Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 5 (1970)

Artikel: Effect of creep on the flexual strength and deformation of structural concrete

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DOI: https://doi.org/10.5169/seals-6925

Effect of Creep on the Flexural Strength and Deformation of Structural Concrete

Influence du fluage sur la résistance et déformation en flexion des ossatures en béton armé

Der Einfluss des Kriechens auf die Biegefestigkeit und die Verformung von Stahlbetontragwerken

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1. INTRODUCTION

A considerable amount of basic research on the creep laws and factors affecting it has been conducted to-date (1). Comparatively less work has been devoted to the effect of long-term loading on the strength and deformations of beams, columns and frames (2), (3), (4), (5).

Recently various aspects of the non-linear response of structural concrete, including creep, have been investigated both theoretically and experimentally at the University of Waterloo (6), (7), (8), (9), (10). The paper develops an approach to the non-linear analysis of structural concrete (6), (7), with particular reference to the strength and deformations of reinforced concrete beams and long columns under long-term loading. The study is based on a time-dependent stress-strain relation for concrete, in which the creep, lateral reinforcement and strain-gradient are some of the major factors considered. A computer program is used in order to develop a general time-dependent moment-curvature relation, which subsequently is used for the prediction of the long-term load, moments and deflections of beams and slender columns.

2. STRESS-STRAIN RELATIONS FOR CONCRETE AND STEEL

2.1 Time-Dependent Stress-Strain Relation for Concrete in Compression

The main factors affecting the concrete behavior are the concrete strength, lateral reinforcement, creep, strain gradient, size of specimens and type of loading. A stress-strain relation for concrete in compression, proposed by Sargin (6), takes all these factors into account by a proper choice of five governing parameters: the concrete cylinder strength f_c', the initial Young modulus, E_c, the ratio of maximum stress to cylinder strength, k_3, the strain corresponding to maximum stress, ε_o, and a parameter, D, that affects the descending branch of the stress-strain curve. By denoting $A = E_c\, \varepsilon_o / k_3 f_c'$ and $x = \varepsilon / \varepsilon_o$ Sargin's relation is expressed as:

$$\sigma_c = k_3 f_c' \frac{A x + (D - 1) x^2}{1 + (A - 2) x + D x^2} \tag{1}$$

In creep analysis the major factors are the rate of loading, loading duration and age of concrete at the time of loading. Because the control of the rate of loading is very difficult in actual structures, a conventional loading rate has to be adopted. A practical proposal, due to Rüsch (2), is to assume the load is applied in about 20 minutes at constant rates and sustained subsequently up to failure. The loading duration and the concrete age at the time of loading are reflected through parameters E_c, k_3 and ε_o, whose expressions are detailed in (6) and are not reproduced here, for the sake of brevity.

Typical stress-strain relations of concrete cylinders loaded in compression at 28 days are shown in Fig. 1 for three loading durations. As Rüsch has shown (2), the changes in concrete under long-term loads consist of the continued cement hydration and the effects of sustained loads. While an advanced hydration results in a strength increase, sustained load effects result in a reduction of strength and an increase of deformation. Because the influence of f_c' on ε_o is negligible, the values of ε_o in Fig. 1 increase in time, due to the sustained load effects. However, the values of k_3 decrease initially due to these effects, but increase subsequently, when the effect of the continued cement hydration is prevailing.

2.2 Time-Dependent Stress-Strain Relation for Concrete in Tension

The behavior of concrete in tension is represented by the following elastic-brittle relations:

$$\begin{aligned} \sigma_t &= E_c\, \varepsilon \quad (\text{for } \varepsilon < \varepsilon_t) \\ \sigma_t &= 0 \quad (\text{for } \varepsilon > \varepsilon_t) \end{aligned} \tag{2}$$

where $\varepsilon_t = \sigma_{tr} / E_c$ is the cracking strain and σ_{tr} is the concrete modulus of rupture. The stress-strain relation (2) is time-dependent because E_c and σ_{tr} are defined in terms of the duration of loading and concrete age at the time of loading.

2.3 Stress-Strain Relations for Reinforcing Steels

The stress-strain relations of most reinforcing steels consist of three branches corresponding to the elastic, yield and strain-hardening ranges, respectively. The following idealized relations, considered applicable to most American steel grades with a yield limit not in excess of 75 ksi, are adopted in this study:

$$
\begin{aligned}
\sigma_s &= E_s\,\varepsilon && (\text{for } 0 \leq \varepsilon \leq \varepsilon_y)\\
\sigma_s &= f_y && (\text{for } \varepsilon_y \leq \varepsilon \leq \varepsilon_h)\\
\sigma_s &= f_y + E_h(\varepsilon - \varepsilon_h)[1 - E_h(\varepsilon - \varepsilon_h)/4(\sigma_{su} - f_y)] && (\text{for } \varepsilon > \varepsilon_h)
\end{aligned}
\qquad (3)
$$

where E_s is the Young modulus for steel, f_y is the yield limit, ε_h is the strain at the onset of hardening, E_h is the strain-hardening modulus and σ_{su} is the ultimate stress. Typical stress-strain curves for steel, Eqs. (3), are illustrated in Fig. 2.

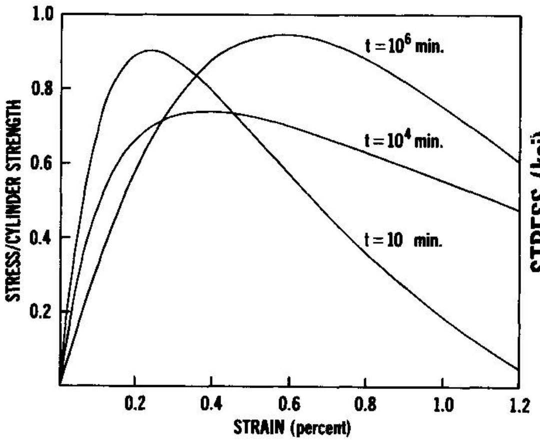


Fig. 1

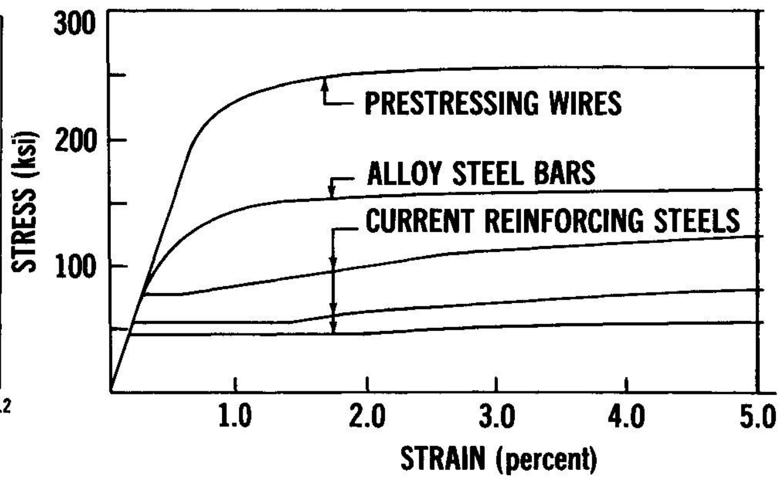


Fig. 2

For high strength steels (alloy steel bars, pre-stressing wires) other equations consisting of an initial linear range and a subsequent non-linear range, can be introduced, without any change in the analysis.

3. NON-LINEAR ANALYSIS OF REINFORCED CONCRETE SECTIONS AND MEMBERS

3.1 Section Analysis

With the notations and assumptions in Fig. 3 the force and moment equations of equilibrium for a reinforced concrete section symmetrical about the vertical axis, can be written as follows:

$$
\int_0^{kd} \sigma(\varepsilon)b(y)dy + A_s'\,\sigma_s' - \int_0^{kd} \sigma_t(\varepsilon)b(y)dy - A_s\sigma_s = P \qquad (4)
$$

$$
\int_0^{kd} \sigma(\varepsilon)b(y)(d-kd+y)dy + A_s'\,\sigma_s'\,(d-d') - \int_0^{kd} \sigma_t(\varepsilon)b(y)(d-kd-y)dy = Pe + M \qquad (5)
$$

where σ_s is the stress in the compression steel.

The assumption of linear strain distribution implies:

$$\frac{\varepsilon_c}{kd} = \frac{\varepsilon_s'}{kd-d'} = \frac{\varepsilon_s}{d-kd} = \frac{\varepsilon}{y} = \frac{\varepsilon_t}{y_t} \qquad (6)$$

Eqs. (1), (2), (3) are used to eliminate σ_c, σ_t, σ_s and σ_s', respectively, and Eqs. (6) to eliminate y and y_t from Eqs. (4) and (5). A numerical method is developed to solve Eqs. (4) and (5) simultaneously in the following steps (Fig. 4a.):

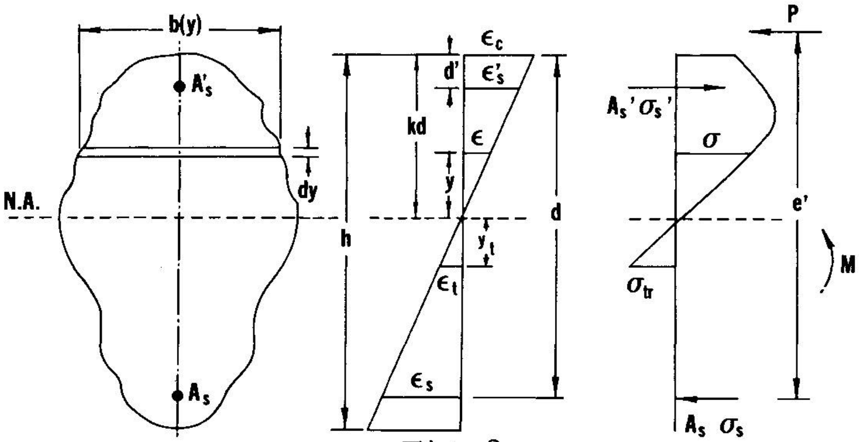


Fig. 3

(a) Starting from zero increase ε_c at some chosen interval.
(b) For any given value of ε_c find a value of k by successive approximations such that Eq. (4) be satisfied within a specified tolerance.
(c) Solve Eq. (5) for M with the known values of ε_c, k and the given P.
(d) Calculate all other behavior parameters ϕ , EI, etc.
(e) Continue to increase ε_c up to the value ε_u at which the moment reaches a maximum. This is the ultimate of the section.

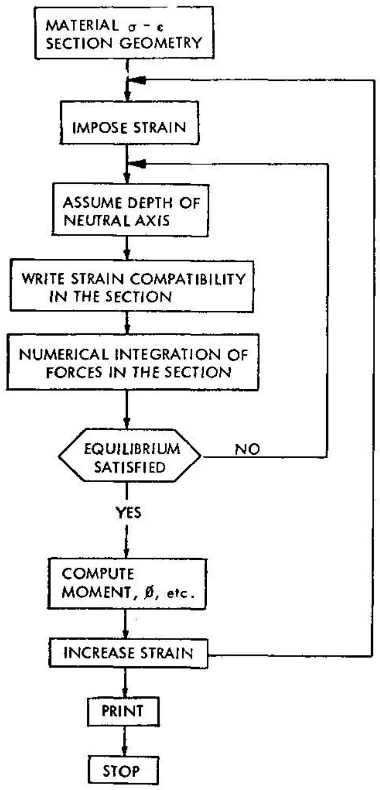


Fig. 4(a)

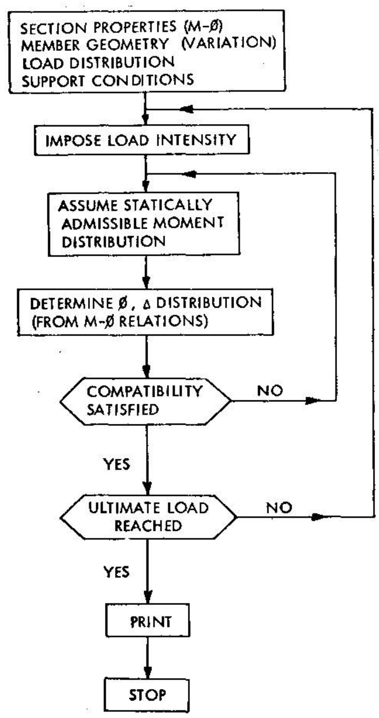


Fig. 4(b)

3.2 Member Analysis

The non-linear analysis of reinforced concrete members can be performed by following the steps outlined in the diagram, Fig. 4b. Loads of some known intensity are first imposed on the member and a statically admissible moment field is assumed. The associated curvature field is computed by using the moment-curvature relations derived as indicated in Section 3.1. Slopes and deflections along the member are calculated from the known curvature field and the boundary conditions are checked. A statically admissible moment field that satisfies all boundary conditions is found by trial and error. The intensity of imposed loads is increased at desired intervals and the above process is repeated until the ultimate moment capacity is reached in at least one section along the member.

4. TIME-DEPENDENT MOMENT-CURVATURE RELATIONS FOR REINFORCED CONCRETE SECTIONS

The moment-curvature relation of a rectangular singly reinforced concrete section in pure bending has been derived for six different loading durations by using the approach of section 3.1. The results, plotted in Fig. 5, show five sets of curves, each corresponding to a particular steel percentage. It is noted that larger durations of loading have a more favorable effect on the ductility than on the strength of reinforced concrete sections.

The maximum moment, M_u, and the associated curvature, ϕ_u, are plotted against the time in Figs. 6 and 7, respectively, where each line corresponds to a set of curves in Fig. 5. These diagrams show that time has but a minor influence on the ultimate moments and curvatures. Fig. 6 illustrates the initial stress drop due to sustained load effects and the subsequent stress increase that occurs when the continued hydration of the cement paste becomes predominant. However, from Fig. 7, it is seen that the peak curvatures tend to increase in time because of the sustained load effects.

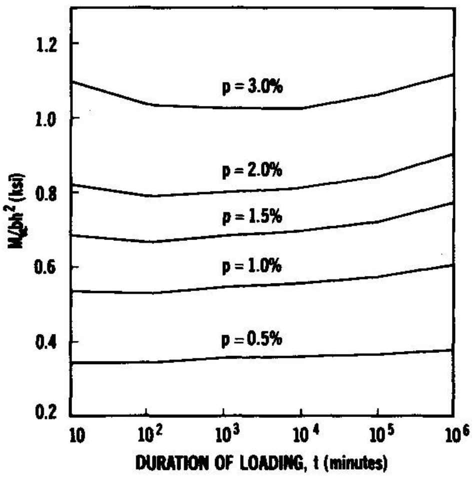


Fig. 6

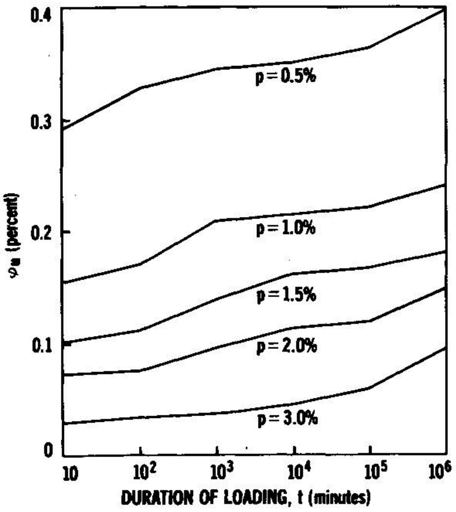
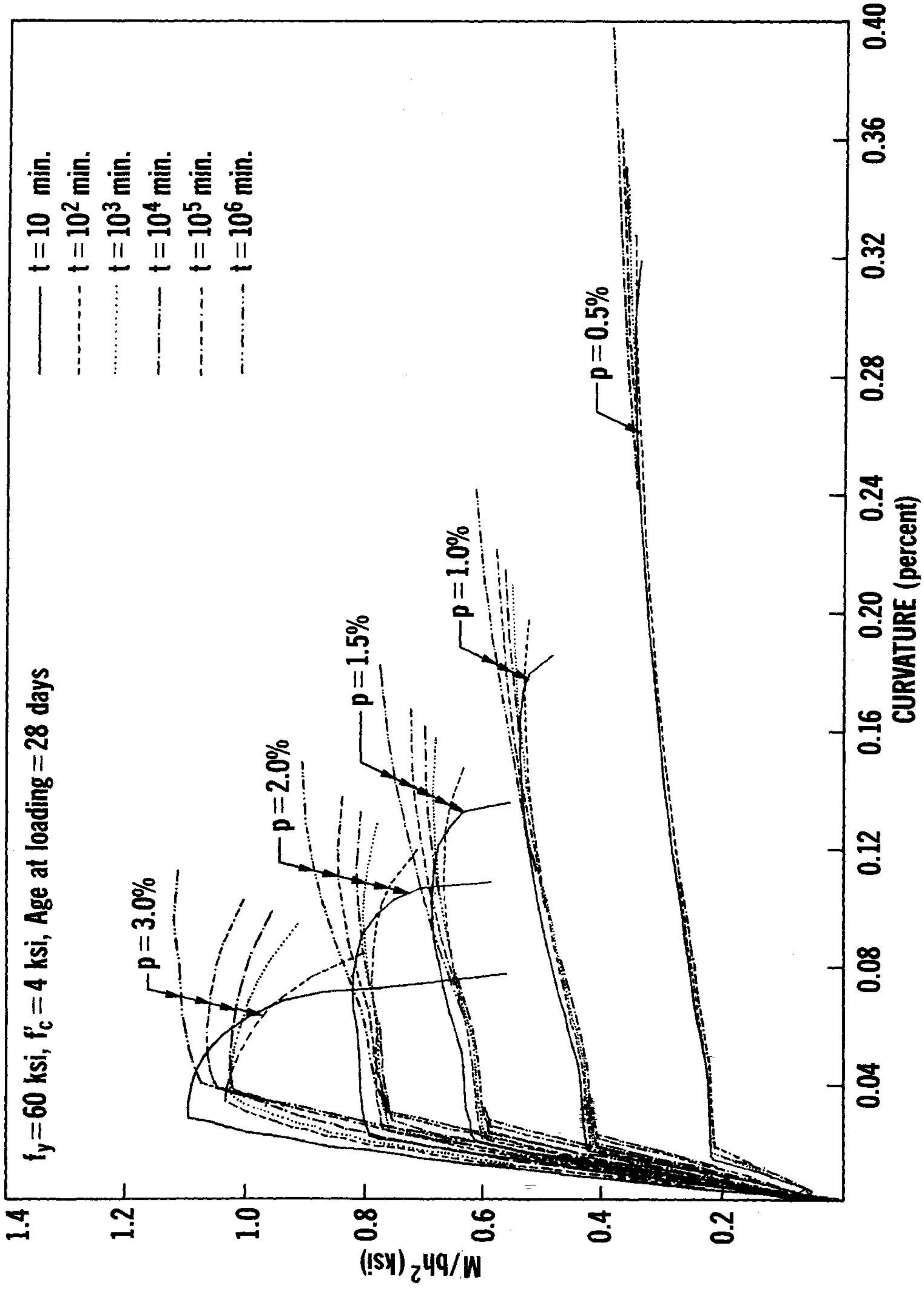


Fig. 7



f_y = 60 ksi, f'_c = 4 ksi, Age at loading = 28 days
t = 10 min.
t = 10^2 min.
t = 10^3 min.
t = 10^4 min.
t = 10^5 min.
t = 10^6 min.
p = 3.0%
p = 2.0%
p = 1.5%
p = 1.0%
p = 0.5%
M/bh^2 (ksi)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
0.36
0.40
CURVATURE (percent)

Fig. 5

5. TIME-DEPENDENT BEHAVIOR OF A REINFORCED CONCRETE CONTINUOUS BEAM

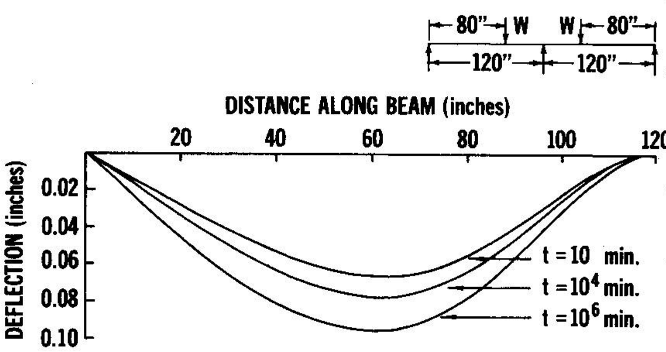


Fig. 8

The analysis described in Section 3 was applied to predict the long-term behavior of a two-span continuous beam previously studied both theoretically (7) and experimentally (9) for short-time loading. The section sizes, steel reinforcing and other details are the same as in Ref. (9). The deflected shape of the beam under service loads is shown in Fig. 8 for loading durations of 10 minutes, approximately one week and two years.

Assuming the service load to correspond to about one third of the theoretical ultimate load, it is seen from Fig. 8 that the maximum deflection after two-years of loading is about 50% higher than the short-time maximum deflection.

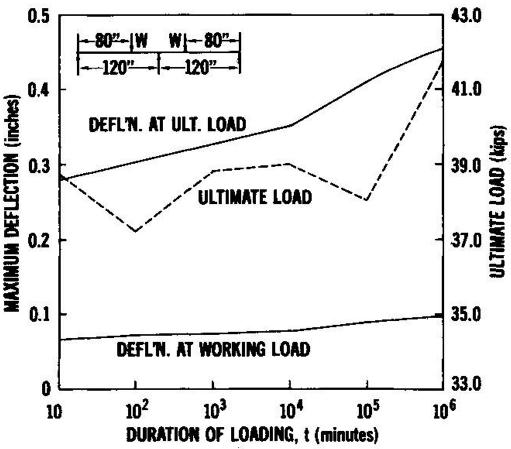


Fig. 9

The maximum deflections at service and ultimate loads as well as the ultimate load of the beam investigated are plotted against the load duration in Fig. 9. It is seen that whereas the ultimate load is negligibly affected by creep, the deflections at both service and ultimate loads increase considerably in time.

6. TIME-DEPENDENT BEHAVIOR OF A LONG REINFORCED CONCRETE COLUMN

Fig. 10 shows the deflected shapes of a long reinforced concrete column corresponding to loading durations of 40 minutes, 5 days and 70 days. The applied concentric load is 15% of the ultimate load computed by the ACI 318-63 code formula. The geometry, concrete sizes, steel reinforcement and detailing are the same as those used in Hellesland's tests (10), the results of which are represented by dots in Fig. 10. It is seen that the agreement between theoretical and experimental data is very good.

A similar correlation is noted in the comparison of analytical and test data of mid-height deflections, illustrated in Fig. 11. The maximum deviation between predicted and measured deflections does not exceed 10%.

7. CONCLUSIONS

A general, non-linear analysis program, based on a realistic time-dependent stress-strain relation for concrete, was used to predict the behavior of reinforced concrete structures under long-term loading.

The study confirms that creep has little effect on the strength of flexural members and illustrates its major influence on the deflections and strength of slender columns.

Consideration of adequate bond characteristics, and extensions to structures with higher degrees of static indeterminacy, appear to be significant potential developments of the analytical approach presented in the paper.

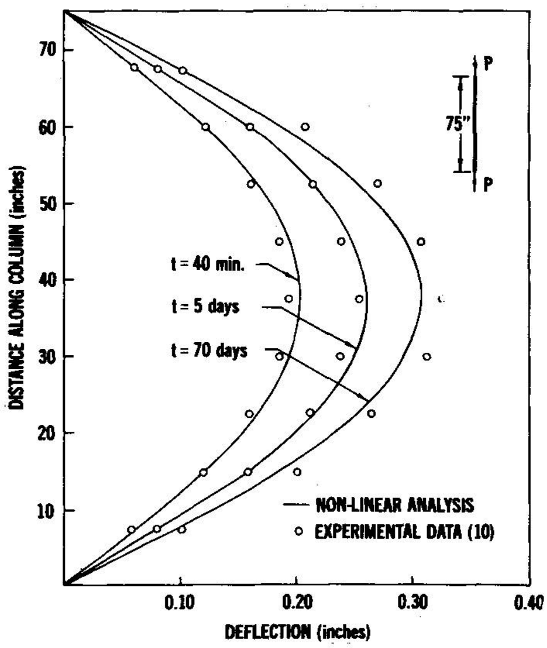


Fig. 10

ACKNOWLEDGMENTS

The authors are indebted to Dr. M. Sargin, formerly a Post-Doctoral Fellow at the University of Waterloo, for his contribution in the early stages of this work. Also, thanks are offered to Mr. S. Srivastava and Dr. J. Hellesland, of the University of Waterloo, for providing some data of their M.A. Sc. and Ph.D. dissertations, which were used in Figs. 8-9 and 10-11, respectively.

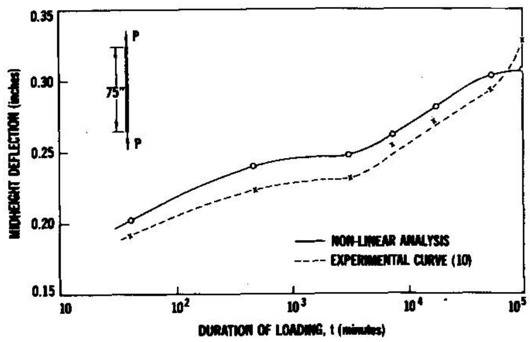


Fig. 11

The work reported in the paper is part of a comprehensive investigation on the "Inelastic Behavior of Reinforced Concrete Structures", in progress at the University of Waterloo with the financial support of the National Research Council of Canada, under Grant A-4789.

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SUMMARY

A non-linear analysis of structural concrete is developed on the basis of a time-dependent stress-strain relation of concrete, in which creep is a major factor, along with the lateral reinforcement, strain-gradient, etc.

A computer program enables the flexural strength and deformation of reinforced concrete sections and moment-curvature relationships under long-term loading to be predicted.

The analytical procedure is applied to the investigation of the creep effects on reinforced concrete beams and columns and corresponding results are compared with some recent experimental data.

RESUME

En partant d'une relation contrainte-déformation dont les principaux facteurs sont: fluage, armature transversale, gradient de déformation etc., on présente une méthode d'étude du comportement non linéaire des ossatures en béton armé.

La résistance et la déformation à la flexion des sections en béton armé ainsi que les relations moment-courbure sous charge de longue durée sont obtenues à l'aide d'un programme d'ordinateur.

La méthode est appliquée à l'analyse du fluage sur les poutres et poteaux en béton armé et les résultats sont comparés avec quelques données expérimentales récentes.

ZUSAMMENFASSUNG

Ein nichtlineares Berechnungsverfahren für Stahlbetonbauwerke ist entwickelt worden aufgrund einer zeitabhängigen Spannung-Dehnungs-Beziehung des Betons, in welcher das Kriechen die Hauptvariabel neben der Schubbewehrung, dem Dehnungsgradienten usw. darstellt.

Zur Ermittlung der Biegefestigkeit und -verformung der Stahlbetonquerschnitte sowie der Moment-Krümmungs-Beziehungen unter Langzeitbelastung ist ein elektronisches Rechenprogramm ausgearbeitet worden.

Das Berechnungsverfahren wird für die Untersuchung der Kriechwirkung von Stahlbetonbalken und -stützen angewendet, und die Ergebnisse sind mit vorliegenden Versuchsdaten verglichen worden.