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Creep Effects in Continuous Reinforced Concrete Beams

Effets du fluage dans les poutres continues en béton armé

Kriechen in durchlaufenden Stahlbetonbalken

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Introduction

It has long been recognized (Ref. 1) that creep effects are important in an evaluation of the stiffness of concrete structures. Until the recent interest in slender and continuous concrete structures however an accurate treatment of a concrete structure's stiffness has not been important since designs were usually stress limited. An "Equivalent Modulus" method (Ref. 2) has been the accepted method of checking deflections for these designs. This method is based upon an elastic analysis employing a modulus variable in time to account for the reduced stiffness of a member as creep occurs.

The writer and Guralnick (Ref. 3 and 4) have shown that this method is only adequate in predicting peak deflections of statically determinate reinforced concrete beams subjected to a sustained loading. Errors were found to result in the application of this method to the prediction of deformation histories or the behavior of a beam subjected to cyclic loading. A numerical method was presented in Ref. 3 which was based upon a viscoelastic solution; results obtained by this method were found to be in agreement with experimental data for statically determinate beams subjected to cyclic loading. The objective of this paper is to present results based upon this procedure and to apply these results to hyperstatic structures.

The engineering significance of the effect of creep on indeterminate reinforced concrete beams was clearly established in an experimental study conducted by Washa and Fluck (Ref. 5). During this study two span continuous reinforced concrete beams were subjected to a sustained uniform loading for a period of two and one half years. Over this period 5 percent increases in the center reaction were observed. This change in reaction corresponds to a 15 percent decrease in the maximum positive moment for the beam and a 25 percent increase in the maximum negative loading moment at the center support. Bending moment changes of this order of magnitude are clearly of concern to the designer.

Analytical Model

The theory developed in Ref. 3 is summarized here for ease of reference. Bernoulli-Euler beam theory is assumed to apply throughout creep and the conventional reinforced concrete beam assumptions relating to zero tensile strength concrete, and perfect bond are made.

The stress-strain behavior of the plain concrete is represented with a viscoelastic model consisting of a linear spring in series with a single Kelvin-Voigt element. For a singly reinforced rectangular cross section the governing equations are then of the form:

$$\frac{1}{\bar{R}} = \bar{\sigma}^* + \frac{\exp(-Kt/\phi)}{\phi}\int_0^t E_c\,\bar{\sigma}^* \exp(K\tau/\phi)\,d\tau$$

$$\frac{k^2\bar{\sigma}^*}{2} + \frac{pn}{\bar{R}}(k-1) = 0 \qquad (1)$$

$$\frac{k^3\bar{\sigma}^*}{3} + \frac{pn}{\bar{R}}(k-1)^2 = \bar{M}$$

and

$$\frac{\partial^2\bar{y}}{\partial\bar{x}^2} = \left(\frac{L}{d}\right)^2\frac{1}{\bar{R}}$$

Closed form solutions to this system of equations are not possible because of the time dependence of the neutral axis location. An iterative numerical solution was outlined in Ref. 3. The computational procedure is carried out for a complete range of parameters of practical interest with a constant load maintained on the beam.

A review of the numerical solutions indicates that the peak values of curvature, stress, and neutral axis locations are functions of the parameter (E_c/K) and (pn) only and are independent of the parameter (K/ϕ).

The ratios of peak value of deflection, concrete stress, and neutral axis location to corresponding initial values are shown in Fig. 1 through 3. For many practical problems, these charts are sufficient in themselves. It is of considerable importance to note from Fig. 3 that the neutral axis undergoes a considerable shift with time. Hence creep theories based upon the assumption that the neutral axis location remains invariant with time may be expected to err considerably when utilized for deflection or stress predictions.

It has been found that neutral axis location, stress and curvature are each different functions of time and that each of these functions of time is influenced by all three of the beam parameters (i.e., K/ϕ, pn, and E_c/K). It is possible, however, to fit the computed data with exponential expressions. To this end the following functional forms are assumed:

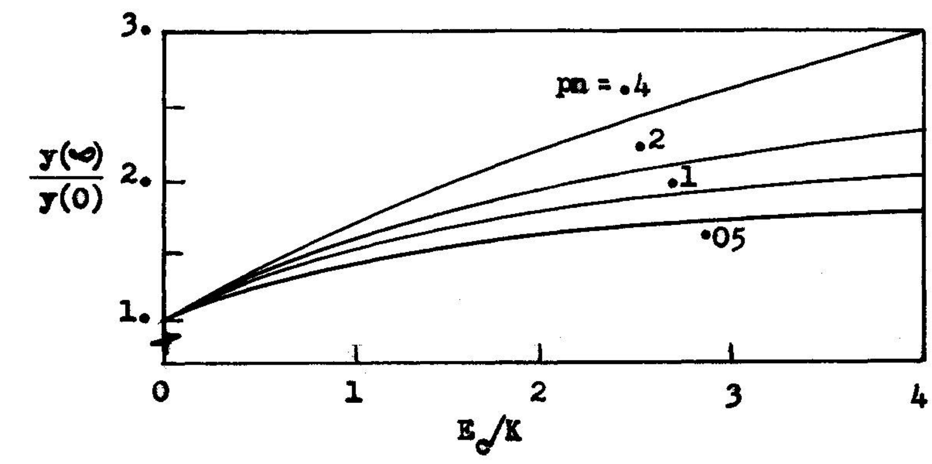

Fig.1 PEAK CREEP DEFLECTION

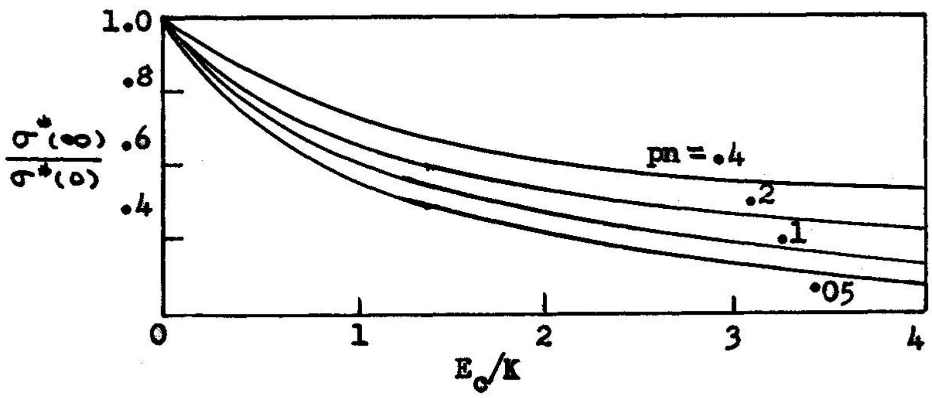

Fig.2 PEAK CREEP STRESS

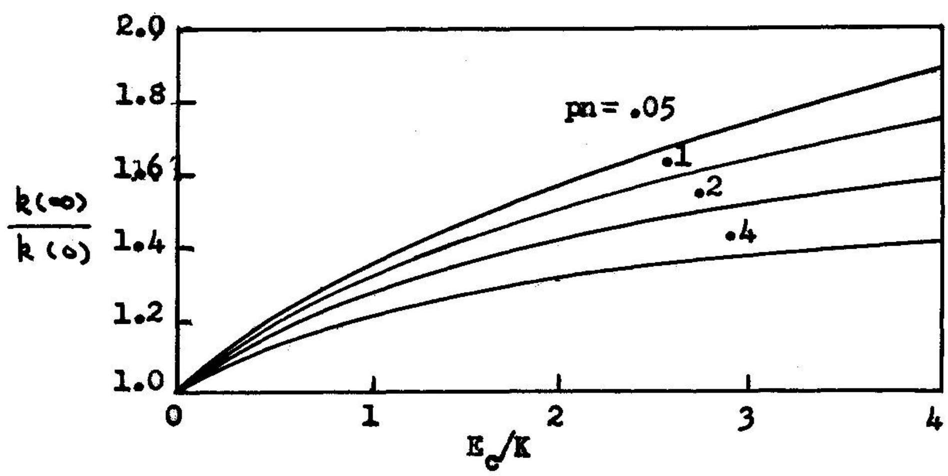

Fig.3 NEUTRAL AXIS LOCATION

$$\frac{k(t)}{k(0)} = 1 + \left[\frac{k(\infty)}{k(0)} - 1\right]\left[1 - \exp(-i_k t)\right]$$

$$\frac{\bar{\sigma}^*(t)}{\bar{\sigma}^*(0)} = 1 - \left[1 - \frac{\bar{\sigma}^*(\infty)}{\bar{\sigma}^*(0)}\right]\left[1 - \exp(-i_\sigma t)\right] \quad (2)$$

$$\frac{y(t)}{y(0)} = \frac{\bar{R}(0)}{\bar{R}(t)} \cdot 1 + \left[\frac{y(\infty)}{y(0)} - 1\right]\left[1 - \exp(-i_R t)\right]$$

in which,

i_k, i_σ, i_R are the parameters to be determined. A least squared-error criterion is used to compute values of the parameters i_k, i_σ, and i_R which best fit the numerical solutions for each combination of the beam parameters.

Values of the exponents can be conveniently presented in chart form as shown in Fig. 4 through 6. Note that the exponent for the curvature variations (i_R) is generally similar to the exponent for plain concrete strains (K/ø) while the stress and neutral axis exponents are dissimilar.

Application to Continuous Beams

Creep effects in continuous beams may be approached in the same way as that discussed above for determinate beams. The only difference now would be that a set of Eqs. (1) at discrete points along the beam would have to be solved simultaneously at each time step.

A promising alternate approach, however, is to consider a solution based upon modified slope deflection equations. A general expression of the slope deflection equation is

$$\bar{M} = C(2\theta_1 + \theta_2 - 3\Delta/L) \quad (3)$$

where $\theta_1, \theta_2, \Delta$ = rotations and relative displacement of end points of beam

C = flexural stiffness of beam

The last of Eq. 2 indicates that the stiffness of the beam varies with time according to

$$C = \frac{C_0}{1 + \left[\frac{y(\infty)}{y(0)} - 1\right]\left[1 - \exp(-i_R t)\right]} \quad (4)$$

where C_0 = initial elastic stiffness

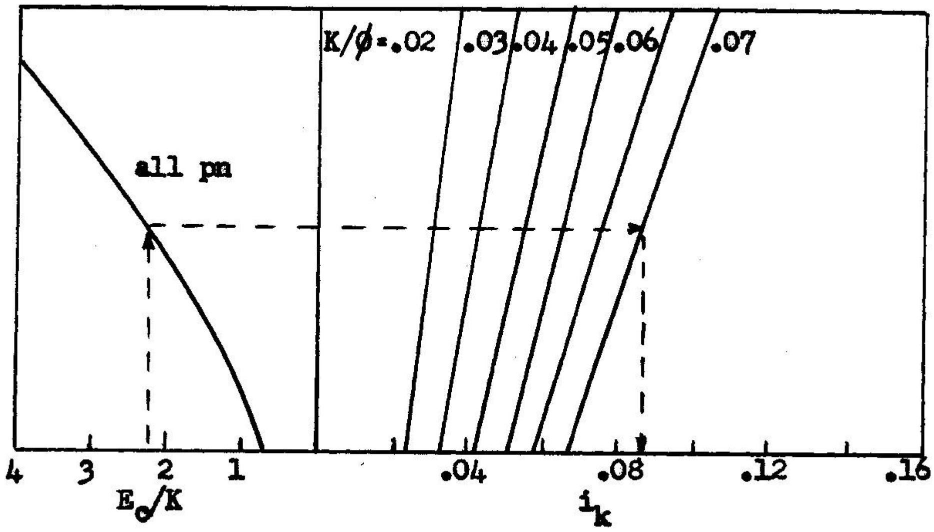

Fig.4 TIME VARIATION OF NEUTRAL AXIS LOCATION

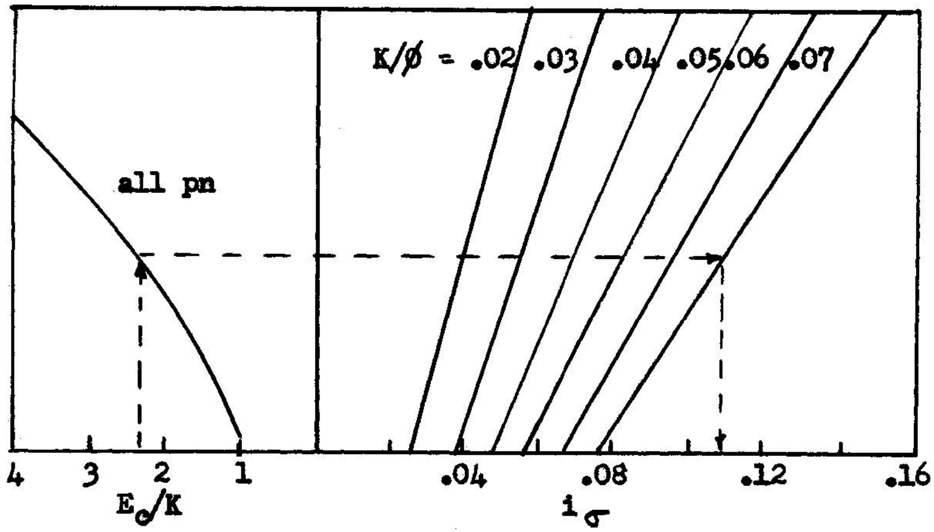

Fig.5 TIME VARIATION OF STRESS

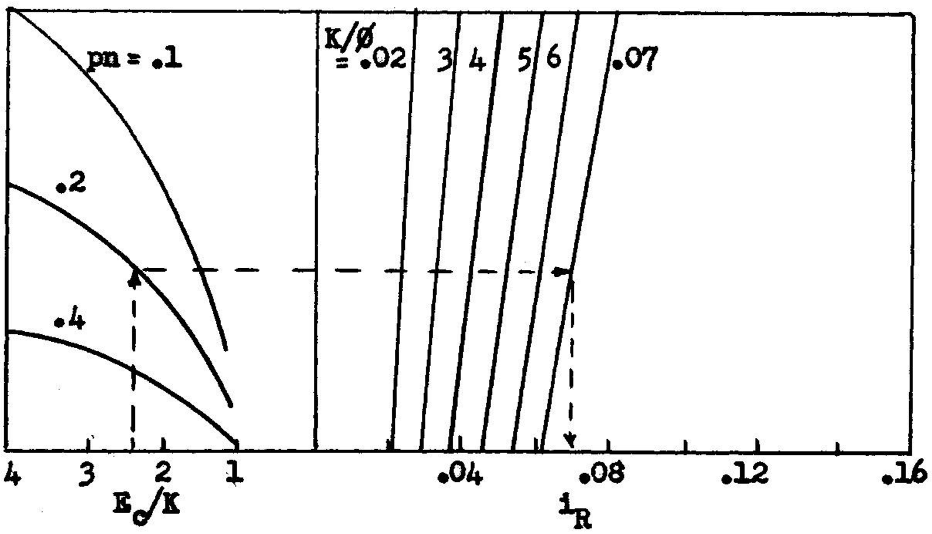

Fig.6 TIME VARIATION OF CURVATURE

One may use Eq. (3) in the same way that classical slope deflection equations are used to analyze hyperstatic elastic beams. Applying the modified slope deflection equations to the analysis of a continuous beam is tantamount to analyzing a beam whose stiffness varies with time thereby causing a continuous distribution of moments with time.

The creep deformation of hyperstatic structures tends to change the end moment of the beams. As an example of the effect of creep on the response of a statically-indeterminate beam, consider a two span beam which is subjected to a single end moment. The slope deflection equations for this beam are,

$$
\begin{aligned}
M_{AB} &= \bar{M}_o = C_1\ (2\ \theta_A + \theta_B) \\
M_{BA} + M_{BC} &= 0 = C_1\ (\theta_A + 2\theta_B) + C_2\ (2\theta_B + \theta_C) \\
M_{CB} &= 0 = C_2\ (2\ \theta_C + \theta_B)
\end{aligned}
\tag{5}
$$

Equations (5) have the solution

$$
\begin{aligned}
\theta_A &= \frac{\bar{M}_o}{6C_1}\ \frac{(4+3\xi)}{(1+\xi)} \\
\theta_B &= -\frac{\bar{M}_o}{3C_1}\ \frac{1}{(1+\xi)} \\
\theta_C &= \frac{\bar{M}_o}{6C_1}\ \frac{1}{(1+\xi)}
\end{aligned}
\tag{6}
$$

where $\xi = C_2/C_1$ = ratio of flexural stiffness span 1 to span 2

The moment at the center support is then

$$\bar{M}_s = \bar{M}_o\ \xi / 2(1+\xi) \tag{7}$$

Using Eq. (2) the ratio of final to initial value of $\bar{M}_s$ is,

$$\frac{\bar{M}_s(\infty)}{\bar{M}_s(0)} = \frac{\alpha\ (1 + C_{o_2}/C_{o_1})}{1 + \alpha\ C_{o_2}/C_{o_1}} \tag{8}$$

where

$$\alpha = \left[y(\infty)/y(0) \right]_1 \Big/ \left[y(\infty)/y(0) \right]_2$$

C_{o_1}, C_{o_2} = elastic flexural stiffness of beam 1 and 2

Consider the case where both spans are equal except that the second span has been poured later than the first. In the first span then $\frac{E_c}{K}$ may equal 2 and $\frac{E_c}{K}$ equal 4 in the second span. Then from Fig. 1, α = 0.8 and $\bar{M}_s(\infty)/\bar{M}_s(o)$=0.89. Of course more severe redistributions may occur for larger differences in the properties of the two spans.

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NOMENCLATURE

b = Width of Beam (inches)

d = Depth of beam to centroid of steel (inches)

E_c = Young's modulus of concrete (psi)

k = Depth of neutral axis from outermost fiber in compression/d

K = Spring constant in Kelvin-Voight model of concrete (psi)

L = Beam span (inches)

$\bar{M}$ = Bending moment/$E_c bd^2$

n = Ratio of steel to concrete moduli

p = Area of steel reinforcement/bd

$\bar{R}$ = Radius of curvature/d

t = Time after loading (days)

$\bar{x}$ = Coordinate along length of beam/L

$\bar{y}$ = Beam deflection/d

$\bar{\sigma}^*$ = Ratio of stress to distance from neutral axis times d/E_c

ϕ = Constant of proportionality for dashpot of Kelvin-Voight unit (psi per day)

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SUMMARY

Procedures are developed in this paper for calculating the deformation and stress histories in reinforced concrete beams including the effects of concrete creep.

Beam deflection is found to be a minimum at the time of initial loading and may increase by a factor of three with time. The steel stress is at a minimum initially and may increase by 15 percent. Flexural normal stresses in the concrete of a beam may reduce to 30 percent of initial values with time.

RESUME

Dans ce mémoire, on présente une méthode permettant le calcul de la variation des déformations en fonction du temps et des contraintes sollicitant des poutres en béton armé, tout en tenant compte du fluage du béton.

La flèche de la poutre est minimale au moment même de la mise en charge et peut augmenter avec le temps d'un facteur allant jusqu'à trois. Les contraintes des armatures sont minimales au début et peuvent, par la suite, augmenter de 15 %. Les contraintes de flexion dans le béton peuvent être réduites, avec le temps, de 30 % par rapport aux valeurs initiales.

ZUSAMMENFASSUNG

In diesem Beitrag wird eine Prozedur entwickelt, welche die Berechnung der Verformung und der Spannungsgeschichte in Stahlbetonbalken einschliesslich Kriechen erlaubt.

Es ergab sich, dass die Durchbiegung im Zeitpunkt der Anfangsbelastung am kleinsten ist und mit der Zeit dreimal anwächst. Die Stahlspannung, am Anfang am kleinsten, wächst hingegen nur um 15 %. Die Biegenormalspannungen im Beton können um 30 % gegenüber dem Ursprungswert mit der Zeit abfallen.