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III

Design of Concrete Structures for Creep, Shrinkage and Temperature Changes

Etudes concernant le fluage, le retrait et les changements de température pour les structures en béton

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1. INTRODUCTION

Concrete is a dynamic structural material which is in constant transition from its initally plastic state to an ultimately stable state. To design or analyze concrete as a stable material, a state that can only be approached but never completely achieved, can result in serious misconceptions as to the capacity and behavior of the structure in which it is used. Of particular importance is the self-straining process caused by the shrinkage that concrete undergoes throughout a structure's existence. The internal straining caused by the shrinkage process can result in large tensile stresses at the exposed surfaces of a structural member during its early history. This can, under certain circumstances, significantly affect a member's load carrying characteristics.

It should be noted that when concrete is treated as a static material and the stresses induced in it due to its own internal straining processes are not considered, much of the reserve capacity of the concrete structure is being ignored. As the concrete ages, the initially induced tensile stresses at the surface of a member's cross-section will be completely replaced by a state of compression. By considering this aging process, unnecessary strengthening of many structures to carry new loading requirements imposed later in the life of the structure may be eliminated.

The difficulty with incorporating the effects of the self-straining process of concrete in normal design is that the only solution currently available requires the solution of second-order partial differential equations (1). The solution of these equations is beyond the mathematical techniques ordinarily available to the design engineer. In addition, although the properties of concrete are well known, it is difficult to guarantee these properties. Modulus of elasticity and shrinkage characteristics can vary quite a bit for the same concrete mix. However the properties of concrete can be fixed with adequate accuracy to make the approach outlined herein valid. The purpose of this paper is to bridge the gap between these theoretically exact solutions and the actual design situation. It provides an approximate solution to the problem of the self-straining of concrete which is within the scope of the mathematical techniques usually employed by a design engineer.

Two related questions are raised in solving the self-straining problem. The questions pertain to how the distribution of self-induced strains, particularly shrinkage strains, vary with depth and time; and what, if any, stress relaxation will occur to relieve the stresses induced in the concrete. This paper provides all the necessary data to consider both shrinkage distribution and stress relaxation. First, to make this information more meaningful, a basic solution for self-strained members is presented.

2. BACKGROUND

The background of this paper was developed as a result of research performed for the McDonnell Douglas Aircraft Corporation. This research was used to investigate the structural behavior and capacity of the LaGuardia Airport Runway deck located in New York City, New York, U.S.A. This deck consisted of a composite deck of precast concrete used as formwork and to carry the dead load of a poured-in-place slab.

In addition, consultation to the American insurance companies has led to the authors involvement in the study of many shrinkage and creep induced failures in structures. Two examples are a twenty-story apartment which exhibited severe distress of its facing due to shortening of its columns and a parking garage that was experiencing significant cracking of its continous main girders although it had been designed within accepted structural standards. The results of these studies are included in an illustrative example presented later in this paper.

As a result of these investigations, the conclusion was reached that the internal straining of the concrete can have a dominant role on a structure's behavior and that there is widespread misconception as to the nature of the internal straining of concrete. In order to help clear up this problem and to provide a design oriented approach, this paper has been prepared.

3. DERIVATION OF BASIC SOLUTION

The solution of shrinkage, creep and thermal problems in concrete structures can be divided into three areas.

1. Determination of the build-up of internal strains in concrete through time.
2. Analysis of stress due to the induced strain pattern.
3. Relaxation of the induced stresses.

The general governing differential equation for the distribution of strain in three dimensions is:

$$\frac{\partial S}{\partial t} = K\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2}\right) \qquad \text{(a)}$$

where

S is the unrestrained unit linear shrinkage strain or thermal strain for the concrete.

K is the diffusion coefficient of shrinkage or temperature in square inches per day.

t is the time in days.

For the simple case of one directional diffusion the problem reduces to

$$\frac{dS}{dt} = K \frac{d^2S}{dy^2} \tag{b}$$

The solution for this strain distribution has been plotted in Figure 1 and an explanation of how to use the chart is included in the next section. With the induced strain pattern thus determined, the next step is the determination of the resultant equilibrium stress pattern.

An important point to note at this stage of the analysis is that the induced strain pattern is set in the member. Its shape is set and any type of equilibrium transformation can only result in the strain changing sign from tension to compression. To illustrate this point assume that a typical strain pattern, namely the most common one for diffusion, a parabola with a strain 100×10^{-6} in/in on one surface and zero strain on the other surface is chosen as illustrated in Figure 2. The equivalent unrestrained stress diagram can be found by multiplying the strains by the modulus of elasticity (3×10^6) as shown in Figure 3. Since the diagram is parabolic the centroid is 3/4 the depth away from the bottom and the area is 1/3 the maximum stress times the crossectional area of the diagram. This area can be considered as an equivalent unrestrained force (P_u) acting at that centroid.

Using the principle of plane sections remaining plane the restrained stress distribution supplied by the section (Fig.3b) is

$$\sigma_y = \frac{P_u}{A} \pm \frac{P_u e y}{I} \tag{c}$$

Where P_u is the unrestrained equivalent force, e is the eccentricity of the equivalent force, y is the distance to and from the sections centroid, A is the area, and I is the moment of inertia. This stress distribution must be superimposed on the unrestrained diagram to achieve the final stress diagram shown in Figure 3 (c).

This simplified technique has been verified by matrix compatibility schemes and found perfectly general. If reinforcing steel is included then the properties of the transformed section are used in the analysis.

4. DETERMINATION OF SHRINKAGE STRAINS IN CONCRETE

The use of the equivalent force solution for stresses induced by the internal straining of concrete requires that the distribution of internal strain with respect to the cross section of a member be determined as the concrete ages. Such a relationship was found to exist between the moisture distribution in a member and shrinkage strain. On this basis Carlson (2) developed a chart of the percent shrinkage remaining in a member versus time. This chart depended on three factors:

1. The rate of diffusion "R" of the concrete (usually taken as .0001 ft^2./day).
2. The distance "a" from the face to the center line of the member (normally one half the depth of the member in ft.).
3. The time " ϕ " in days since the pouring of the concrete.

The results of this relationship are plotted in Fig.1 in terms of a non-dimensional factor $1000\, R\phi/a^2$ and the distance away from the surface in terms of "a".

Once the diffusion constant and the depth are chosen, the shrinkage distribution through the depth of the member is only dependent on the age of the structure. Then, the non-dimensional parameter can be written as $K\phi$ where K equals 1000 R/a^2.

With this information, the induced strain in a concrete member at any depth at any time can be found. The only other value needed is the ultimate shrinkage for the type of concrete under consideration. This is normally chosen as the twenty-year shrinkage value. Typical values for the twenty-year shrinkage for different classes of cement have been provided in Table 1.

A more accurate solution of the problem includes the build-up of the modulus of elasticity with respect to time. Such a refinement is justifiable during the early history of the structure since a significant percentage of the external shrinkage occurs during the period when the modulus of elasticity is rapidly building up. Although very little direct information on the build-up of the modulus of elasticity exists, it is possible to use readily available information on the build-up of compressive strength of concrete which is proportional to modulus of elasticity. By using the empirical ACI relationship between the compressive strength of the concrete and its modulus of elasticity, it is possible to translate the build-up of compressive strength into an equivalent build-up of the modulus of elasticity. The empirical ACI formula is

$$E=33w^{3/2}\sqrt{f'c} \tag{d}$$

where w is the weight of the concrete in pounds per cubic foot; and f'c is the compressive strength of the concrete during any time interval.

5. DETERMINATION OF EFFECT OF RELAXATION

The problem of determining the behavior of concrete due to its internal straining process is not complete after the initial stress pattern due to the build-up of shrinkage strain in each time interval has been found. One final phenomena of concrete behavior, stress relaxation, must be accounted for. To clarify this action, two definitions of concrete behavior will be presented; namely, that of creep and stress relaxation. Creep is the increase in strain due to a constant stress while stress relaxation is the decrease in stress due a constant strain. As might be expected from these definitions, one has a reciprocal relation to the other. It is thus possible to analyze for stress relaxation by means of readily available information on the creep properties of the concrete being considered.

Early work on stress relaxation was performed by Whitney (5), but the most significant work was recently performed by Ross (4). Based on Ross's work, Mattock (3) developed a simple rate of creep formula for stress relaxation which has the following relations:

$$R = (1-e^{-\theta}) / \theta \tag{e}$$

where R= residual stress factor and $\theta = e_c/e_e$ is ratio of creep strain, e, to elastic strain, ϵ, per psi of stress.

The value of creep strain, and therefore θ, is highly dependent on the age of the structure when the stress is applied. An example of how the creep and elastic strains vary as stresses are induced at different points in the life of the structure is illustrated in Fig.4. As can be seen from this illustration, the magnitude of creep strain varies inversely with the age of the concrete. Some average values of θ and the modulus of elasticity as related to the 28-day strength (f'c) of the concrete have been plotted in Fig.5 and Mattock's formula for different values of θ has been illustrated in Fig. 6. Based on Fig. 4 and 6, a normal stress relaxation curve at various ages can be determined. This relationship has been presented in Fig. 7. The important result of this plot is that it can be seen that stress relaxation decreases with age until a stable state is reached beyond which time the stress relaxation will closely approximate a constant factor. A further note on creep and stress relaxation can be made as to the effect of compression and tension. Illston (6) found no significant difference between total creep in tension or compression. The only difference that was noticed was that tensile creep builds up faster than compressive creep although both will have the same ultimate value.

6. PROCEDURE FOR ANALYSING CONCRETE FOR SELF-INDUCED STRAINS.

The approach used in this paper is an incremental time method. The step by step procedure for the total solution is as follows:

1. The first step in the analysis is to decide on the proper time increments to be used. This is based on two criteria: the way in which the modulus of elasticity and the stress relaxation approach their ultimate steady state conditions as the concrete ages.

2. With the proper time increments chosen, the change and distribution of shrinkage strains (or any other strains) over the depth of the member for each time interval are determined from Fig.1.

3. The initial state of stress for each time interval can be determined by using the equivalent force approach.

4. With the initial state of stress determined for each time interval, the stresses are then relaxed in the subsequent time intervals in accordance with the age of the structure when the initial strain was induced. This is accomplished using Fig.6. The value of θ is obtained by only accounting for the amount of creep strain that would have occurred during the time since the initial strain was induced. By superimposing consecutive time intervals, a complete history of the behavior of a structure due to the self-induced strains can be obtained.

7. ILLUSTRATIVE EXAMPLE

A problem similar to one investigated by the authors of this paper was a multi-bay concrete frame illustrated in Figure 8a. The main concrete girder was showing severe cracking, and questions were raised as to the extent shrinkage had and would continue to contribute to the cracking.

For ease of illustration only, the steel reinforcement was considered to be the equivalent of six 1-inch round bars symetrically placed along the girder's entire length. The girder was of standard weight concrete with minimum 28-day strength of 4000 psi.

Using the smaller 14-inch width of the girder as the controlling dimension, the shrinkage constant "K" is equal to 0.292 and the average shrinkage strain can be determined from Table 1. Using the procedure just outlined, a complete history of the concrete girder due to its internal straining process was obtained. This history has been listed in Table 2, and complete stress distributions at three months, one year, three years and twenty years have been illustrated in Figure 8b.

The following conclusions were reached from the illustration: First, concrete shrinkage cannot be prevented by reinforcement within normal percentage of steel for beams or columns. Concrete stresses will reverse themselves due to the exterior of a concrete member "drying-out" earlier than the interior while the concrete is in a more plastic state. Of particular importance is the outside face of the concrete girder which changes from over 400 psi tension at three months to over 600 psi compression at twenty years.

8. CONCLUSION

Up to now concrete has only been successfully analyzed as a static material, a material whose own internal processes did not affect the behavior of the structure in which it was used. The few attempts at analyzing concrete for its internal straining processes were either too complex for practical application or simplified the problem by omitting the way in which shrinkage is distributed over the member's depth as it ages or the way in which concrete relaxes stresses through time. These ommissions could lead to large inconsistencies as to the behavior of a structure through time. The time dependent analysis presented in this paper provides a framework not only for shrinkage strains but any self-induced strains in a structure whose properties vary with time. The analysis has been set up by the use of simultaneous equations which are normally used in engineering and which can be solved either by hand or rapidly on computer.

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10. TABLES

TABLE 1

TYPICAL ULTIMATE (20yr.) SHRINKAGE VALUES (10^{-6} in/in)

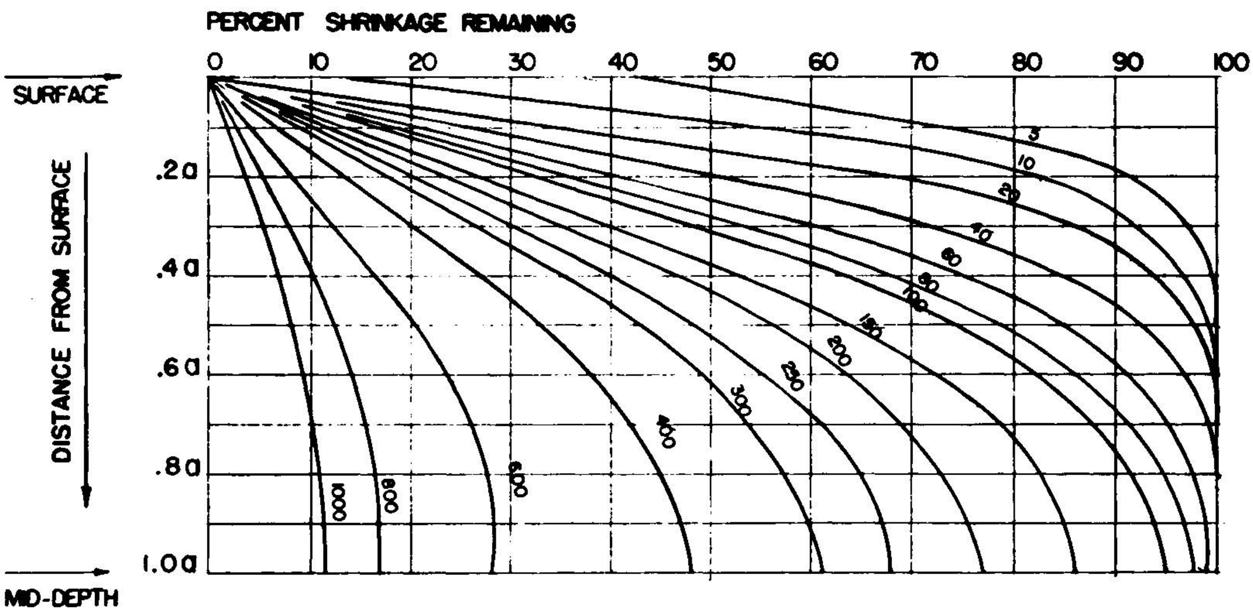
Classification of Concrete	Shrinkage Strain 10^{-6} High.	Avg.	Low.
Light-weight	1010	850	790
Standard	770	660	600
Finely Ground	810	680	630
High Early Strength	820	700	640

TABLE 2.

	Unrestrained Concrete		Restrained by Reinforcement and Columns		
Age of Concrete	Maximum Compression	Maximum Tension	Increase in Tension of Concrete	Compressive Stress Induced in Reinforcement	Deflection Inwards of Ends of Girder (from Center Line of Frame)
3 months*	-120 psi	+440 psi	+ 38 psi	- 2.8 ksi	.07"
6 months	-130	+270	+ 49	- 3.7	.09"
1 year*	-200	+190	+ 78	- 5.9	.15"
2 years	-140	+120	+117	- 8.3	.21"
3 years	-130	+100	+135	-10.1	.25"
10 years	-520	+140	+203	-15.1	.38"
20 years	-600	+200	+218	-16.2	.41"

* Complete Stress Distribution Illustrated in Figure 8.

11. FIGURES



Ø = TIME IN DAYS
K = $1000\,R/a^2$

Fig. 1 Shrinkage Distribution Through Time

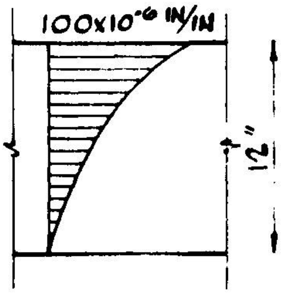


Fig. 2
Strain Pattern

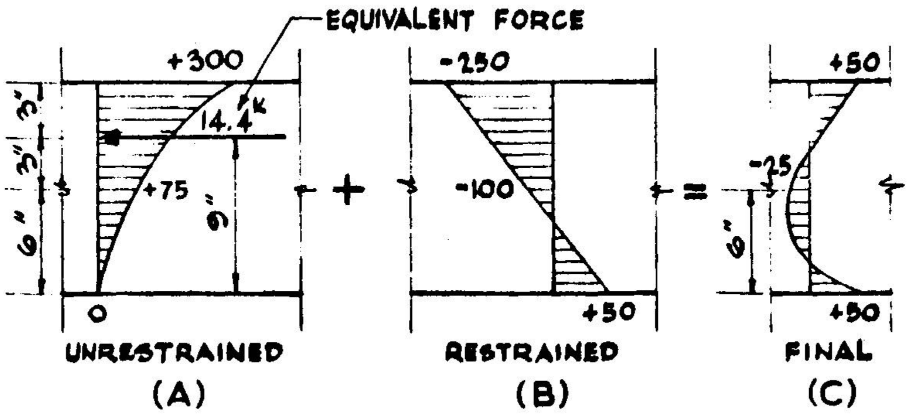


Fig. 3
Stress Distribution

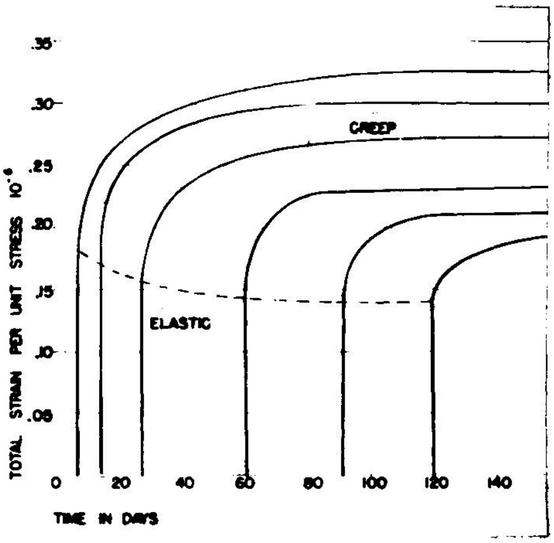


Fig. 4
Total Strains Due to Constant Sustained Stresses Applied at Various Ages

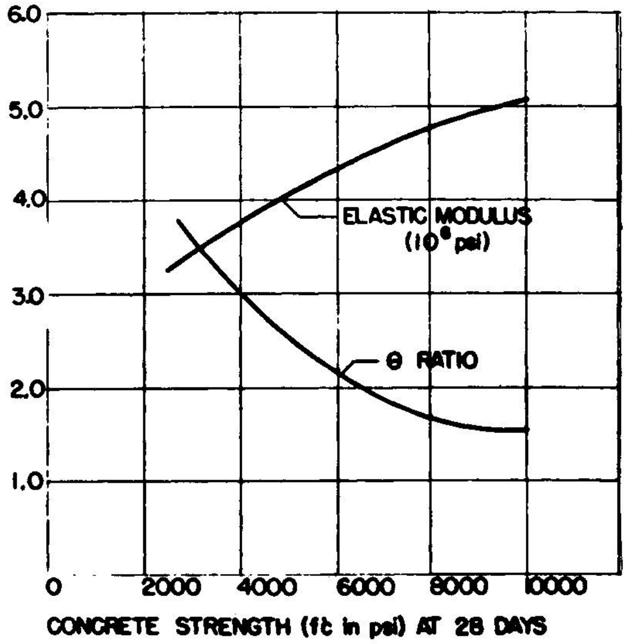
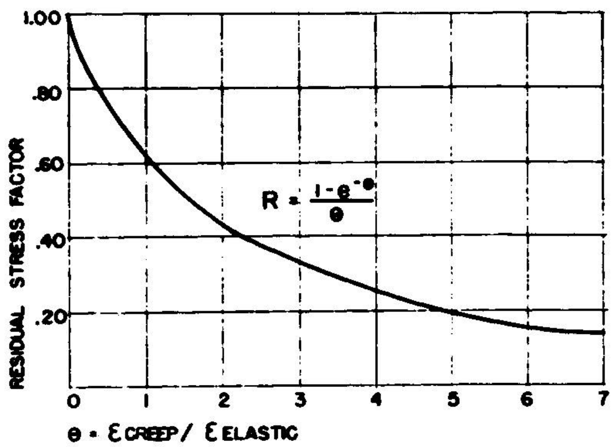


Fig. 5
Typical Values of θ and Elastic Modulus

Fig. 6
Residual Stress Relationship



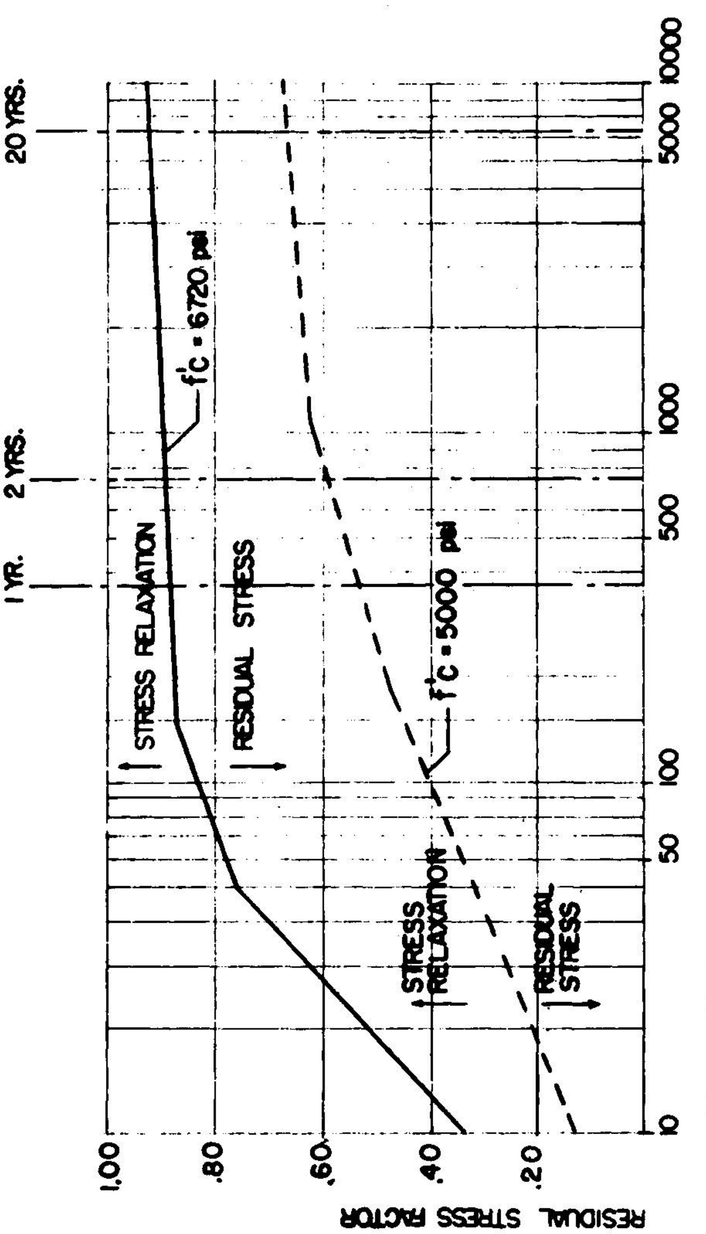


Fig. 7

Stress Relaxation Through Time

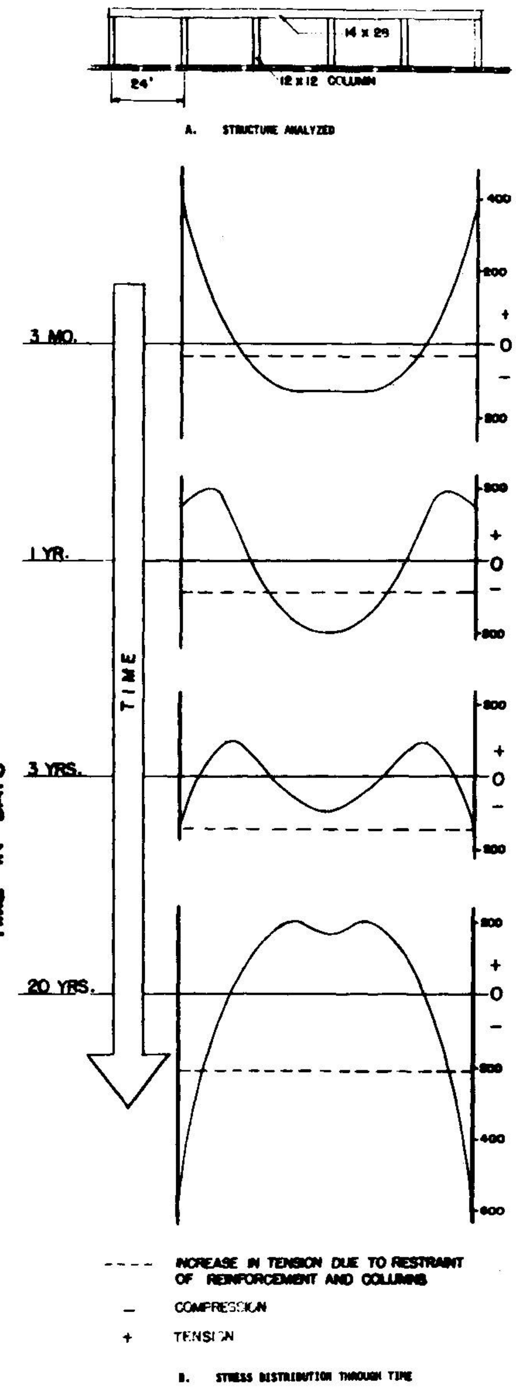


Fig. 8
Illustrative Example

APPENDIX

Examples of Existing Structures.

The internal straining of the concrete can have a dominant role on a structure's behavior. The magnitude of stresses developed by the internal straining of concrete can only be appreciated by observing cases in which shrinkage or thermal movements are constrained. For example, if a unit contraction equal to 200×10^{-6} in/in is induced in a concrete structure, the corresponding stresses are quite high when the contraction is constrained. These stresses are equal to the modulus of elasticity times the unit contraction, or 3×10^6 (lbs. per square inch) times 200×10^{-6} in/in which equals 600 psi. Multiplying this stress by the area of concrete determines the force developed by restraining the member from contracting. Using an area of 500 square inches, typical for large prestressed members, a restraining force of 300,000 lbs. is developed.

An example of this restraining force is illustrated in Figure 9. In this case the prestressed beam is connected by welding at both ends to the supporting bracket. The beams were thus partially restrained. This resulted in the development of large restraining forces. These restraining forces were sufficient to cause the bracket to crack open due to the tensile stresses.

A second example of the effect of internal straining of concrete, is the case of a twenty story building with brick facing. In this case shrinkage, creep and temperature contributed to the contraction of the structural concrete framework. If adequate provision is not made in the brick work to allow for this contraction the brick will restrain the movement. Expansion joints in the horizontal joints of the brick work should have been provided. Figure 10 illustrates a case where adequate expansion or contraction joints were not provided. The result was severe distress and eventual buckling of the brick facing. The contraction of the concrete frame was aggravated in this case by the use of lightweight concrete.

The main lesson of these two examples is that ample provisions must be allowed for the expansion and contraction of concrete. Any attempt at trying to restrain such movements will result in large and potentially destructive restraining forces.

The internal straining of concrete can be beneficial instead of destructive if the designer takes proper account of the time dependent behavior of concrete. This was particularly evident in the investigation of the structural capacity of the LaGuardia Airport runway deck in New York City, in the United States, as illustrated in Figure 11. The deck is supported over the water and consists of composite construction of precast inverted double tees that served as form work to carry the dead load of a poured-in-place slab. Since the shrinkage had partially occured in the precast tees before the poured-in-place slab was installed, a differential shrinkage between the two sections existed. This differntial shrinkage induced compressive stresses across the bottom face as illustrated in Figure 12. This differential shrinkage effect played a major role in determining that the structure had developed sufficient reserve capacity to land a plane twice as heavy as the original design called for. This was subsequently verified by load tests. A similar result wouldbe found even if the deck were not composite as was seen in Figure 8. The development of reserve capacity as concrete ages can be particularly important to structures where load requirements increase with time, such as highways and airports.

Each of these examples illustrate the dominant role that the internal straining of concrete can have on a structures performance. Concrete can no longer always be treated as a static material whose own internal straining processes do not effect the behavior of the structure in which it is used. The purpose of the paper was to provide an approach that will allow the designer to predict the behavior of structure under internal straining.

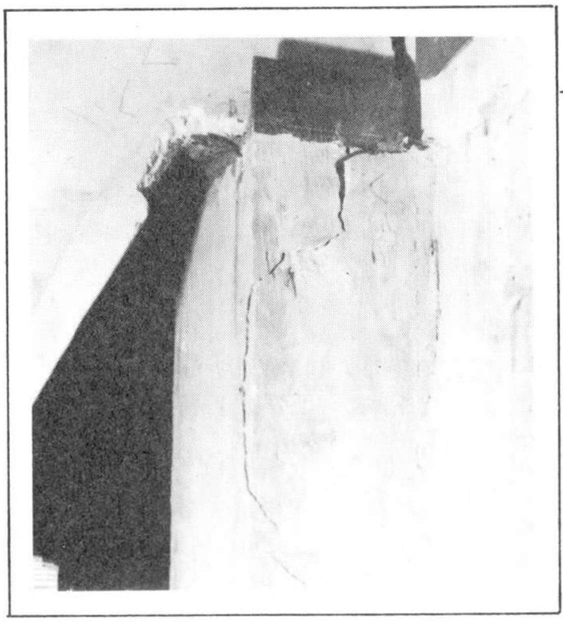


Fig. 9

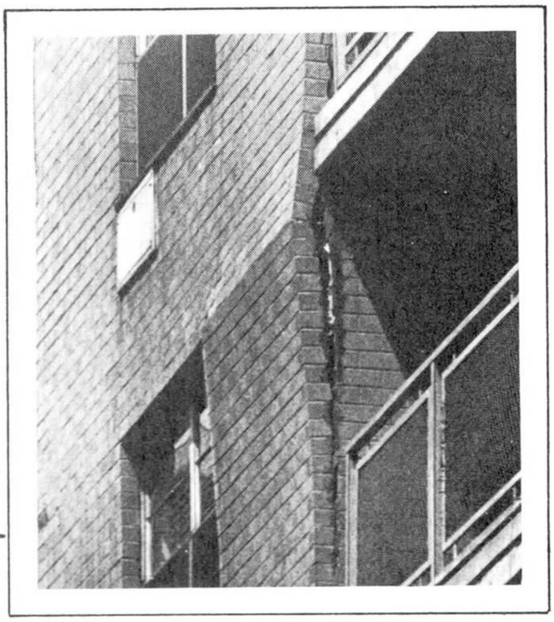
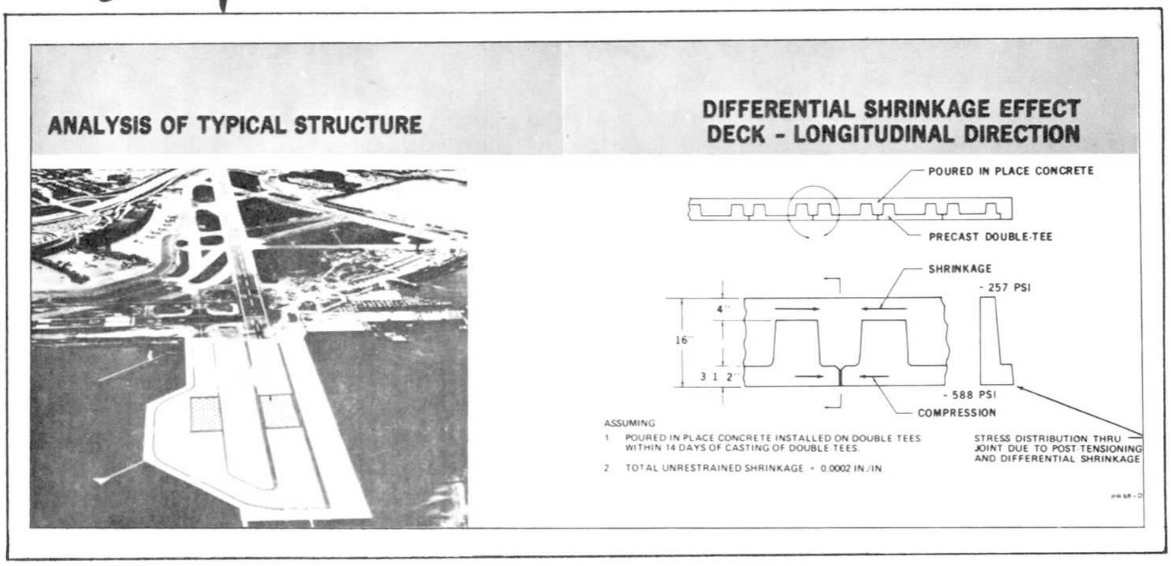


Fig. 10

Fig. 11



SUMMARY

The internal straining process of concrete due to shrinkage can be either beneficial or detrimental to the performance of a structure. Because of this phenomena the productive course of action is to design all structures utilizing this or at least minimizing its deleterious effects.

A brief outline of the mathematical approach to the problem and examples of actual structures built in the Unites States where the effects of shrinkage were predominant in the design and behavior are presented.

RESUME

Le retrait, qui entraîne dans le béton l'apparition de tensions internes, peut être un avantage ou un inconvénient pour l'exécution d'une structure. En conséquence, l'étude de toute structure se fera en tenant compte de ce phénomène, ou tout au moins en diminuant ses effets néfastes.

On présente un aperçu de l'approche mathématique du problème ainsi que des exemples de structures construites aux Etats-Unis d'Amérique dont l'étude et la réalisation furent intéressées par ce phénomène.

ZUSAMMENFASSUNG

Der Vorgang der inneren Stauchung infolge Schwinden kann sich entweder vor- oder nachteilhaft auf das Verhalten des Bauwerkes auswirken. Es ist daher vorteilhaft, in der Planung eines Bauwerkes dieses Phänomen zu beachten, oder zumindest die Nachteile zu minimieren.

Eine kurze mathematische Uebersicht der Lösung dieses Problems sowie Beispiele für in den Vereinigten Staaten errichtete Bauten, bei welchen Schwinden im Entwurf und Verhalten überwiegend war, werden angegeben.