rectangulaire ou en T, en béton armé et en béton précontraint. L'effet du fluage a été introduit de deux manières: en utilisant le concept du module réduit et par intégration de la courbe de fluage spécifique.

On a pu conclure que l'histoire du fluage a une très petite influence sur la résistance à la flexion de la poutre armée.

ZUSAMMENFASSUNG

Der Einfluss des Kriechens auf statisch bestimmte Stahlbetonträger mit und ohne Vorspannung wird untersucht. Der Einfluss der zeitabhängigen Verformungen wurde mit Hilfe einer reduzierten Elastizitätskonstante und durch numerische Integrierung von spezifischen Kriechkurven berücksichtigt.

Es wurde die Schlussfolgerung gezogen, dass das vorhergehende Kriechen eine sehr kleine Auswirkung auf die Biegefestigkeit von Stahlbetonträgern hat.