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## Measurements and Observations of Creep and Shrinkage in Prestressed Reinforced Concrete Bridges

Mesures et observations des effets du fluage et du retrait sur les ponts en béton précontraint

Kriech- und Schwindmessungen sowie -beobachtungen an Spannbetonbrücken

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### 1. Introduction

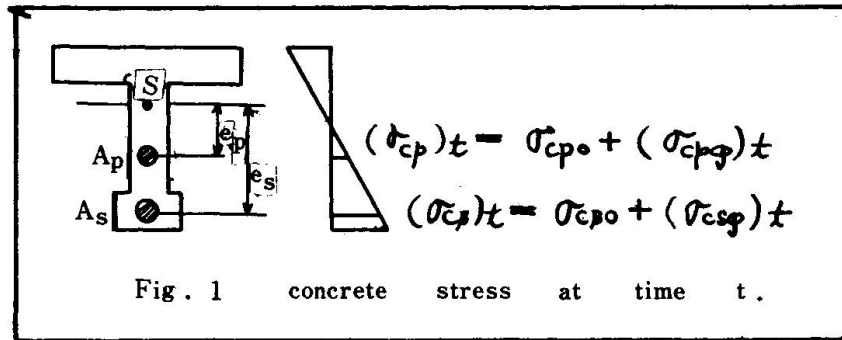
The "Prestressed Reinforced Concrete (PRC)" system (1) is defined as a system in which relatively small prestressing forces are introduced into reinforced concrete (RC) to reduce the opening of cracks. In this case, the RC is designed to have the required safety factor against collapse. The conditions to be met for tolerable crack opening in PRC is the same as that in RC. In other words, PRC is a partially prestressed concrete with steel reinforcements.

The term "Prestressed Reinforced Concrete (PRC)" was proposed by the senior author's team in 1961 to distinguish PRC from the full or limited prestress system used to date. The main difference between the two systems is as follows; the basic system of PRC is reinforced concrete while that of PC is plain concrete in the absence of prestressing forces.

In this paper, the behaviors of creep and shrinkage in PRC system is discussed on the basis of the proposed method of analysis and measurements on Kamihimekawa bridge which was designed as the proposed system.

### 2. Derivation of the Equations for Creep and Shrinkage Stresses in PRC system

The following equations are obtained for creep and shrinkage stresses in PRC system which section has comparatively large sectional area of tendon.



Concrete stresses at time  $t$  are shown in Fig. 1 and concrete strain due to creep and shrinkage at the position of reinforcement for  $dt$  time can be expressed by

$$\left\{ (\sigma_{cs})_t / E_c + w / \varphi \right\} dy$$

On the other hand, the axial stress due to the restraint of reinforcements can be obtained as follows:

$$-\left\{ (\sigma_{cs})_t + w E_c / \varphi \right\} dy$$

In the similar manner, the following equations can be derived at the position of tendon:

$$\left. \begin{aligned} d(\sigma_{cs})_t / dy + \alpha_{ss} \{ (\sigma_{cs})_t + w E_c / \varphi \} + \alpha_{sp} \{ (\sigma_{cp})_t + w E_c / \varphi \} &= 0 \\ d(\sigma_{cp})_t / dy + \alpha_{ps} \{ (\sigma_{cs})_t + w E_c / \varphi \} + \alpha_{pp} \{ (\sigma_{cp})_t + w E_c / \varphi \} &= 0 \end{aligned} \right\} (1)$$

$$\left. \begin{aligned} \text{where } \alpha_{ss} &= n A_s / A_i (1 + e_s^2 / \rho_i^2), \quad \alpha_{sp} = n_p A_p / A_i (1 + e_s e_p / \rho_i^2) \\ \alpha_{ps} &= n A_s / A_i (1 + e_s e_p / \rho_i^2), \quad \alpha_{pp} = n_p A_p / A_i (1 + e_p^2 / \rho_i^2) \end{aligned} \right\} (2)$$

where  $\rho_i$  is radius of gyration about  $A_i$ .

Substituting the initial conditions that  $(\sigma_{cs})_{t=0} = \sigma_{cs0}$

and  $(\sigma_{cp})_{t=0} = \sigma_{cp0}$  into Eq. (1)

$$\left. \begin{aligned} \sigma_{cs} &= K_1 e^{Ay} + K_2 e^{By} - w E_c / \varphi, \quad \sigma_{cp} = K_3 e^{Ay} + K_4 e^{By} - w E_c / \varphi \\ \therefore \sigma_{csy} &= \sigma_{cs} - \sigma_{cs0}, \quad \sigma_{cpy} = \sigma_{cp} - \sigma_{cp0} \end{aligned} \right\} (3)$$

where

$$\left. \begin{aligned}
 A, B &= \frac{\alpha_{SS} + \alpha_{PP}}{2} \pm \sqrt{\left(\frac{\alpha_{SS} + \alpha_{PP}}{2}\right)^2 - (\alpha_{SS}\alpha_{PP} - \alpha_{SP}\alpha_{PS})} \\
 K_1 &= \frac{1}{A-B} \left\{ (A + \alpha_{PP})(\sigma_{CS0} + w E_c / \varphi) - \alpha_{SP}(\sigma_{CP0} + w E_c / \varphi) \right\} \\
 K_2 &= \frac{-1}{A-B} \left\{ (B + \alpha_{PP})(\sigma_{CS0} + w E_c / \varphi) - \alpha_{SP}(\sigma_{CP0} + w E_c / \varphi) \right\} \\
 K_3 &= \frac{1}{A-B} \left\{ (A + \alpha_{SS})(\sigma_{CP0} + w E_c / \varphi) - \alpha_{PS}(\sigma_{CS0} + w E_c / \varphi) \right\} \\
 K_4 &= \frac{-1}{A-B} \left\{ (B + \alpha_{SS})(\sigma_{CP0} + w E_c / \varphi) - \alpha_{PS}(\sigma_{CS0} + w E_c / \varphi) \right\}
 \end{aligned} \right\} (4)$$

Solving for steel stress

$$\sigma_{sy} = - \int_0^y n \left\{ (1 - \alpha_{SS})(\sigma_{CS} + w E_c / \varphi) - \alpha_{SP}(\sigma_{CP} + w E_c / \varphi) \right\} dy$$

There results

$$\left. \begin{aligned}
 \sigma_{sy} &= n(1 - \alpha_{SS})T_1 - n\alpha_{SP}T_2 \\
 \sigma_{py} &= n\alpha_{PS}T_1 - n(1 - \alpha_{PP})T_2
 \end{aligned} \right\} (5)$$

where  $T_1 = \frac{K_1}{A}(1 - e^{Ay}) + \frac{K_2}{B}(1 - e^{By})$ ,  $T_2 = \frac{K_3}{A}(1 - e^{Ay}) + \frac{K_4}{B}(1 - e^{By})$

The increment of radius due to creep and shrinkage can be expressed as

$$\left\{ \frac{\sigma_{sy}}{E_s} - \frac{\sigma_{py}}{E_p} \right\} / (e_s - e_p)$$

Therefore, the deflection and the angle of slope are obtained by integrating the above term throughout the span length.

The above derivations are based on the uncracked whole concrete section.

However, for the cracked sections, the deflection and the increment of radius due to creep and shrinkage can be also obtained by substituting the section modulus and stresses, computed from the concrete section with the neglected tension zone, into above equations.

### 3. Kamihimekawa Bridge

The Kamihimekawa bridge is the first PRC (prestressed reinforced concrete) bridge constructed in Japan.

The bridge has the main span length of 48m forming a -shaped rigid frame structure with the total span length of 80m. The carriageway is 7m wide and horizontally curved with the radius of 400m, Fig. 2.

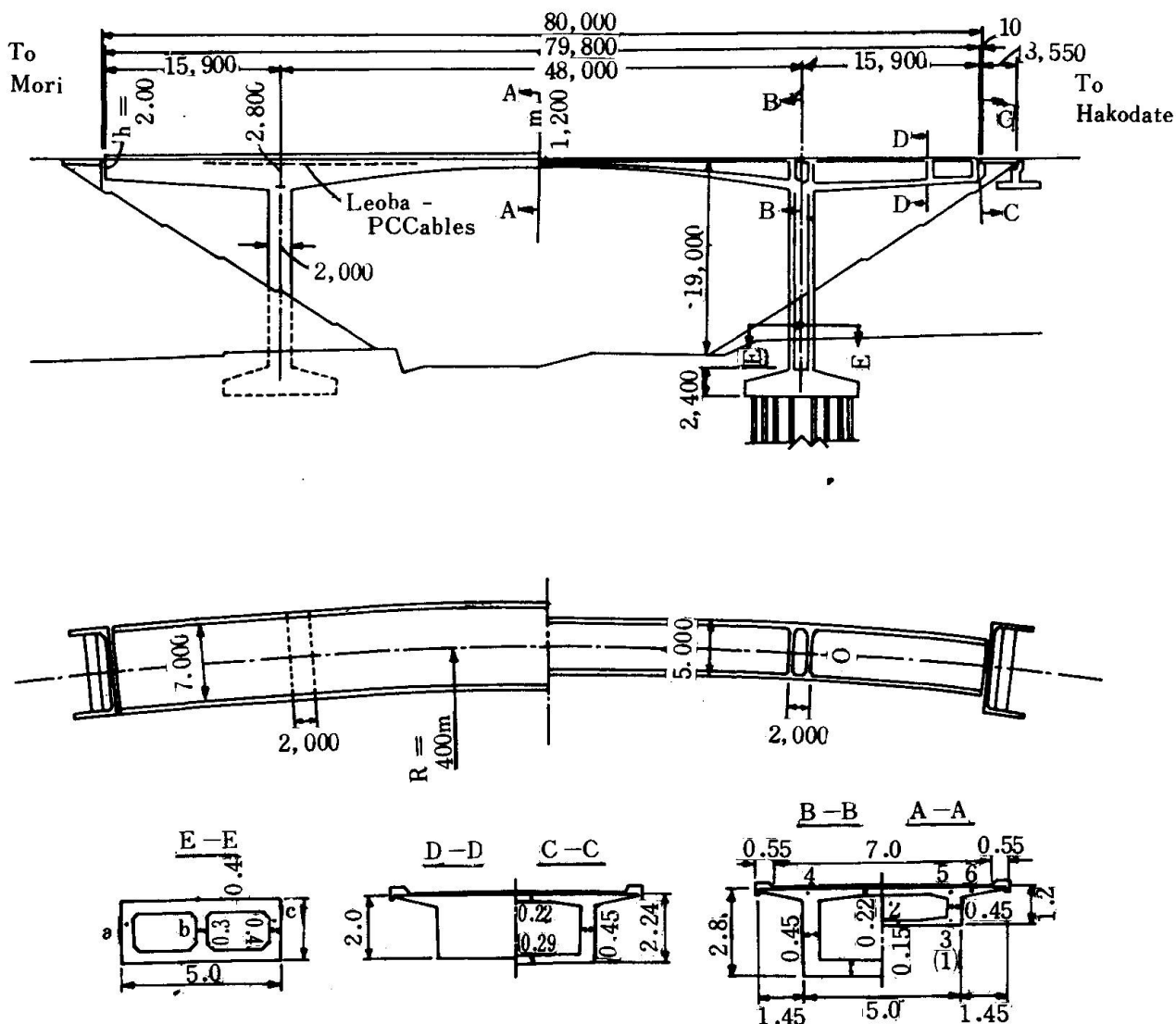


FIG. 2 Kamihimekawa Bridge (Japan)

Each half span and the cantilever girder were constructed by successive-corbellings from the piers in a symmetrical manner. The static calculations of

the structure were performed for the two states of structural systems, i.e. a system of cantilevers during the construction and that of rigid frame after closing.

The structure was designed as a RC system, using 32mm cold-twisted bars of steel SDC 40 as the main reinforcement and concrete of  $\sigma_{28}=360 \text{ kg/cm}^2$  quality for the main girder and  $\sigma_{28}=210$  for the piers.

The total of 470 ton of prestressing force were longitudinally introduced during the corbelling construction to the portions of deck girders of 22m length in order to control crack widths, Fig. 2. The prestressing was performed in accordance with the Leoba system in the two webs of box girder by means of prestressing tendons which consist of 6 x S66 (16/8) and 2 x S24 (12/5)

The super structures was built by corbelling in successive stages without centering. The length of the successive advances, which numbered 20 in all, was 3.0m. In accordance with the progress of the girders, the non-prestressed main bars  $\phi 32$  were screwed to the next bars of 6m length by means of FY-couplers.

The FY couplar is a special type of screw-threaded coupler which was developed by the senior author's team and experimentally tested to ensure the transmitting of the entire strength of the bar to be joined.

The main structure was completed in November, 1965 and the bridge was opened to traffic in June, 1966.

4. Measurements of Creep and Shrinkage in Kamihimekawa Bridge

The strain due to shrinkage are measured at the section D-D of side span, shown in Fig. 2, where the prestressing force is not introduced.

Fig. 3 shows the results of the investigations on shrinkage.

The dotted points indicate the calculated values and folded lines show the measured values.

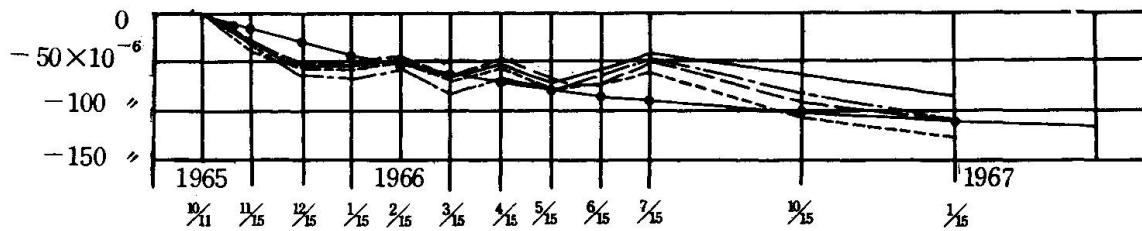


Fig. 3  $E_{el}$  at section D-D  
 $E_{el}$  : elastic strain  
 $E_g$  : creep strain  
 $E_w$  : shrinkage strain

The shrinkage strains observed for the first three months are greater than the calculated ones, since the developments of shrinkage is almost same as the assumption that  $\epsilon_s = 13 \times 10^{-5}$ .

Fig. 4 shows the total value of elastic creep and shrinkage strains, and Fig. 5 showw the total value of elastic, creep, shrinkage and secondary strains.

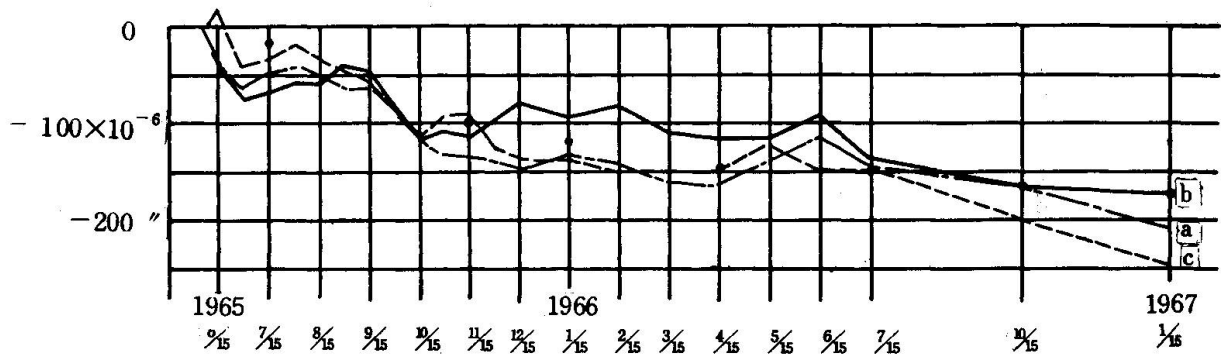


Fig. 4  $E_{el} + E_g + E_w$  at section E-E

The observed final values are almost same as the calculated value based on the assumptions that  $\varphi_{\infty} = 2.5$  and  $\varepsilon_s = 13 \times 10^{-5}$ .

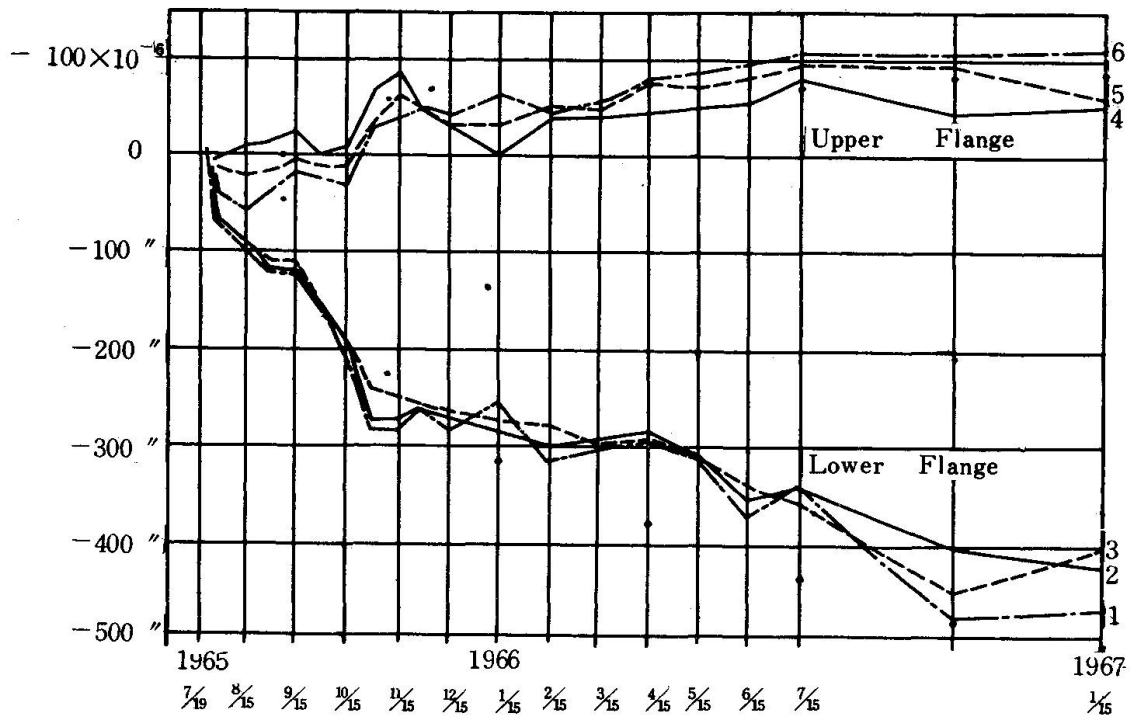


Fig. 5  $E_{el} + E_g + E_w$  at section A-A

## 5. Acknowledgement

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## 6. Reference

- (1) Hideo Yokomichi, "Prestressed Reinforced Concrete System", the final report, the 8th Congress of IABSE, New York, September, 1968. PP 901-912.

**SUMMARY**

The behaviors of creep and shrinkage in PRC system is described on the basis of the proposed method of analysis and measurements on Kamihimekawa bridge which was designed and constructed as the proposed system.

**RESUME**

On décrit les influences du fluage et du retrait dans le béton armé précontraint (PRC), sur la base des calculs et des mesures du pont de Kamihimekawa conçu et exécuté selon le système proposé.

**ZUSAMMENFASSUNG**

Beschrieben wird das Kriech- und Schwindverhalten in bewehrten Spannbetonbauten aufgrund eines vorgeschlagenen Rechenverfahrens sowie Messungen an der Kamihimekawa Brücke, welche nach diesem Verfahren entworfen und ausgeführt worden ist.