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Prestressed Concrete Columns under Long-time Loading

Poteaux de béton précontraint sous charge permanente

Vorgespannter Betonpfeiler bei Dauerbelastung

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1. Introduction

The use of prestressed concrete columns is of a more recent date than prestressed concrete beams. The first reports on prestressed columns were published in USA in the early 1950's. The research in this field has mainly been concentrated to USA and Australia [1], [2]. With the exception of a few series of tests reported from Russia, only short-time loading has been dealt with.

Whereas the constructional advantages of prestressing beams are obvious, it is more questionable to use prestressing for concrete columns. To artificially introduce a compressive force into a member which at a later stage mainly will be loaded with compressive forces, seems at a first glance very objectionable. However, with the exception of thick centrically loaded columns prestressing seems to increase the carrying capacity for short-time loading. This beneficial effect is more accentuated for eccentrically loaded slender columns. The prestressing also means increased stiffness as long as the columns works in the uncracked stage but for long-time loading this advantage is somewhat reduced by the increased creep-deformation due to the higher stress-level. Another advantage is that a suitable prestressing force can eliminate the risk of cracking during handling a transportation. Many of these advantages have been appreciated by the prefabrication industry in Sweden and nowadays prestressed columns are often used.

The object of this investigation is to find a calculation routine which makes it possible to predict the behaviour of long-time loaded columns.

The analysis of the columns has been made with one *general method*, which can be used for all kinds of reinforced concrete columns and with one *special method* the use of which is more restricted to fully prestressed columns. This special method is based upon the observations that tensile cracking means a radical change in behaviour. It could be observed in the tests that cracking meant a rapid increase in deflection and within a short time final collapse. As a start for the long-time analysis the short-time analysis is discussed and compared with some test results.

2. Test program

The investigation consists of two series A and B. The section area and the reinforcement arrangement have been varied in the two series according to Fig. 1. The reinforcement consisted of two and four high tensile steel wires,

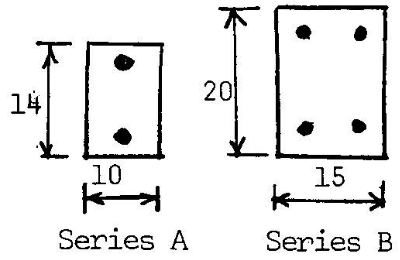

Fig. 1. Dimensions and reinforcement arrangement for columns in series A and series B.

respectively. The amount of prestress σ_p (= prestressing force divided with concrete area), the length of the columns L, and the eccentricity e of the normal force are presented in Table 1.

The concrete was proportioned for a cube strength (15 × 15 cm) of 400 kp/cm^2. Together with each column was also made a couple of testprisms of the same sectional-dimension as the column and a number of test-cubes. Some of the prisms were used to determine the stress-strain relationship and some to evaluate the shrinkage.

Table 1. Outline of test program

Column No.[1)]	σ_p kp/cm^2	L cm	e cm	Column No.[1)]	σ_p kp/cm^2	L cm	e cm	Column No.[1)]	σ_p kp/cm^2	L cm	e cm
A-1 L	100	390	1,5	B-1 S	100	480	2,5	B-6 S	160	390	5,0
A-2 L	100	390	3,0	B-2 S	100	480	5,0	B-7 L	100	480	2,5
A-3 L	160	390	1,5	B-3 S	160	480	2,5	B-8 L	100	480	5,0
A-4 L	160	390	3,0	B-4 S	160	480	5,0	B-9 L	160	480	2,5
A-5 S	100	390	1,5	B-5 S	160	390	2,5	B-10 L	160	480	5,0

1) L = columns under long-time loading
S = columns under short-time loading

3. Analysis of short-time loading

3.1 Outline of analysis

The analysis can be summarized according to Fig. 2, where the operations numbered means the following:

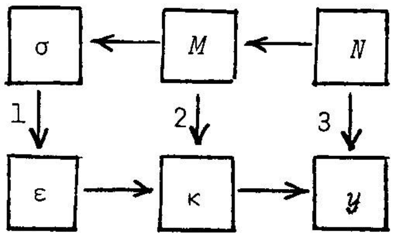

Fig. 2. Operational routine for analysis of short-time loaded columns.

1: Evaluation of stress-strain relationships for steel and concrete (σ-ε diagram).

2: Calculation of the relationship between bending moment M and curvature κ for a small column element (M-κ diagram).

3: Calculation of the relationship between external normal force N and mid-point deflection y. Stability analysis for the whole column.

Some points of interest in this analysis will be somewhat more commented.

3.2 Stress-strain relationship

It was found during the analysis that the σ-ε relationship evaluated from the prism-tests had to be adjusted due to the time-effect of the prestress. This correction has been made according to H.J. Brettle [3] and implies an increase in the modulus of the elasticity. This phenomenon could perhaps be compared with the consolidation effect of clays.

No special tension tests were made but from observations of the columns themselves it could be concluded that tensile cracking occurred at $\varepsilon_t \approx 0{,}40$ ‰ and the calculations are based on this value together with a triangular stress-strain relationship with σ_t = 50 kp/cm^2. (σ_t = tensile strength).

3.3 Moment-curvature relationship

For this operation a program has been made which makes it possible to treat materials with an arbitrary stress-strain relationship.

3.4 Stability analysis

For the calculation of the buckling load three methods have been used. One of these methods (method I) is very accurate and is based upon numerical integration along the column, whereby the column is divided into a number of finite elements. The other two methods are approximate and in this analysis the deflection-curve is assumed to be a part of a half cosine-wave (method II) or exactly half a cosine-wave (method III) [2].

No significant difference was obtained between method I and II. Method II is more accurate than method III but method III which is the most simple, seems to be accurate enough for practical calculations. Some theoretical results are compared with tests in Table 2.

Table 2. Comparison between theoretical and experimental buckling loads.

Column No.	Buckling load, Mp		
	Tested N^0_{test}	Calculated N^0_{calc} Method II	Method III
A-2	10,6	9,8	9,5
A-5	12,0	11,0	11,7
B-1	25,9	21,4	20,5
B-2	17,4	16,5	15,5
B-4	23,6	24,5	26,0
B-6	28,0	31,8	33,5

The agreement between tested and calculated buckling loads is in most cases satisfactory. Some disagreement can be explained by the choice of the σ-ε relationship which for these analysed columns is obtained with prism-tests performed under shorter time than the column-tests. The successful performance of this theoretical analysis is very sensitive to the choice of the stress-strain relationship (the nature of all buckling problems) and it is recommended to choose values on the safe side in practical problems.

4. Analysis of long-time loading

4.1 Creep-function

The creep-function for uniaxial compression has been evaluated from strain measurements in the column ends. Measurements on the shrinkage prisms have made it possible to separate creep ε_c and shrinkage ε_s. The creep-function is thus defined by the ratio

$$\phi = \varepsilon_c/\varepsilon_{el} \qquad (1)$$

where ε_{el} = elastic strain.

Some adjustments have been made to compensate for small time-dependent movements of the neutral axis x, see Fig. 3.

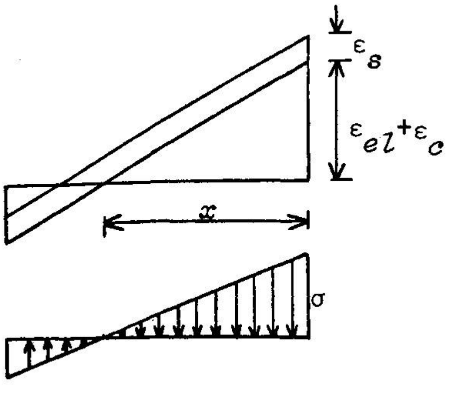

Fig. 3. Separation of shrinkage and creep. Evaluation of ϕ from measurements in the column ends.

The appearance of the creep-function is given in Fig. 4 for column A-2. An analytical expression for the creep-function ϕ can be rather well adapted to the test results:

$$\phi = \phi_\infty \frac{t}{T + t} \qquad (2)$$

where ϕ_∞ = final creep value
t = loading time
T = loading time for $\phi = \phi_\infty/2$

In Fig. 4 two curves of the form (2) have been drawn. The upper curve corresponds to ϕ_∞ = 1,6 and T = 4 days. The creep-deformations are rather rapid because of the small dimensions of the specimens.

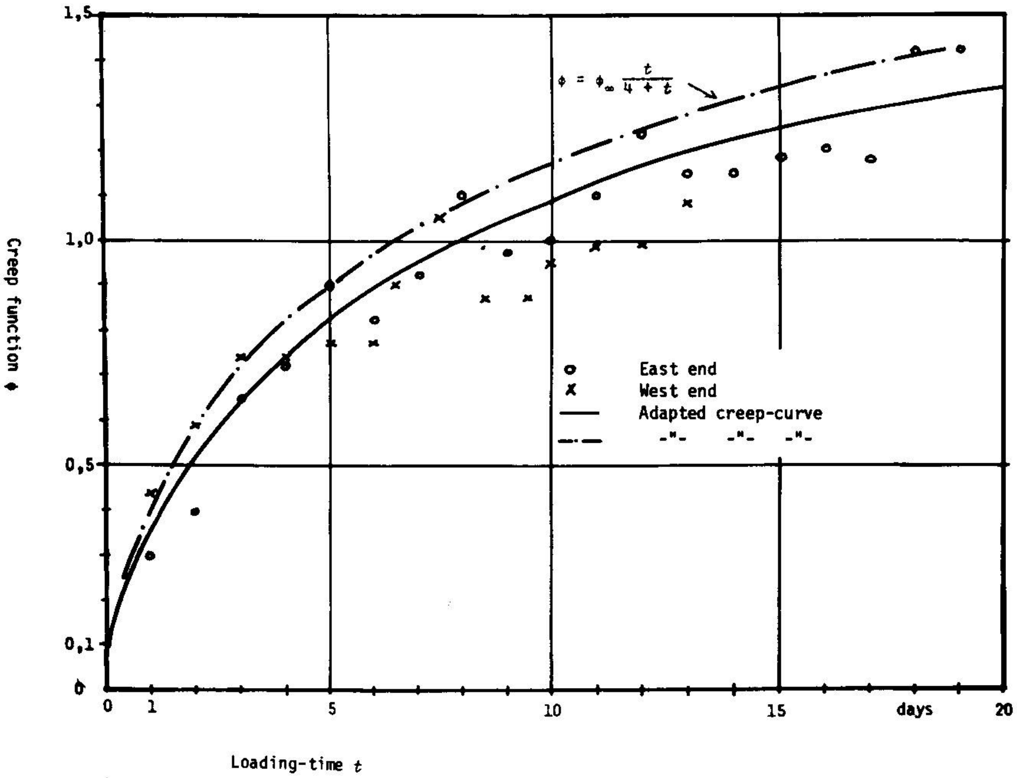

Fig. 4. Column A-2. Creep-function ϕ.

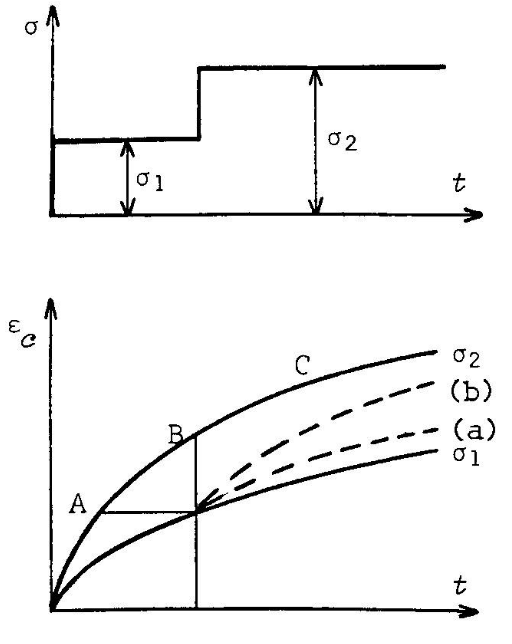

Fig. 5. A sudden increase of the stress level from σ_1 to σ_2 gives a strain increase following (a) when (3) is used. For concrete it would be more correct if the strain increase followed (b).
Curve (a) is parallell to BC (time--hardening) and curve (b) is parallell to AB (strain-hardening).

4.2 Rheological model

In this paper it has been chosen to use a Maxwell body with time-dependent (growing) viscosity. The relationship between uniaxial strain and stress can be formulated as:

$$\frac{d\varepsilon_c}{dt} = \frac{\sigma}{E}\frac{d\phi}{dt} + \frac{d\sigma}{E} \qquad (3)$$

The equation (3) is known as Dischingers basic equation [4].

The use of (3) implies that at a sudden increase of the stress level from σ_1 to σ_2 according to Fig. 5 the time-dependent strain increase follows curve (a). For concrete it would be more correct if the strain increase followed the curve (b). In the general method presented below, measures are taken to compensate for the error introduced by (3). Equation (3) is justified because of its mathematical simplicity.

4.3 General method

As mentioned in the introduction this procedure can be used for concrete columns with all kinds of reinforcement. The application will here be demonstrated on fully prestressed columns.

The stress-strain relationship obtained from the prism-tests represents the curve $\phi = 0$ in Fig. 6. From this basic curve the curves $\phi = 0{,}2$, $\phi = 0{,}4$ etc. (representing steps in the calculation routine) have been constructed according to Fig. 6. Thus the curve $\phi = \phi_1$ represents the stress-strain relationship after a loading time:

$$t_1 = T \frac{\phi_1/\phi_\infty}{1-\phi_1/\phi_\infty} \qquad (2a)$$

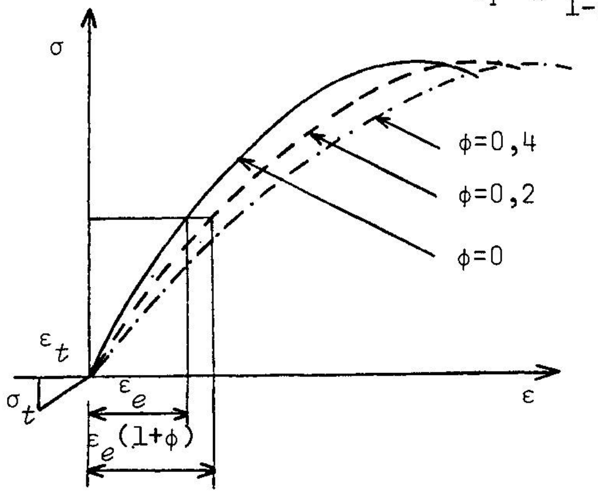

Fig. 6. Construction of stress-strain relationship for different loading times.

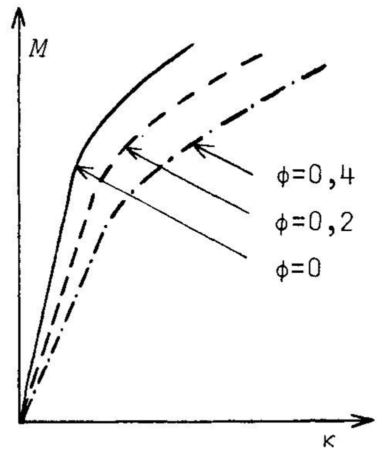

Fig. 7. Moment-curvature relationship for different loading times for a constant external normal force *N*.

The creep of concrete in tension has been neglected. Observations from the column tests seem to justify this.

With the described σ-ε diagrams as a basis moment-curvature diagrams have been drawn according to Fig. 7. All curves in Fig. 7 represent the same external normal force N (but different loading times and different prestressing forces due to shrinkage losses).

The calculation procedure is divided in the following steps:

(a) Elastic initial deflection y in the middle section is calculated according to method III, described in section 3.4 above:

$$y = e \frac{N}{N_E - N} \quad \text{where} \quad N_E = \pi^2 EI/L^2 \qquad (4)$$

The stiffness EI is obtained from the curve $\phi = 0$ in Fig. 7 as $EI = dM/d\kappa$ at the point where $M = N(e + y)$.

(b) The time-dependent additional deflection Δa obtained during a short time Δt represented by $\Delta\phi = 0{,}2$ is calculated. This additional deflection Δa consists of two parts $\Delta a \doteq \Delta e + \Delta y$. One part Δe corresponds the creep deflection under a constant bending moment $M = N(e + y)$. On account of creep one obtains an increase in the curvature $\Delta\kappa$ according to Fig. 8. This curvature increase corresponds to a midpoint deflection:

$$\Delta e = \left(\frac{L}{\pi}\right)^2 \Delta\kappa \qquad (5)$$

Δe according to (5) can be interpreted as an additional initial deflection.

The deflection Δe means an increase

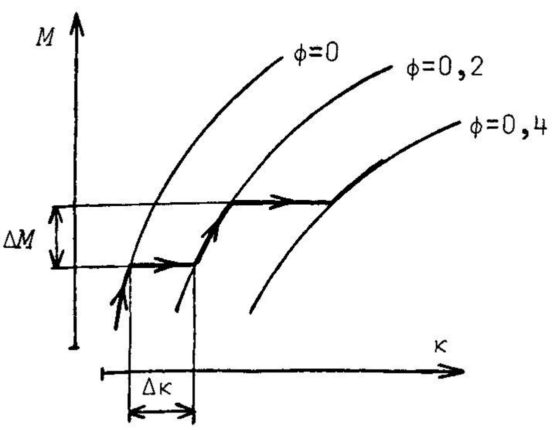

Fig. 8. Stepwise calculation of the additional time-dependent midpoint deflection.

in the lever arm for the force N and this gives an additional elastic deflection Δy. This is the other part of the additional deflection. Δy can be written:

$$\Delta y = \Delta e \frac{N}{N_{E,\phi} - N} \qquad (6)$$

where

$$N_{E,\phi} = \frac{\pi^2}{L^2} \left(\frac{dM}{d\kappa}\right)_{\phi=0,2} \qquad (7)$$

The stiffness has thus been taken from the curve $\phi = 0,2$ in Fig. 8. According to (3) one should take the stiffness from the curve $\phi = 0$, but in order to compensate for the error introduced by (3) and mentioned above, it has been considered more correct to use the stiffness for the curve $\phi = 0,2$. Numerical experiments have shown that only a minor error is introduced by this routine when it is assumed that the column obtains creep deflection for a constant bending moment under a short time-element Δt.

(c) At the loading time represented by $\Delta\phi = 0,2$ the moment has increased with the amount $\Delta M = N\Delta a = N(\Delta e + \Delta y)$ according to Fig. 8. The procedure is repeated until $\phi = \Sigma\Delta\phi = \phi_\infty$ or stopped when the moment has increased to a point where the ultimate moment-carrying capacity of the column is reached at the axial load N considered. In this latter case the lifetime of the column can be estimated. In general it can be concluded that the accuracy in the first case is greater than in the latter since a rheological model of this kind is not valid close to crushing failure. An alternative way of calculating the long-time buckling load is presented in section 4.4 below.

A comparison between calculated and measured deflection is presented in Fig. 9. The upper curve corresponds to the upper adapted creep-function in Fig. 4 and vice versa. The agreement is very good.

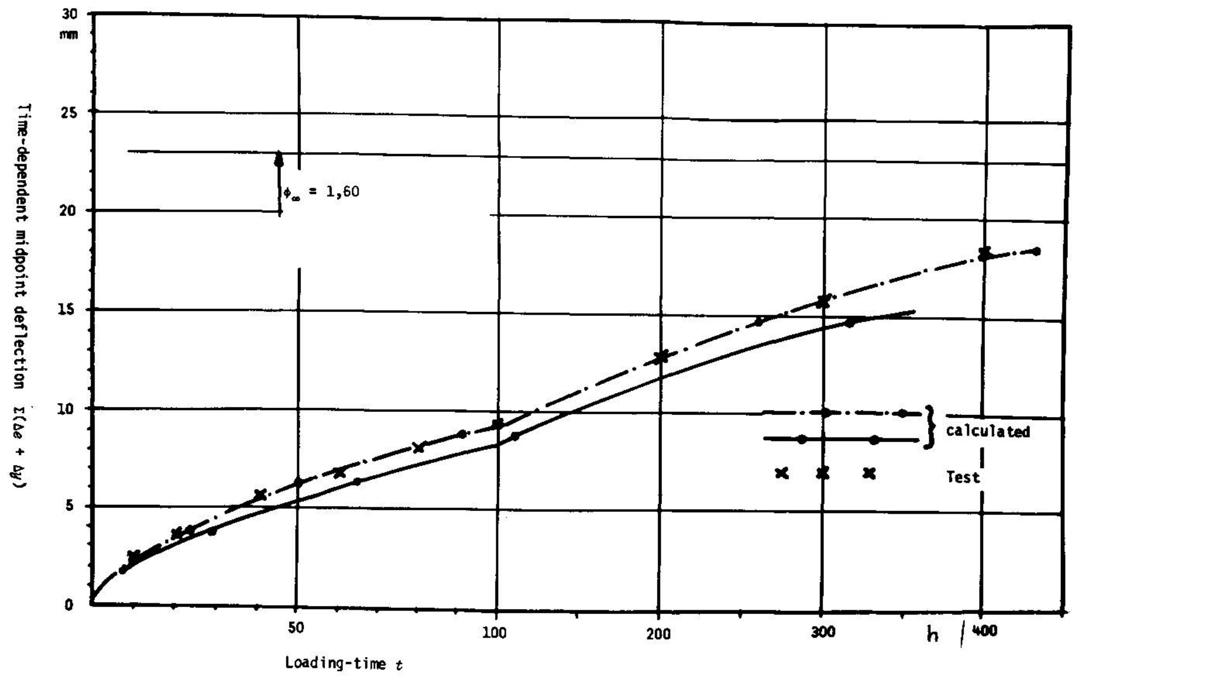

Fig. 9. Comparison between calculated and measured deflections for column A-2.

In Fig. 10 a load-deflection diagram for the same column has been drawn. It is clear that the tested load N_{test} = 4,36 Mp gives a total deformation smaller than the critical.

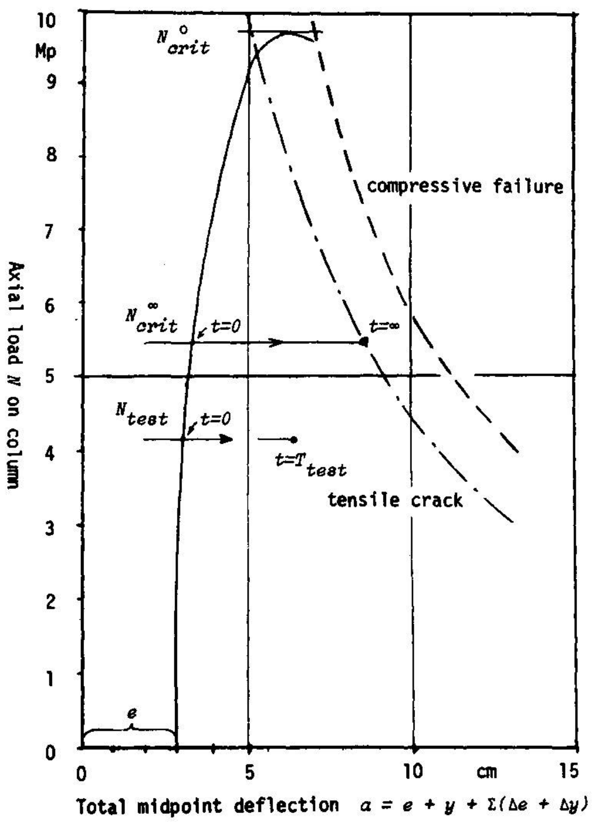

Fig. 10. Load-deflection diagram. Column A-2.

It is concluded that the described method for calculation of time-dependent deflections is sufficiently accurate for practical application.

4.4 Special method

The study of the behaviour of the tested fully prestressed columns has resulted in a formula for calculating the long-time buckling load. It has already been mentioned in the introduction that the transition from uncracked to cracked stage means a radical change in behaviour and it therefore seems reasonable to take this transition stage as a failure criteria. In Table 3 below the loading time has been presented for cracking, T_{crack}, and for final collapse, T_{fail}. In some cases the final failure will not come directly after cracking but the deflections increase rapidly.

For a constant stiffness EI equation (3) can be integrated and gives for the total deflection a the expression:

$$a = a(t) = e + y + \Sigma(\Delta e + \Delta y) =$$

$$= e \frac{\nu}{\nu-1} 2{,}718^{\frac{\phi}{\nu-1}} \qquad (8)$$

where $\nu = N_E/N$ (formal buckling safety) and 2,718 is the base in the natural logarithm system.

Equation (9) is valid at cracking

$$\sigma = -\frac{P_\phi}{A} - \frac{N}{A} + \frac{Na}{W} = \sigma_t \qquad (9)$$

where P_ϕ = prestressing force with regard to losses at the time represented by ϕ and A = sectional area, W = bending resistance, and σ_t = tensile strength. With $\phi = \phi_\infty$ and a_∞ according to (8) it is possible to calculate the long-time buckling load N^∞_{crit}:

$$N^\infty_{crit} = \frac{W(\sigma_t + P_\phi/A)}{e \frac{\nu}{\nu-1} \cdot 2{,}718^{\frac{\phi_\infty}{\nu-1}} - \frac{W}{A}} \qquad (10)$$

N^∞_{crit} according to (10) has been calculated for some tested columns and theoretical and calculated loads have been compared in Table 3. The loading time for the tested columns is of course smaller than the theoretical infinite loading time and in order to make the comparison more relevant equation (10) has also been used with the values of ϕ, that corresponds to the real loading time $t = T_{crack}$ for the tested columns. The comparison has been made with two values on ϕ_∞ namely $\phi_\infty = 1{,}6$ and $\phi_\infty = 2{,}0$ ($\phi_\infty = 1{,}6$ agrees more closely with the tests). These calculated values N^t_{crit} for $t = T_{crack}$ should be compared with the test results N_{test}.

Table 3. Comparison between tested and calculated long-time buckling loads.

Column No.	Experimental			Calculated	
	N_{test}	T_{crack}	T_{fail}	N_{crit}^{∞}	N_{crit}^{t}
	Mp	days	days	Mp	Mp
A-1	8,72	5,0	5,5	7,7 7,0	9,0 8,4
A-2	4,36	--	--	6,1 5,6	-- --
A-3	12,35	1,7	2,8	8,8 8,2	12,1 11,6
A-4	5,81	40	62	5,9 5,5	6,0 5,6

The agreement is satisfactory and the method could possibly be used for calculating the long-time buckling load for prestressed columns.

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SUMMARY

This report deals with an experimental and theoretical investigation of slender, excentrically loaded, prestressed concrete columns under both short--time and long-time loading.

A theoretical analysis for the behaviour of the columns under both short--time and long-time loading has been worked out. The theoretical analysis gives a realistic description of the behaviour of the columns over the full range of loading and loading time.

RESUME

La présente communication traite d'une étude expérimentale et théorique de poteaux élancés en béton précontraint soumis à une charge excentrique soit de courte durée, soit permanente.

Les auteurs présentent une analyse théorique du comportement des poteaux s'appliquant aussi bien aux charges de durée prolongée qu'aux charges de brève durée. Cette analyse comporte une description réaliste du comportement des poteaux pour toute la gamme d'importance et de durée des charges.

ZUSAMMENFASSUNG

Dieser Bericht behandelt experimentelle und theoretische Untersuchungen schmaler exzentrisch belasteter vorgespannter Betonpfeiler sowohl bei kurzwährender Belastung als auch bei Dauerbelastung.

Eine theoretische Analyse über das Verhalten der Pfeiler sowohl bei Kurz- als auch bei Dauerbelastung wurde ausgearbeitet. Die theoretische Analyse gibt eine realistische Beschreibung des Verhaltens der Pfeiler für die gesamte Belastungssteigerung und die Belastungszeit.

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