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## Strength Characteristics of Reinforced Concrete Columns under Sustained Loading

Influence d'un effort permanent sur la résistance des colonnes en béton armé

Tragfähigkeitseigenschaften von Stahlbeton-Stützen unter Dauerlast

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### 1. Introduction

The problem of creep instability of eccentrically loaded reinforced concrete columns become increasingly important with the use of more slender columns in building and bridge structures. Most of the available analytical (usually qualitative only) and experimental studies are aimed at determining the maximum load a column can sustain for an indefinite period of time. Less attention has been paid to the equally important problem of the influence of a sustained loading period on the residual strength or load carrying capacity ( $P_{u2}$ , Fig. 1) as obtained in a short time test following a sustained load period. Some experimental and analytical results pertaining to this problem are available [1, 2, 3, 4, 5, 8].

The present study is analytical and was undertaken to gain further insight into strength characteristics of columns under sustained loading (Fig. 1) and hopefully to provide a basis for rational design procedures for reinforced concrete columns under such load actions. The columns considered are unrestrained and loaded at constant end eccentricities (Fig. 2). The cross section is rectangular and symmetrically reinforced (Fig. 3). Further, only bending in one plane is considered.

### 2. Analysis

#### 2.1 General

Only the main features of the rather involved rheological model and the analytical procedure employed are outlined in the following. More detailed information is available elsewhere [7].

Concrete is considered to be nonlinear visco-elastic material with time variant material properties, and also exhibiting shrinkage. The instantaneous or shorttime stress strain diagram is approximated by an exponential relationship, Fig. 4, in which unloading and reloading take place along a path given by the 'modulus of elasticity',  $E_0 = E_0 (f_c'', \epsilon_s)$ . The effects on modulus of elasticity and concrete strength ( $f_c''$ ) of hydration (beneficial effects) and of high sustained stresses (detrimental effects, noted previously by Rüs ch [9]), are considered through a nondimensional 'strength variation

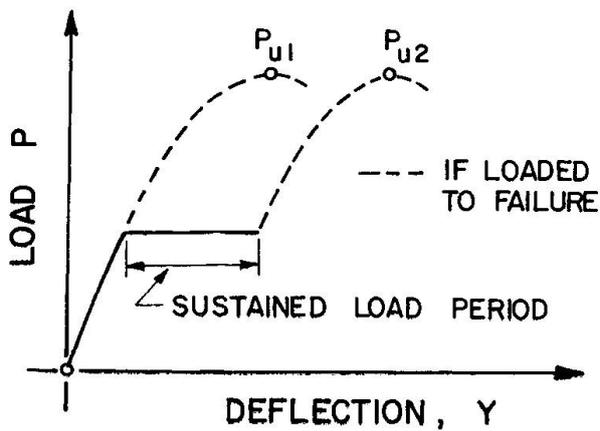


Fig. 1

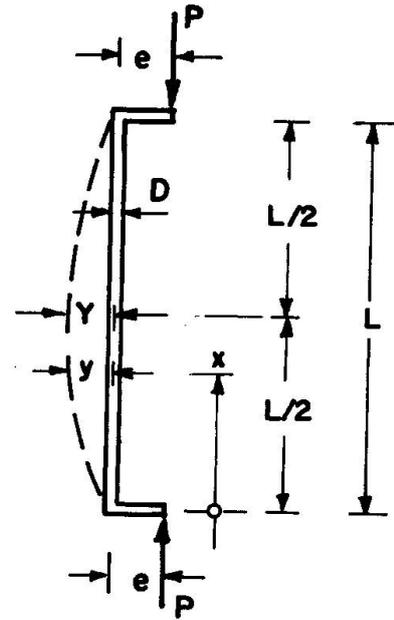


Fig. 2

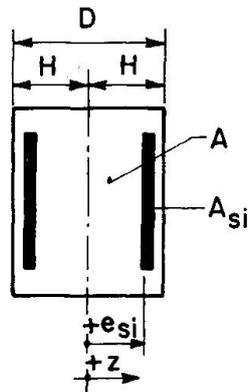


Fig. 3

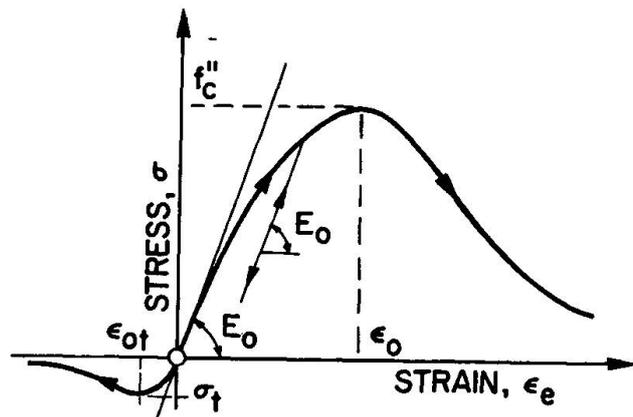


Fig. 4

function',  $\theta$ . The adverse effects are taken a function of the stress intensity and the time over which the stress is maintained. Only stresses in excess of 70 per cent of the standard short time strength (at the time considered in the stress history) are assumed to have adverse effects on the strength. The strength variation model may be applied to any stress history, constant or variable.

The creep strains are accounted for by associating the specific creep (creep per unit stress as obtained at low stress levels) with a creep non-linearity function, dependent upon stress level and time (through the strength variation  $\theta$ ), and chosen so as to represent the large creep strains at high stress levels and stresses on the descending branch of the stress-strain diagram. Micro cracking beyond that taking place in a standard 2-minute test, is considered a part of the creep strain. The 'rate of creep' method is used to predict creep under variable stresses.

Predictions using the rheological model compare favourably with experimental results [9, 10] of plain concrete in uniform compression.

2.2 Problem Formulation

The equation of state representing the rheological model is (with all arguments deleted for convenience),

$$\dot{\epsilon} = \dot{\epsilon}_e + \dot{\epsilon}_c + \dot{\epsilon}_{sh} \tag{1}$$

where  $\epsilon$ ,  $\epsilon_e$ ,  $\epsilon_c$  and  $\epsilon_{sh}$  are total, instantaneous, creep and shrinkage strains respectively. Dots indicate differentiation with respect to time.

The equilibrium equations governing the quasi-static problem become

$$P = \int_A \dot{\sigma}(z,x,t) dA + \sum_{i=1}^2 \dot{\sigma}_{si}(x,t) A_{si} \tag{2}$$

$$P (y(x,t) + e) + P\dot{y}(x,t) = \int_A \dot{\sigma}(z,x,t) z dA + \sum_{i=1}^2 \dot{\sigma}_{si}(x,t) e_{si} A_{si} \tag{3}$$

Here  $\sigma$  and  $\sigma_{si}$  are the concrete and steel stress respectively, the load rate  $\dot{P}$  has a given value and coordinates  $y$ ,  $x$  and  $z$  are defined in Figs. 2 and 3. Navier-Bernoulli's hypotheses is assumed for total strains,

$$\phi(x,t) = \frac{\dot{\epsilon}(z,x,t) - \dot{\epsilon}(z=0,x,t)}{z} \tag{4}$$

where  $\phi$  is the curvature.

Eqs. 1 through 4 are reduced, in a similar fashion previously employed by Mauch [6], to a set of linear equations in discrete stress rates over the column midheight section. They may be written in matrix form as

$$[F] [\dot{\sigma}] = [U] \tag{5}$$

Small displacements were assumed ( $(y'')^2 \ll 1$ ) and further no 'slip' in the interface concrete-steel. The integral terms were approximated by Simpson's rule and half a sine wave was assumed for the deflected shape (one point collocation). Eq. 5 was solved, on an IBM 360/75 digital computer, as a propagation problem in time using Euler's extrapolation formulae to yield both the short time and sustained load response. A set of initial conditions were obtained from a static analysis at a small load value. The preloading strains (induced by shrinkage prior to loading) were included in that analysis.

Instability, indicated by negative deflection rate, was checked after each propagation step.

Predictions using the analysis compare favourably with experimental results [3, 7].

3. Numerical results

3.1 Input Data

The input data include a concrete strength at initial loading of  $f'_c = 4600$  psi (324 kg/cm<sup>2</sup>) at an instantaneous strain  $\epsilon_o = 0.00225$  (Fig. 4). The corresponding values in tension were taken  $\sigma_t = 0.05 f'_c$  and  $\epsilon_{ot} = 0.1 \epsilon_o$ . The ideally elasto-plastic steel considered had a yield stress of  $f_y = 59000$  psi (4150 kg/cm<sup>2</sup>) and a modulus of elasticity  $E_s = 30.6 \times 10^6$  psi ( $2.15 \times 10^6$  kg/cm<sup>2</sup>). The steel is placed at a distance of 0.325D from the centroid. The creep magnitudes chosen yield a creep coefficient (ratio of limiting creep to initial instantaneous strain) of approximately 2.5 at low stress levels.

The complete set of input data for the material laws are available [7].

### 3.2 Load Carrying Capacities

The variables considered were

- end eccentricity of applied load ( $e/D = 0.1$  and  $0.4$ ),
- slenderness of the column ( $L/D = 15$  and  $30$ ),
- steel percentage ( $p = 1.26$  and  $3.54$  per cent),
- concrete age at initial loading ( $t_0 = 14$  and  $56$  days),
- sustained load period ( $t_s = 131$  and  $3000$  days) and
- sustained load intensity ( $P_s$ ).

The influence on a column's load carrying capacity of intensity and duration of a sustained load is illustrated, for some of the columns investigated, in Figs. 5 through 8. The load capacity,  $P_{u2}$  (Fig. 1), after a sustained load period,  $t_s$ , is plotted versus the corresponding sustained load,  $P_s$ . The coordinates are nondimensionalized with respect to the short time capacity,  $P_{u1}$ , at initial loading. The curves are limited to the right by the line  $P_{u2} = P_s$ , i.e. by the case of creep instability taking place under the constant load  $P_s$ . The change in  $P_{u2}$  with increasing  $t_s$  at  $P_s = 0.0$  is mainly due to the continued hydration alone (effects of preloading strains are negligible).

Hydration effects, causing a concrete strength increase, may, depending upon column geometry, yield a considerable increase in load carrying capacity ( $P_{u2}$ ) at low and intermediate load levels. On the other hand, a significant decrease in load carrying capacity results at high sustained load levels, this being mainly due to the large deflection increase at these load levels (see Fig 10). This decrease is, for all columns except those becoming unstable at low stress levels ( $e/D = 0.1$ ,  $L/D = 30$ ), further accentuated by adverse effects to the concrete strength from high sustained stress levels at the most strained portions of the column section.

With larger steel percentage, more stress transfer (due to creep and shrinkage) from concrete to steel takes place. High concrete stresses thus prevail for shorter periods of time with adverse effects to  $P_{u2}$  becoming less pronounced.

It was found [7], for the columns investigated, that the ratio of the sustained load capacity, defined as the load that causes creep instability at  $t_s = \infty$  (here approximated by  $t_s = 3000$  days), to  $P_{u1}$  decreases with increasing slenderness, increases with increasing end eccentricity, decreases with decreasing steel percentage, and is not significantly influenced by the age of loading.

## 4. Design Considerations

### 4.1 Safety Aspects

The load capacity,  $P_{u2}$ , of a column was found to vary with time under a given sustained load. Design procedures may be based on the lower bound values,  $P_{u \min}$  (Schematically illustrated in Fig. 9) and related to the allowable service loads by a set of load factors, A and B, such that

$$P_{u \min} = A P_D + B P_L \quad (6)$$

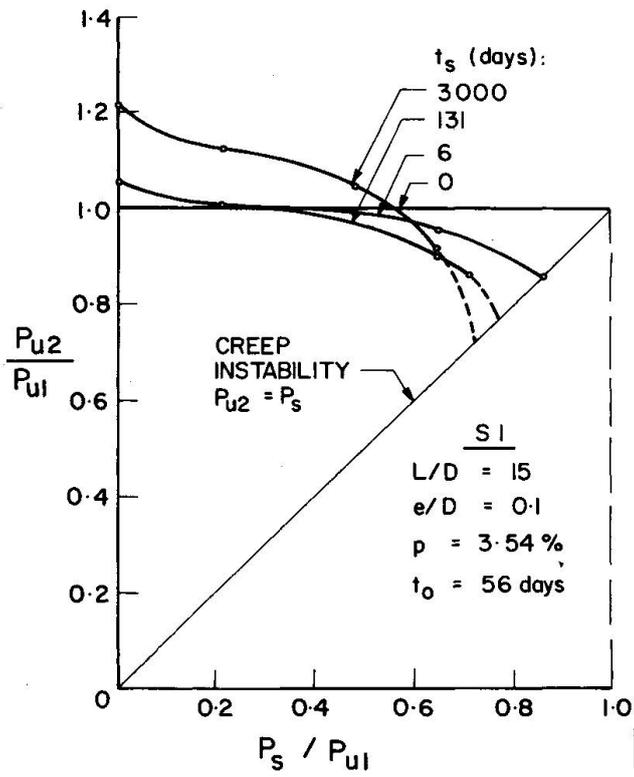


Fig. 5

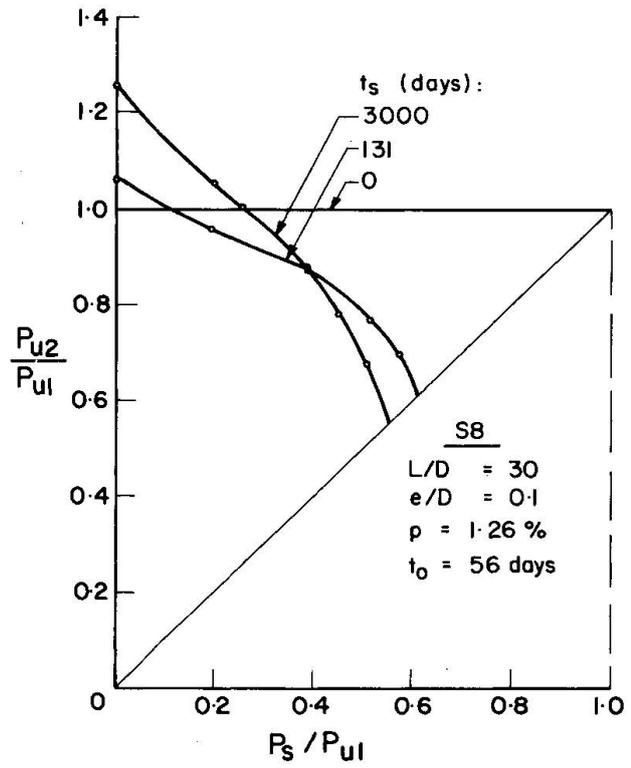


Fig. 6

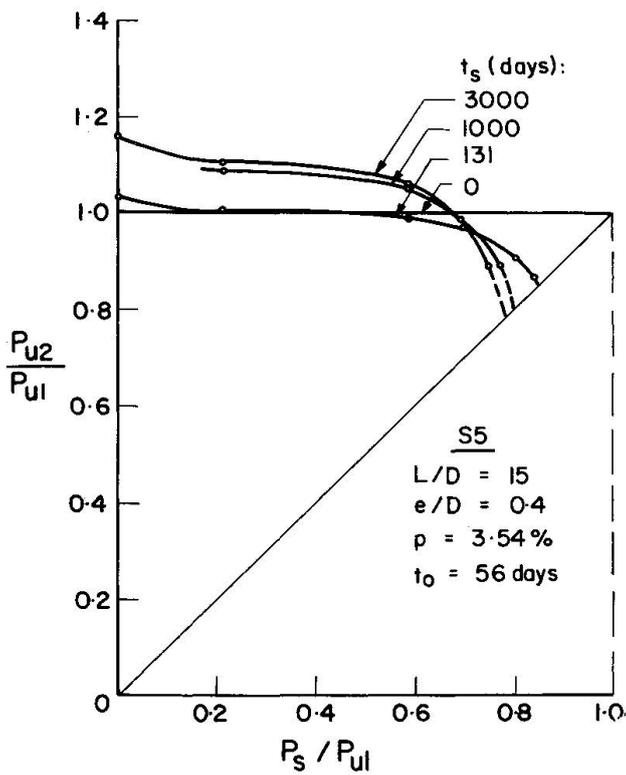


Fig. 7

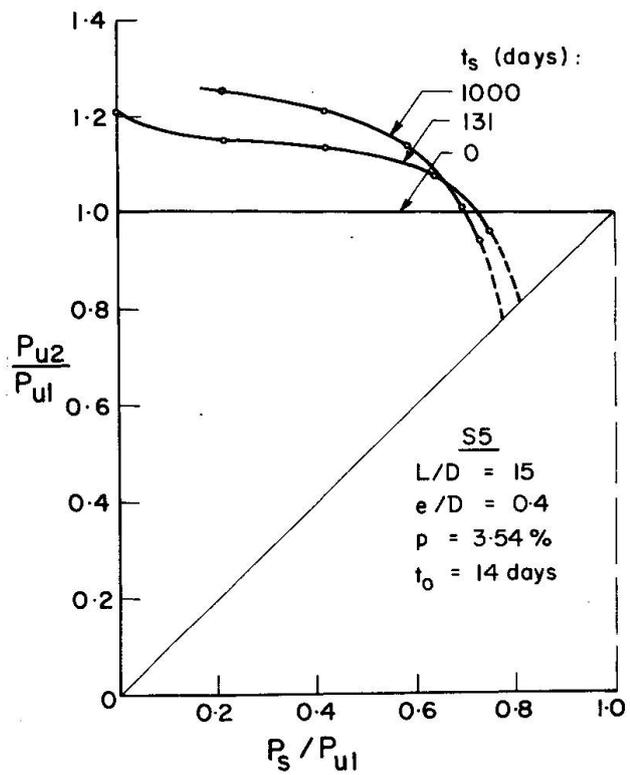


Fig. 8

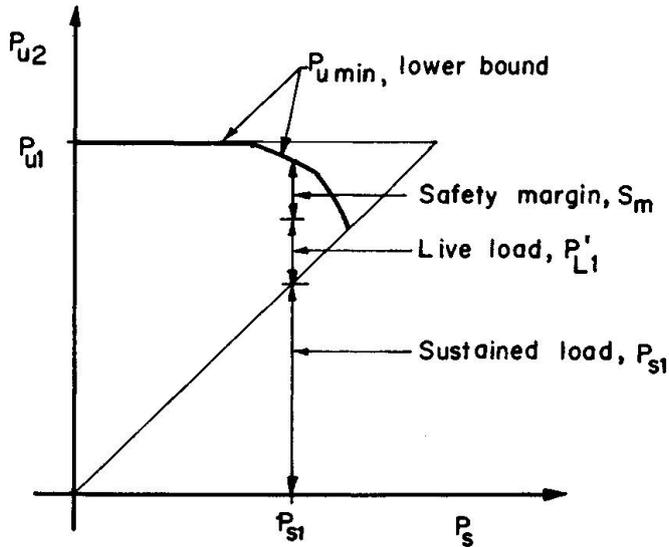


Fig. 9

where  $P_D$  is the dead load and  $P_L$  the live load.

These load factors, or more conveniently an 'overall load factor' (LF),

$$LF = P_{u \min} / (P_D + P_L) \quad (7)$$

can be examined in terms of a set of given 'nominal' factors,  $a$  and  $b$ , related to the short time capacity by

$$P_{u1} = aP_D + bP_L \quad (8)$$

A 'nominal overall load factor' is obtained by replacing  $P_{u \min}$  with  $P_{u1}$  in Eq. 7.

If a typical building structure is considered with the dead load equal to the live load ( $P_L = P_D$ ) and with half the live load essentially permanently applied throughout the life of the building, the total sustained load becomes  $P_s = P_D + P_L/2 = 0.48 P_{u1}$  (Eq. 8), with load factors  $a = 1.4$  and  $b = 1.7$  suggested in the proposed revision of ACI 318-63 [11]. With  $P_s = 0.48 P_{u1}$  and the column in Fig. 6 as an example,  $P_{u \min}$  becomes  $0.73 P_{u1}$  (at  $t_s = 3000$  days) and the overall load factor 1.14 (Eq. 7). This represents a 26 per cent reduction of the nominal overall load factor of 1.55.

In long span bridges the dead load is the predominant load. Using a live load of 6 per cent of the dead load, the dead load becomes  $P_D = 0.60 P_{u1}$ , Eq. 8, with  $a=1.5$  and  $b=2.5$  (AASHTO load factors [12]). For the column in Fig. 6, failure (creep instability) takes place under this dead load only at about  $t_s = 131$  days. Thus while the nominal overall load factor is 1.56, the one based on  $P_{u \min}$  is below unity. Thus, the load factors used above may not be sufficient for the sustained loading case of this slender column. Comparison with the other columns investigated [7] indicates that all the columns but those very slender ( $L/D = 30$ ) and with small end eccentricities ( $e/D = 0.1$ ) would remain stable under the allowable loads, although with actual load factors smaller than the nominal ones.

These examples reinforce the need for careful consideration of sustained load effects in column design.

#### 4.2 Serviceability and Aesthetic Aspects

It may be desirable, both from serviceability requirements and aesthetic viewpoints, to limit the column deflection to an amount at which it is not particularly apparent to the eye. Deflections in excess of about  $L/360$  to  $L/300$  are noticeable.

Typical results showing sustained loads versus the corresponding midheight deflections after various time periods are given in Fig. 10. A deflection of  $L/360$  is indicated.

It was found, for all the columns investigated, that a sustained load  $P_s \geq 0.48 P_{u1}$  resulted in deflections at  $t_s = 3000$  days in excess of  $L/360$ . Thus, deflection limitations rather than safety aspects may tend to become a design criterion for this class of columns.

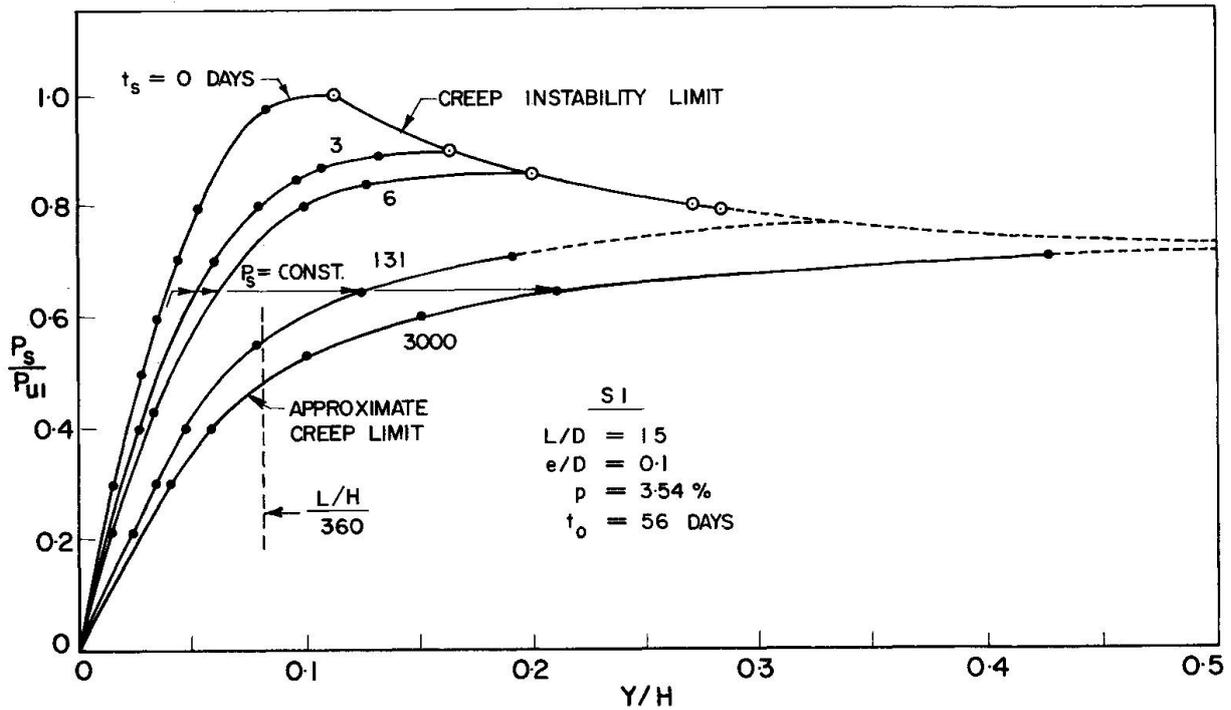


Fig. 10

5. Conclusions

Perhaps the principal conclusion to be drawn from this study is that the load carrying capacity of a column may be significantly reduced under sustained overload conditions and also under service loads for some combinations of end eccentricities and slenderness ratios.

Indications were further obtained that present design provisions in terms of load factors, may not provide adequate safety for very slender columns ( $L/D = 30$ ) with relatively small end eccentricities ( $e/D = 0.1$ ). It should be noted, however, that columns in actual structures are usually subjected to more favourable boundary conditions than those considered here (unrestrained). End restraints provided by beams and footings will usually allow a gradual moment transfer from the column to the restraining elements during the sustained loading period.

Lastly, serviceability and aesthetic considerations, seeking to limit deflections, may call for smaller service loads than those obtained from load capacity considerations.

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REFERENCES

1. VIEST, I.M., ELSTNER, R.C. and HOGNESTAD, E., "Sustained Load Strength of Eccentrically Loaded Short Reinforced Concrete Columns", Journ. A.C.I. Proc. Vol. 54, No. 9, March 1956.

2. RAMU, P., GRENACHER, M., BAUMAN, M. and THÜRLIMANN, B., 'Versuche an gelenkig gelagerten Stahlbeton-stützen unter Dauerlast', Bericht Nr. 6418-1, Institut für Baustatik, ETH, Zürich, May 1969.
3. HELLESLAND, J. and GREEN, R., 'Sustained and Cyclic Load Behavior of Reinforced Concrete Columns', A.S.C.E. National Structural Engineering Meeting, Portland, Oregon, April 1970.
4. BREEN, J.E. and FERGUSON, P.M., 'The Restrained Long Column as a Part of a Rectangular Frame', Journ. A.C.I., Proc. Vol. 61, No. 5, May 1964.
5. FURLONG, R.W. and FERGUSON, P.M., 'Tests of Frames with Columns in Single Curvature', Symposium on Reinforced Concrete Columns, A.C.I. Publication SP-13.
6. MAUCH, S., and HOLLEY, M.J., 'Creep Buckling of Slender Columns', Journ. A.S.C.E., Vol. 89, No. ST4, 1963, pp. 451-481.
7. HELLESLAND, J. 'A Study into the Sustained and Cyclic Load Behaviour of Reinforced Concrete Columns', A Doctoral Dissertation presented to the University of Waterloo, Waterloo, Ontario, Canada, 1970.
8. MANUEL, R.F. and MacGREGOR, J.G., 'Analysis of Restrained Reinforced Concrete Columns under Sustained Load', Journ. A.C.I., Jan. 1967.
9. RÜSCH, H., 'Researches towards a General Flexural Theory for Structural Concrete', Journ. A.C.I., Proc. Vol. 57, No. 1, July 1960, pp. 1-28.
10. RASCH, C., 'Spannungs - Dehnungs - Linien des Betons und Spannungsverteilung in der Biegedruckzone bei konstanter Dehngeschwindigkeit,' Deutscher Ausschuss für Stahlbeton, Heft 154, Sept. 1962, Berlin.
11. A.C.I. COMMITTEE 318, 'Proposed Revision of A.C.I. 318-63: Building Code Requirements for Reinforced Concrete', Journ. A.C.I., Proc. V. 67, No. 2, Febr. 1970.
12. AASHTO, 'Standard Specifications for Highway Bridges', The American Association of State Highway Officials, 1965.

## SUMMARY

This study has been concerned mainly with the influences of a sustained load on a column's load carrying capacity. Attention was restricted to unrestrained, eccentrically loaded columns. Most emphasis was placed on the column strength characteristics at low and intermediate load levels. The limiting case of loads causing creep instability at the end of the time periods considered was also obtained, however.

It was found that the load carrying capacity may be significantly reduced under sustained overloads, and also under sustained service loads for slender columns with small end eccentricities.

## RESUME

Dans cette étude, les auteurs examinent l'influence d'un effort permanent sur la résistance des colonnes en béton armé. Ils examinent seulement les colonnes libres soumises à des charges excentriques. On détermine les caractéristiques de la résistance des colonnes soumises à des efforts réduits ou moyens et on obtient les valeurs des charges dont le fluage du béton entraîne l'instabilité au bout d'un certain temps.

L'étude montre la réduction considérable de la résistance des colonnes soumises à des excès de charges permanentes, et des colonnes élancées soumises à des charges de service légèrement excentriques.

## ZUSAMMENFASSUNG

Die vorstehende Studie gilt hauptsächlich dem Einfluss der Dauerlast auf die Tragfähigkeit einer Säule, wobei die Betrachtungen auf freie exzentrisch belastete Säulen beschränkt waren. Das Hauptgewicht wurde auf die Aenderung der Tragfähigkeit zufolge geringer und mittlerer Dauerlasten gelegt. Ebenso wurde der Grenzfall der Belastungen bestimmt, welche Kriech-Instabilität am Ende des betrachteten Zeitintervalles bewirkten.

Als Ergebnis konnte festgestellt werden, dass die Tragfähigkeit durch Dauerlasten über dem zulässigen Lastwert beträchtlich reduziert wird; dies gilt auch für schlanke Säulen mit kleiner End-Exzentrizität unter geringer Dauerlast, wie etwa übliche Gebrauchsbeanspruchung.

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