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Verification and Application of a New Method of Creep Analysis to Structural Members

Vérification et application d'une nouvelle méthode pour l'analyse du fluage sur des parties d'ouvrage

Überprüfung und Anwendung eines neuen Verfahrens für die Kriechberechnung von Bauteilen

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We propose to show how data on creep and shrinkage of plain concrete can be used to calculate the time-dependent deformations and stresses in reinforced and prestressed concrete members. Using Trost's (1) relaxation coefficient η, we can write a general expression for the strain (including shrinkage $\varepsilon_{sh}(T)$) at time T:

$$\varepsilon(T) = \frac{f_o}{E_o}\left[1 + \phi(T,K_o)\right] + \frac{f(T) - f_o}{E_o}\left[1 + \eta\phi(T,K_o)\right] + \varepsilon_{sh}(T) \qquad (1)$$

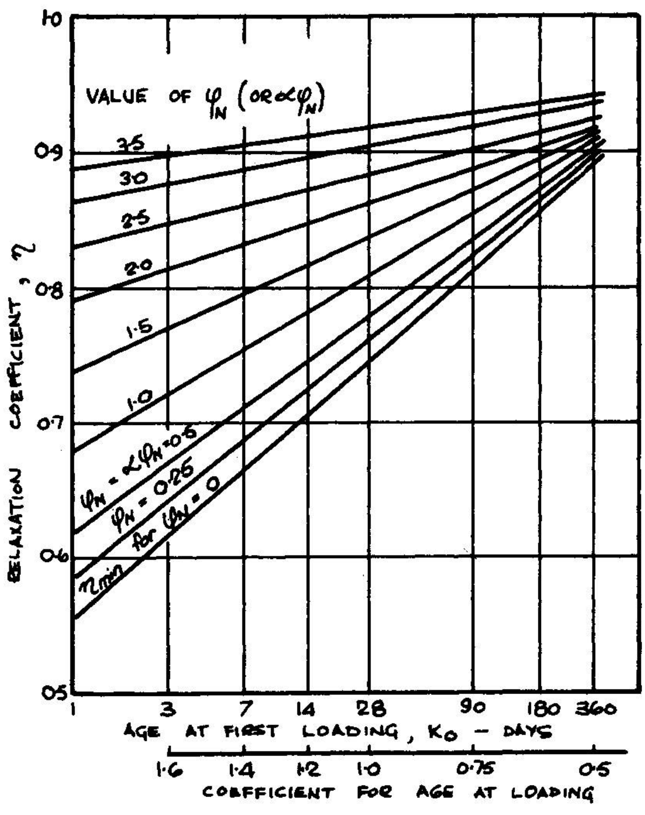

Fig. 1 Variation in relaxation coefficient

where f_o = concrete stress at the instant of loading K_o,
$f(T)$ = concrete stress at time $T > K_o$,
$\phi(T,K_o)$ = creep coefficient for concrete loaded at age K_o with the load sustained till time T.

The modulus of elasticity of concrete, E_o, is assumed to be constant and equal to the value at the age of application of stress f_o. The relaxation coefficient η takes into account the ageing of concrete as well as the variation in stress, which is assumed to follow the creep-time function. The value of η lies between 0.5 and 1.0 and is given in Fig. 1. This figure gives not only the variation of η with the age at loading K_o and with the normal creep coefficient ϕ_N (which is the ultimate creep coefficient for K_o = 28 days) but also for a modified creep coefficient $\alpha\phi_N$. The factor is introduced to account for the influence of reinforcement on creep and is in fact a stiffness coefficient $\alpha = \delta_c/(\delta_c + \delta_s)$. For an eccentrically reinforced uncracked concrete member with one layer of steel at distance y_1 from the centroid of the concrete section, the deformation of the steel due to a unit force is $\delta_s = 1/A_sE_s$ and the deformation of the concrete due to a unit force applied at the level of the reinforcement is $\delta_c = (1 + y_1^2/r^2)/(A_cE_o)$. Thus,

$$\alpha = \frac{pn_o\,(1 + y_1^2/r^2)}{1 + pn_o\,(1 + y_1^2/r^2)} \tag{2}$$

where p is the ratio of the steel area A_s to the net concrete area A_c; n_o is the modular ratio (E_s/E_o), and r the radius of gyration of the net concrete section. For most practical cases the minimum value of η in Fig. 1 can be used because α is small. Using equilibrium and compatibility conditions and Eq. (1) to solve the problem of time-dependent change in stress in an uncracked reinforced or prestressed concrete member with top and bottom reinforcement (at y_1 and y_2 respectively from the centroid of the net concrete section) subjected to the forces of Fig. 2, we find the change in steel stress (2) in fibres 1 and 2:

$$\bar{f}_{s1}(T) = \frac{(1 + b_{22} - b_{21})\,\varepsilon_{sh}(T)\,E_s + \left[(1 + b_{22})\,f_1 - b_{21}\,f_2\right]\,n_o\phi(T,K_o)}{(1 + b_{11})\,(1 + b_{22}) - b_{12}b_{21}} \tag{3a}$$

$$\bar{f}_{s2}(T) = \frac{(1 + b_{11} - b_{12})\,\varepsilon_{sh}(T)\,E_s + \left[(1 + b_{11})\,f_2 - b_{12}\,f_1\right]\,n_o\phi(T,K_o)}{(1 + b_{11})\,(1 + b_{22}) - b_{12}b_{21}} \tag{3b}$$

where f_1 = initial concrete stress in fibre 1; f_2 = initial concrete stress in fibre 2.

$$b_{11} = p_1n_o\,(1 + y_1^2/r^2)\,(1 + \eta\phi);\quad b_{12} = p_1n_o\,(1 + y_1y_2/r^2)\,(1 + \eta\phi)$$

$$b_{22} = p_2n_o\,(1 + y_2^2/r^2)\,(1 + \eta\phi);\quad b_{21} = p_2n_o\,(1 + y_1y_2/r^2)\,(1 + \eta\phi)$$

$p_1 = A_{s1}/A_c$ where A_{s1} = area of steel in fibre 1

and $p_2 = A_{s2}/A_c$ where A_{s2} = area of steel in fibre 2 (see Fig. 2).

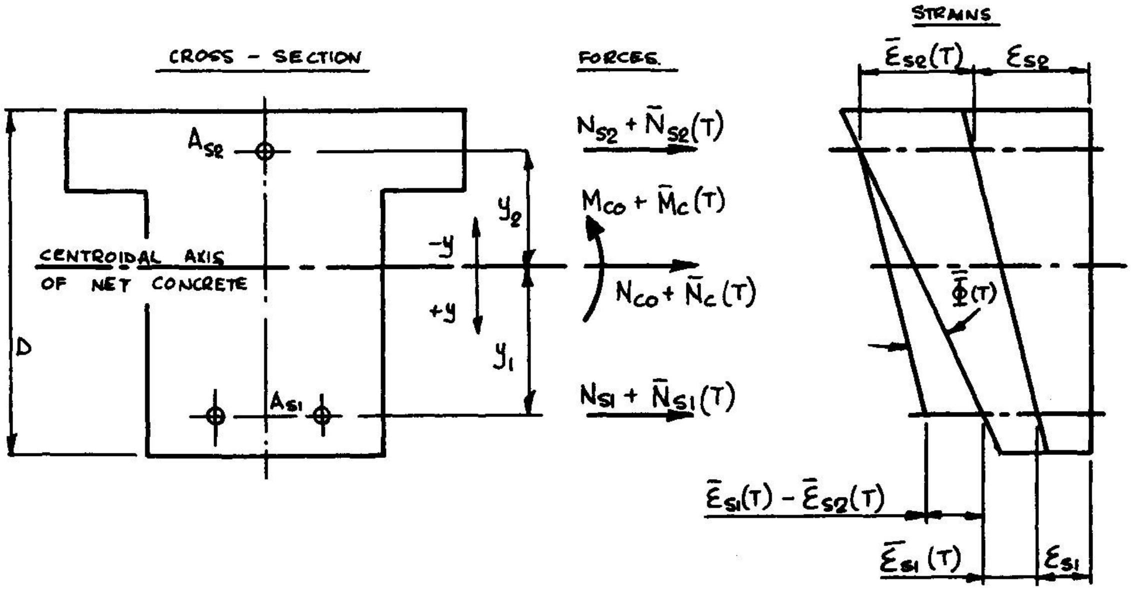

Fig. 2 Forces and strains in a section with two layers of reinforcement (A bar on top of a symbol denotes time-dependent change in force, strain or curvature)

The change in strains in the two fibres can be computed by dividing Eq. 3 by the modulus of elasticity of the steel, E_s. Knowing the time-dependent change in strain in the two fibres, we can compute the change in curvature from

$$\bar{\Phi}(T) = \frac{\bar{f}_{s1}(T) - \bar{f}_{s2}(T)}{E_s\ (y_1 - y_2)} \tag{4}$$

(Note that y is positive below the centroid.)

If the section under consideration is symmetrical and symmetrically reinforced (i.e. $y_1 = -y_2$, $A_{s1} = A_{s2}$), then Eq. 3 can be simplified considerably and we obtain for fibre 1:

$$\bar{f}_{s1}(T) = \frac{f_1^N\ n_o\phi(T,K_o) + \varepsilon_{sh}(T)\ E_s}{1 + pn_o\ (1 + \eta\phi)} + \frac{f_1^M\ n_o\phi(T,K_o)}{1 + pn_o\ (y_1^2/r^2)\ [1 + \eta\phi(T,K_o)]} \tag{5}$$

where f_1^N and f_1^M are respectively the normal and bending stress in fibre 1. Introducing the creep reduction coefficients

$$a_1 = \frac{1}{1 + pn_o\ [1 + \eta\phi(T,K_o)]} \quad \text{and} \quad a_3 = \frac{1}{1 + pn_o\ [1 + \eta\phi(T,K_o)]\ y_1^2/r^2} \tag{6}$$

where $p = (A_{s1} + A_{s2})/A_c$, we can write Eq. 5 in the form:

$$\bar{f}_{s1}(T) = (a_1\ f_1^N + a_3 f_1^M)\ n_o\phi(T,K_o) + a_1\varepsilon_{sh}(T)\ E_s \tag{7a}$$

Similarly, for fibre 2

$$\bar{f}_{s2}(T) = (a_1 f_1^N - a_3 f_1^M)\ \ n_o\phi(T,K_o) + a_1\varepsilon_{sh}(T)\ E_s \tag{7b}$$

The change in curvature can be expressed by

$$\bar{\Phi}(T) = a_3\Phi_o\phi(T,K_o) \tag{8}$$

where Φ_o is the initial curvature. Thus the total curvature (initial plus time-dependent) Φ (T) is

$$\Phi(T) = \Phi_o\ [1 + a_3\phi(T,K_o)] \tag{9}$$

and the total deflection at time T can be written as

$$u(T) = u_o\ [1 + a_3\phi(T,K_o)] \tag{10}$$

where u_o is the initial deflection.

In the case of a symmetrically reinforced member subjected to an axial load, Eq. 7a further simplifies to yield for the time-dependent steel stress

$$\bar{f}_s(T) = a_1\ [n_o f\phi(T,K_o) + \varepsilon_{sh}(T)E_s] = \frac{n_o f\phi(T,K_o) + \varepsilon_{sh}(T)E_s}{1 + pn_o\ [1 + \eta\phi(T,K_o)]} \tag{11}$$

(The subscript of stress can be omitted.)

If there is only one eccentric layer of reinforcement (or tendon) Eq. 3 reduces to

$$\bar{f}_s(T) = \frac{n_o f_1\phi(T,K_o) + \varepsilon_{sh}(T)\ E_s}{1 + pn_o(1 + y_1^2/r^2)\ [1 + \eta\phi(T,K_o)]} \tag{12}$$

Introducing the creep reduction coefficient

$$a_2 = \frac{1}{1 + pn_o(1 + y_1^2/r^2)\ [1 + \eta\phi(T,K_o)]} \tag{13}$$

we can write Eq. 12 as

$$\bar{f}_s(T) = a_2[n_o f_1\phi(T,K_o) + \varepsilon_{sh}\ (T)\ E_s] \tag{14}$$

For design purposes, the creep coefficients are available in chart form (3) for various values of the parameters pn_o, y_1/r and $\eta\phi(T,K_o)$. Since the creep reduction coefficients indicate the effect of reinforcement on creep, by using a reduced creep coefficient $a\phi(T,K_o)$, reinforced concrete can be treated in the same way as plain concrete.

BIAXIALLY LOADED COLUMNS

For symmetrically reinforced, biaxially loaded columns, Eq. 7 can be suitably expanded to

$$\bar{f}_s(T) = (a_1 n_o f^N \pm a_3^y n_o f^{M_x} \pm a_3^x n_o f^{M_y})\ \phi(T,K_o) + a_1 \varepsilon_{sh}(T)\ E_s \qquad (15)$$

where $f^N = \frac{N_o}{A'_c}$ = normal stress,

$f^{M_x} = \frac{M_{ox}}{I'_{cx}} y_{s1}$ = stress in concrete due to moment M_{ox} in a fibre distant y_{s1} from the centroid of the concrete section,

$f^{M_y} = \frac{M_{oy}}{I'_{cy}} x_{s1}$ = stress in concrete due to moment M_{oy} in a fibre distant x_{s1} from the centroid of the concrete section,

A'_c = cross-sectional area of the transformed section,

I'_c = second moment of area of the transformed section, the second subscript denoting the axis about which the moment is taken,

x_{s1} and y_{s1} are distances from centroidal axis to the outer layer of reinforcement,

and a_1, a_3^x, a_3^y are creep reduction coefficients, given by Eq. 6. Since the last two coefficients involve y_1 = distance from the centroidal axis to the centroid of steel area on each side, we require this distance in the x and y directions for a_3^x and a_3^y respectively.

PRESTRESS LOSSES

As mentioned before, all the equations apply equally to reinforced and to prestressed concrete. If Eq. 3 is used to determine the loss of prestress in a member with top and bottom layers of tendons and with additional non-prestressed reinforcement, the terms p_1 and p_2 have to include all the reinforcement, and y_1 and y_2 are the distances of the centroids of the bottom and top steel (prestressed and non-prestressed taken together) respectively.

If we have one eccentric layer of prestressed steel only, we find the prestress loss (including the effect of steel relaxation) from

$$\bar{f}_s(T) = a_2\ [n_o f_o \phi(T,K_o) + \varepsilon_{sh}(T)\ E_s + f_r(T)] \qquad (16)$$

where $f_r(T)$ is the intrinsic relaxation loss of steel kept under a constant strain for $(T-K_o)$ days, and f_o is the stress in concrete at the level of the tendon due to dead load and to prestress.

If only one layer of tendon is used in combination with non-prestressed reinforcement uniformly distributed across the section, then $a_1 \approx a_3$. Eq. 12 then takes the form (steel relaxation included)

$$\bar{f}_s(T) = \frac{a_1\ [(n_o f_o \phi(T,K_o) + \varepsilon_{sh}(T)\ E_s] + f_r(T)}{1 + pn_o(1 + y_1^2/r^2)\ [1 + a_1 n\phi(T,K_o)]} \qquad (17)$$

If the non-prestressed steel is symmetrically disposed in two layers $a_1 \neq a_3$, and we can find the prestress loss from

$$\bar{f}_s(T) = \frac{(a_1 f_o^N + a_3 f_o^M)\, n_o \phi(T,K_o) + a_1 \varepsilon_{sh}(T)\, E_s + f_r(T)}{1 + p n_o (1 + y_1^2/r^2)\left[1 + \frac{a_1 + a_3}{2}\, \eta\phi(T,K_o)\right]} \qquad (18)$$

In Eq. 17 and 18 the creep reduction coefficients are determined for the non-prestressed reinforcement only and the term p is the ratio of the prestressing steel area to the concrete area.

VERIFICATION AND APPLICATION OF THE METHOD

Graf's tests (4) on columns and tests on prestressed concrete members by Ban et al. (5) are well suited to verify the approach presented. However, only two of Graf's columns (No. 587 and 591) can be compared with the theory, as the others were stressed to 0.60 f_c' at initial loading so that creep cannot be considered to be proportional to stress. The following data are available.

TABLE 1

Column N°	587	591
Steel area, A_s(cm^2)	24.3	24.3
Net concrete area, A_c(cm^2)	875.7	875.7
$p = A_s/A_c$	0.028	0.028
Age at loading, K_o(days)	13	13
Time under load ($T-K_o$) (days)	1102	1080
Modulus of elasticity of concrete at time of loading, E_o(kg/cm^2)	191,000	149,000
Modulus of elasticity of steel, E_s(kg/cm^2)	2.1×10^6	2.1×10^6
Modular ratio, n_o	11	14
Applied load, P_o(kg)	72,000	70,000
Shrinkage, $\varepsilon_{sh}(T)$	-450×10^{-6}	-460×10^{-6}
Observed change in steel stress, $\bar{f}_s(T)$(kg/cm^2)	1512	1407
Creep coefficient, $\phi(T,K_o)$	3.20	2.89

With the initial concrete stress computed from the relation $f_o = P_o/A_c(1 + pn_o)$, and the relaxation coefficient $\eta = 0.76$ (determined for $\alpha\phi_N = 0.62$ and $K_o = 13$ days), we find, using Eq. 11, the change in steel stress, in column 587:

$$\bar{f}_s(T) = \frac{11\ (-62.9)\ 3.20 + (-450) \times 10^{-6} \times 2.1 \times 10^6}{1 + 0.028 \times 11.0\ (1 + 0.76 \times 3.20)} = -1530 \text{ kg/cm}^2$$

Using the same procedure, the increase in compressive stress in column 591 is found to be $\bar{f}_s(T) = -1455$ kg/cm^2. Both values agree very well with those observed in the tests (see Table 1).

The tests of Ban et al. (5) will be used to demonstrate the accuracy of the equations when applied to determine the loss of prestress. The tests include members with symmetrical and unsymmetrical non-prestressed steel, and with the prestressing force applied axially (Series A) and eccentrically (Series B). Since no creep tests were performed, we shall use the tests without non-prestressed reinforcement to determine the creep coefficient, solving Eq. 12 for $\phi(T,K_o)$. From the test data in Table 2, the creep coefficient for series A is found to be $\phi = 2.60$ (Test A5). For series B, both test B5 and test B6 yield $\phi = 2.70$ for the period under load (350 days).

TABLE 2

Beam property	Series and beam No.	Eccentricity of tendon	Area ratio of non-prestressed steel		Initial prestress	Measured loss	Calc. loss
		in.	top	bottom	lb	lb	lb
	A-1	-0.04	-	0.31	26,060	6590	6940
$\varepsilon_{sh} = -470 \times 10^{-6}$	A-2	-0.08	-	0.31	26,830	7210	7140
$\phi = 2.60$	A-3	-0.08	0.16	0.16	27,050	6770	6770
$n_o = 6.48$	A-4	0	0.16	0.16	28,330	7140	7100
	A-5*	0.08	-	-	25,130	7960	-
	B-1	0.71	-	0.31	27,290	8070	7330
$\varepsilon_{sh} = -520 \times 10^{-6}$	B-2	1.01	-	0.31	28,920	8460	7750
$\phi = 2.70$	B-3	1.30	0.16	0.16	27,970	8420	8340
$n_o = 6.95$	B-4	1.11	0.16	0.16	29,230	8600	8380
	B-5*	1.02	-	-	26,900	9460	-
	B-6*	0.85	-	-	27,560	9520	-

Tendon area, $A_s = 0.369$ in^2**
Cross-section 4 in. × 8 in. (duct area 0.45 in^2)
Age at loading, $K_o = 28$ days
Time under load: $T-K_o = 350$ days
Modulus of elasticity: prestressing steel $E_s = 27.5 \times 10^6$ psi
non-prestressed steel $E_s' = 29.9 \times 10^6$ psi

Consider Test A1: Eq. 3 is used to compute the loss of prestress. With $p_1 = 0.369/(32.00 - 0.45 - 0.31) = 0.0117$, $p_2 = A'_sE_s'/(A_sE_s) = 0.31 \times 29.9 \times 10^6/(27.5 \times 10^6 \times 31.24) = 0.0108$, $r^2 = 5.33$ in^2, $\eta = \eta_{min} = 0.75$, $y_1 = 0$, $y_2 = 2.75$ in. we obtain the coefficients:

$$b_{11} = b_{12} = 0.0117 \times 6.48\ (2 + 0.75 \times 2.60) = 0.224$$

$$b_{22} = 0.0108 \times 6.48\ (1 + 2.75^2/5.33)\ (1 + 0.75 \times 2.60) = 0.499$$

$$b_{21} = 0.0108 \times 6.48\ (1 + 0.75 \times 2.60) = 0.206$$

The concrete stresses at age $K_o = 28$ days are: $f_1 = -790$ psi, and $f_2 = -860$ psi. Thus,

$$\bar{f}_{s1}(T) = \frac{(1+0.499-0.207)(-470\times10^{-6})27.5\times10^6 + [(1+0.499)(-790) - 0.207\times(-860)]6.48\times2.60}{(1 + 0.224)\ (1 + 0.499) - 0.224 \times 0.207}$$

$= 18,800$ psi

This stress corresponds to a loss in prestress of 6940 lb.

By the same procedure, we obtain $\bar{f}_s(T) = -19350$ psi for Test A2, which corresponds to a prestress loss of 7140 lb.

Consider Test A3: To compute the loss we can either use Eq. 11 or Eq. 17. Using the first of these, we find, with $p = (A_s + A_s'E_s'/E_s)/A_c = (0.369 + 2 \times 0.16 \times 29.9 \times 10^6/27.5 \times 10^6)/31.24 = 0.0231$ and the concrete stress $f = 810$ psi at age $K_o = 28$ days,

$$\bar{f}_{s1}(T) = \frac{6.48(-810) \times 2.60 - 470 \times 10^{-6} \times 27.5 \times 10^6}{1 + 0.0231 \times 6.48\ (1 + 0.75 \times 2.60)} = -18,350 \text{ psi}$$

Notes: * Test used to determine ϕ
** Since the steel is stressed to only 0.5 f_s', there is no steel relaxation loss.

This corresponds to a prestress loss of 6,790 lb.
Using Eq. 17, we find, with $y_1 = 0$ and $f_r(T) = 0$, $a_1 = 1/[1 + 0.0117 \times 6.48 (1 + 0.75 \times 2.60)] = 0.82$,

$$\bar{f}_{s1}(T) = \frac{[6.48(-810) \times 2.60 - 470 \times 10^{-6} \times 2.75 \times 10^{6}]\ 0.82}{1 + 0.0117 \times 6.48\ (1 + 0.82 \times 0.75 \times 2.6)} = -18{,}350\text{psi}$$

The calculated prestress losses of Series B do not agree as well with the measured ones as for Series A. However, the agreement is still good.

EXAMPLE ON A BIAXIALLY LOADED COLUMN

The column shown in Fig. 3 is reinforced by 14 bars, 7/8 in. diameter, so that $A_s = 8.40$ in^2 and $p = 0.0365$. We have $f'_c = 4{,}000$ psi, $n_o = 8.0$, $\epsilon_{sh} = -300 \times 10^{-6}$, $\phi_\infty = 2.5$, $K_o = 60$ days. From Fig. 1 $\eta = \eta_{min} = 0.80$. The section properties are $A_c = 230$ in^2, $A'_c = 297$ in^2, $I_{cx} = 5538$ in^4, $I'_{cx} = 7090$ in^4, $I_{cy} = 3760$ in^4, $I'_{cy} = 4775$ in^4. The forces applied are $N_o = 180{,}000$ lb, $M_{ox} = 250{,}000$ lb in, $M_{oy} = 210{,}000$ lb in.

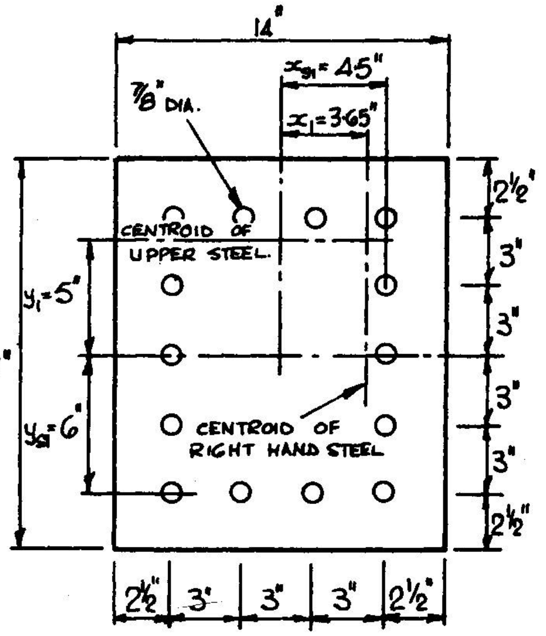

Fig. 3 Cross-section of column

Hence, the stresses in concrete are

$$f^N = -\frac{180{,}000}{297} = -606 \text{ psi},$$

$$f^{M_x} = \pm\frac{250{,}000}{7090} \times 6 = \pm 211 \text{ psi},$$

$$f^{M_y} = \pm\frac{210{,}000}{4775} \times 4.5 = \pm 198 \text{ psi}$$

The creep reduction coefficient a_1 is found from Eq. 6 to be $a_1 = 0.54$. From the same equation, we find the values of a_3. For a_3^x, $x_1 = 3.65$ in. (to replace y_1 in Eq. 6), $r^2 = I_{cy}/A_c = 3760/230 = 16.4$ in^2, so that

$$a_3^x = \frac{1}{1 + 0.29(1 + 2.0)(3.65)^2/16.4} = 0.59$$

Now, $y_1 = 5$ in., $r^2 = I_{cx}/A_c = 5538/230 = 24.1$ in^2, whence

$$a_3^y = \frac{1}{1 + 0.29(1 + 2.0)(5)^2/24.1} = 0.53$$

From Eq. 15, the ultimate change in steel stress in the corner of the column subjected to the highest compression is

$$\bar{f}_{s\infty} = [0.54 \times 8 \times (-606) + 0.53 \times 8 \times (-211) + 0.59 \times 8 \times (-198)]\ 2.5 + 0.54 \times 29 \times 10^{6} \times (-300) \times 10^{-6} = -15{,}820 \text{ psi}$$

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SUMMARY

The paper shows how data on creep and shrinkage of plain concrete can be used to calculate time-dependent deformations and stresses in beams and columns of reinforced and prestressed concrete (with or without non-prestressed steel). Comparison of calculated values with experimental results of other investigations shows very good agreement.

RESUME

Ce document montre comment les données sur le fluage et le retrait du béton peuvent être utilisées pour calculer les déformations dépendant du temps et les tensions dans les poutres et les piliers en béton précontraint ou armé (avec ou sans acier précontraint). Des comparaisons effectuées entre les calculs et les résultats expérimentaux d'autres recherches, montrent de très bonnes concordances.

ZUSAMMENFASSUNG

Dieser Beitrag zeigt, wie das Datenmaterial über Kriechen und Schwinden von Beton gebraucht werden kann, um zeitabhängige Verformungen und Spannungen in Balken und Stützen aus Stahl- und Spannbeton (mit oder ohne Zusatzbewehrung) zu berechnen. Der Vergleich der berechneten Werte mit den experimentellen Ergebnissen anderer Untersuchungen ergibt gute Uebereinstimmung derselben.