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Non-Uniform Drying Shrinkage in Reinforced Concrete

Retrait non uniforme dans le béton armé

Ungleichmässiges Schwinden im Stahlbeton

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INTRODUCTION

Effects of shrinkage and creep in the design of reinforced concrete structures must be given proper consideration. Shrinkage and creep, as well as loading history, significantly influence cracking patterns and consequently affect the distribution of moments and forces in frames. Also, with increasing height of buildings, differential shortening of columns can have considerable effect on the bending moments in the connecting beams and slabs, particularly in the upper stories (1).

In most cases shrinkage is assumed uniformly distributed throughout the section of the member (2, 3). Actually, shrinkage is closely related to moisture diffusion with drying and therefore it varies throughout the section, particularly for large size members in which drying is confined to the exterior portion of the section (4).

The method presented here takes into account non-uniform shrinkage and provides a more accurate evaluation of the effects of member size and shape, and of varying load history on stresses and deformations. The distribution of non-uniform shrinkage is evaluated using diffusion as a mathematical model. Elastic and creep properties of concrete are taken from experimental data. Effects of shrinkage in plain and reinforced concrete prisms calculated by the method presented here agree well with experimental results. A summary of analytical results obtained from a study of shrinkage effects in reinforced concrete columns, in which the principal variables were column size, amount of reinforcement, and loading history, is presented. It is shown that non-uniform shrinkage must be considered if a reasonable evaluation of tension stresses in concrete is desired. Also, restrictions on minimum size and minimum and maximum amounts of reinforcement for concrete columns which are specified to control shrinkage effects are discussed.

EVALUATION OF STRESSES AND DEFORMATIONS

Numerical procedures for evaluating time-dependent internal stresses and deformations are based on geometric discretization of reinforced concrete structural system, material behavior laws, and time-step incremental method of analysis (5, 6, 7, 8, 9, 10). A structural system is subdivided into segments which in turn are subdivided into elements of concrete and reinforcing steel. Initially a stress-free plane strain condition is considered. During a given time-step the increments of "free deformation" due to shrinkage, creep and temperature in each element may violate the plane strain condition as the free deformations are allowed to develop without any restraint from steel reinforcement or adjacent concrete elements. At the end of the time-step four conditions must be satisfied: (1) plane strain, (2) compatibility of all steel and concrete elements and of all segments, (3) equilibrium of internal and external forces, and (4) no tension on the cracked elements. This is accomplished by a set of correction stresses to all the elements and segments which are assumed to be applied instantaneously at the end of each time-step.

In each time-step the increment of "free deformations" in the elements represent the incompatibilities and are removed by applying forces ΔP_i such that the plane deformation which existed at the beginning of the time interval is reinstated. For each element the compatibility correction force is calculated using the known incompatibility deformation, the instantaneous modulus of the material and the effective area of the element. These correction forces disturb equilibrium which is restored by applying correction stresses to both concrete and steel acting in a composite fashion. The equilibrium correction stress-resultant must balance the sum of the compatibility correction resultant and the increment in the external loading. Stresses and deformations associated with the equilibrium correction are calculated using conventional methods.

The stresses and deformation at a given time are determined by summing up all the incremental values in the preceding time intervals. If calculated tensile stress in any concrete element exceeds the cracking limit value, then the element is assumed to crack and henceforth cannot carry any tensile stress. Another cycle of calculations at the end of the time-step must account for removal of tension from all "cracked" elements.

MATERIAL BEHAVIOR AND ENVIRONMENT

In calculating "free" deformations during a given time-step a material behavior model must be used which accounts for the effects of humidity and temperature. Bazant (11) proposed a thermodynamic theory for concrete deformation at variable temperature and humidity, which is the most general model proposed to date. However, lacking experimental data and confirmation, it could not be used in the present study. Therefore, in this study the following assumptions have been made regarding material behavior and the exposure conditions: (1) all elements are maintained at constant temperature throughout the intervals of time considered here (heat of hydration and other transient temperature conditions are neglected); (2) for a given time period the element is cured at 100% relative humidity during which no shrinkage or swelling takes place (thereafter the element is exposed to drying in a constant humidity environment); (3) concrete deforms as a linear viscoelastic material, exhibiting strong ageing in creep deformation and weak ageing in elastic deformation, and the principle of superposition is used to evaluate time-dependent deformation; (4) steel reinforcement behaves as a linear elastic material; and (5) free shrinkage of concrete can be calculated using mathematical model of diffusion.

SHRINKAGE DIFFUSION

Carlson (4) and Pickett (12) proposed that free shrinkage of concrete be treated as a "substance" obeying laws of diffusion, and that the amount of free shrinkage at any given time be determined by the "shrinkage diffusivity" of the material, the shape of the body, the boundary conditions defined by the surface factor, and the initial and the final state of the body. Pickett emphasized that diffusion theory for free shrinkage of concrete is not based on a physical model of shrinkage, but he suggested that shrinkage might be closely predictable by this mathematical model if the material and ambient conditions are characterized by properly selected values of diffusivity and surface factor.

For a rectangular prism drying from four faces but not from the ends the diffusion equation is as follows:

$$K\left(\frac{\partial^2 S}{\partial x^2}+\frac{\partial^2 S}{\partial y^2}\right)=\frac{\partial S}{\partial t} \qquad 1$$

where K = diffusivity coefficient of shrinkage in square inches per day, S = free unrestrained unit linear shrinkage strain under constant ambient conditions, and x, y = rectangular coordinates. The exposed faces of the prism are the planes $y = \pm b$, and $x = \pm c$, and the boundary conditions then become:

$$\frac{\partial S}{\partial y} = \pm\frac{f}{K}(S_\infty - S) \qquad @ \quad y = \pm b \qquad 2a$$

$$\frac{\partial S}{\partial x} = \pm\frac{f}{K}(S_\infty - S) \qquad @ \quad x = \pm c \qquad 2b$$

where S_∞ = final unrestrained shrinkage strain under constant ambient conditions, i.e. value of S when $t = \infty$, and f = surface factor, characteristic of the material and the boundary conditions, in inches per day. The solution satisfying equations 1 and 2 are given in (12).

As drying proceeds the value of K, the coefficient of shrinkage diffusivity, decreases and based on Pickett's study it can be approximated as follows:

$$K = 0.10\sqrt{\frac{2}{2+t}} \text{ sq. in./day.} \qquad 3$$

This coefficient is not the same for all concretes and all exposure conditions, and its variation requires further study. However, for temperatures of about 70°F and atmospheric exposure of about 50 percent relative humidity the values obtained using equation 3 gave reasonably good results in this study as well.

The boundary conditions describe the shrinkage gradient at the boundary. When $S = S_\infty$ the gradient is zero; when $S = 0$ the gradient is proportional to S_∞. The surface factor f also decreases as drying proceeds, although the precise nature of this variation is difficult to determine. Assumption that f is directly proportional to K greatly simplifies the solution of the diffusion equation, and is justified as it apparently provides reasonably good agreement with experimental data. The following equation of proportionality has been used in this study.

$$f = 1.67 \text{ K in./day} \qquad 4$$

The shrinkage S is "free shrinkage" not the "apparent" or "restrained" shrinkage observed in a drying concrete prism. The observed deformation of an unloaded drying prism is the result of "free shrinkage" combined with deformations developing as a result of internal stresses induced by the non-uniform shrinkage.

ELASTIC AND CREEP PROPERTIES OF CONCRETE

Experimental evidence indicates that within the working stress range concrete in compression exhibits essentially linear load-deformation characteristics both with respect to instantaneous and time-dependent deformations. However, during the initial months (often 6 to 9 months) of service these characteristics, particularly creep, are strongly dependent on ageing. Also, experimental evidence indicates that "superposition principle" gives a reasonably good approximation of concrete even under variable loading (13, 14). A mathematical model of this behavior is usually represented by an integral equation:

$$\varepsilon(t) = \int_{t_o}^{t} C(t, \tau)\, d\sigma(\tau) \qquad 5$$

where t_o is the time of initial loading, i.e., the concrete is stress free prior to t_o, $\varepsilon(t)$ is the accumulated strain observed at time t due to a variable stress $\sigma(\tau)$, and the function $C(t, \tau)$ is a characteristic of the material independent of loading.

Experiments performed at University of California, Berkeley, in which cylindrical concrete specimens (6 in. diameter, 18 in. long) were loaded at different ages and the instantaneous and long-time deformations were measured, provided the data for which the following expression gives a close fit and is used in the present study.

$$C(t, \tau) = \left[\frac{0.333\times10^{-3}}{t} - \frac{0.306\times10^{-4}}{\sqrt{t}} + 0.221\times10^{-3}\right] + \left[0.1096 - \frac{0.1715}{\tau^{0.85}} + \frac{34.4316}{\tau^{1.7}} - \frac{433.9611}{\tau^{2.55}} + \frac{1260.859}{\tau^{3.40}}\right]$$
$$\cdot\left\{0.991\left[1 - e^{-0.2(t-\tau)}\right] + 2.06\left[1 - e^{-0.02(t-\tau)}\right] + 1.125\left[1 - e^{-0.002(t-\tau)}\right]\right\} \times 10^{-3} \qquad 6$$

COMPUTER PROGRAM

A program has been written for a CDC 6400 computer for evaluation of stresses and strains in steel and concrete in square columns symmetrically reinforced subjected to time-variable axial load. The input data consists of concrete and steel material properties, specified number of days per time-step, number of time intervals over which the analysis is carried out, size of column, number of elements into which the cross section is divided, area of steel reinforcement, load history and duration of initial curing period. Calculation of relative values of free shrinkage, (S/S_∞), is carried out using a computer program sub-routine based on a diffusion model with drying on four faces. A quadrant of a square section is subdivided into 25 elements and shrinkage values are calculated at the node points of the elements, midway between these, and at the center of each area. The computer output prints out stresses and strains in concrete and steel at each time interval.

COMPUTER SOLUTIONS AND EXPERIMENTAL RESULTS

Plain Concrete Prisms

Free Shrinkage (calculated values). Free shrinkage values were calculated at 5-day intervals up to 185 days after concrete was exposed to drying at age of 15 days. Square sections with 6, 10, 16, and 20 inch sides were investigated. Values of (S/S_∞) along the y-axis (x = 0) and the face of the section (y = b) are shown in Table 1 for 6 in. and 20 in. sections at 5, 35, and 185 days after drying.

Table 1. Relative Values of Free Shrinkage S/S_∞

Location	Prism Axis (x = 0)						Prism Face (y = b)					
Prism Size, in.	6			20			6			20		
Days After Drying	5	35	185	5	35	185	5	35	185	5	35	185
Age, Days	20	50	200	20	50	200	20	50	200	20	50	200
(y/b) or (x/c)												
0.0	.000	.031	.280	.000	.000	.000	.531	.684	.809	.531	.679	.774
0.2	.000	.045	.302	.000	.000	.000	.532	.689	.815	.532	.679	.774
0.4	.004	.095	.369	.000	.000	.001	.534	.705	.832	.532	.679	.774
0.6	.035	.207	.479	.000	.001	.023	.548	.741	.867	.532	.680	.779
0.8	.175	.403	.629	.002	.058	.201	.614	.805	.901	.533	.698	.820
1.0	.531	.684	.809	.531	.679	.774	.781	.897	.949	.780	.897	.945

It is noteworthy that after 5 days drying shrinkage proceeds only to about 1.5 inches from the surface in both 6 and 20 inch prisms. At the age of 200 days substantial drying shrinkage exists over the entire section of the 6-in. prism but in the 20-in. prism only the outer 5 in. of concrete has experienced shrinkage. The shrinkage of the concrete on the surface of the prism initially proceeds very rapidly: in 5 days more than half of the ultimate shrinkage takes place in both the 6-in. and 20-in. prism surfaces. In the interior shrinkage takes place more slowly: for example in the 20-in. prism in the 50 to 200 day interval shrinkage extended a distance about 0.1b, from about 4 inches to about 5 inches from the surface.

Restrained Shrinkage (calculated and observed values). Using equation 6 for the elastic and creep characteristics of concrete and free shrinkage calculated by diffusion equation, values of apparent (restrained) shrinkage were calculated for 6-in. prisms using $S_\infty = 1000 \times 10^{-6}$ and the computer program described above. These were compared with the shrinkage values measured on the unloaded 6-inch cylindrical specimens used as controls for the study of deformation under load. The calculated and observed results are shown in Fig. 1 and seem to be in good agreement. Consequently, the values of diffusivity K and of surface factor f defined by equations 3 and 4 are considered appropriate for the concrete used in this study.

Reinforced Concrete Columns

Calculated and Observed Values. Results obtained analytically for 6-in. square column with 1.17 percent and 4.67 percent reinforcement, cured wet for 28 days and subjected to 0.93 the allowable load at age 28 days are compared in Fig. 2 with the experimental results (15) for 6-in. diameter columns with the same percentages of reinforcement, curing period, and loading conditions. Comparison is made in terms of the load carried by concrete. Considering the differences between the shapes of the specimens and the probable differences in material properties of test specimens and the properties assumed in this study, the agreement appears to be good. Unfortunately no data are available regarding appropriate values of K, f, S_∞, and $C(t, \tau)$ for the test specimens, so that a more reliable evaluation of the analytical method is not possible.

Influence of Column Size, Amount of Reinforcement, and Different Load Histories. Influence of several variables on the stresses in and deformations of square reinforced concretes columns due to non-uniform shrinkage and different load histories was investigated analytically. The principal variables were colum size, amount of reinforcement and loading history. Four column sizes were studied: 6-in., 10-in., 16-in. and 20-in. For each of these sizes two percentages of reinforcement were used (2% and 8%) and two loading histories were considered for each of these eight columns. In one case no external load was imposed on the section up to 200 days, but the columns were exposed to drying at the age of 15 days. This is a condition which may occur in a precast column which is wet cured for 15 days and then stored under drying conditions for a period of time up to 200 days. In the second case one-half of the allowable axial compression load (based on ACI WSD criteria) was applied at the age of 20 days and sustained up to the age of 200 days. The case of incremental increase in external load (9, 15) was not considered in this study, but it is clear that it would give results somewhere in between the two loading cases considered here provided that the maximum sustained load did not exceed the value used in this study.

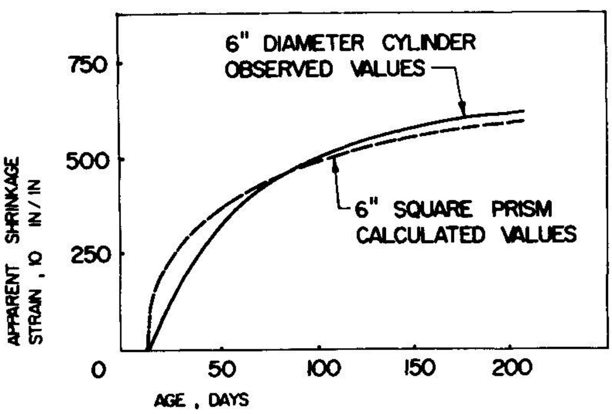

FIG. 1

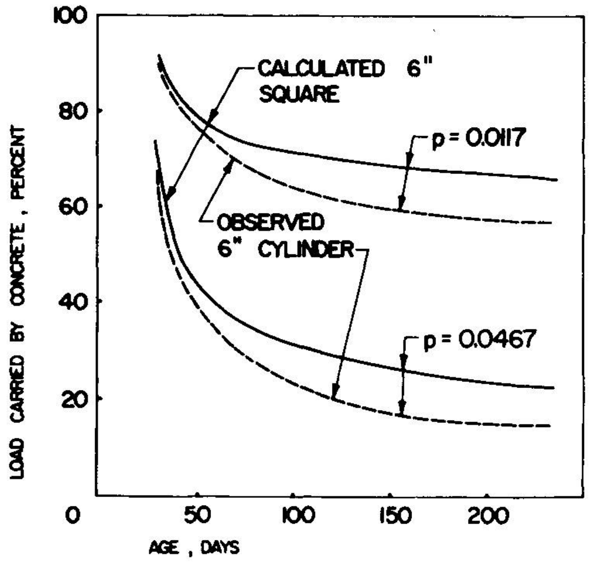

FIG. 2

The effects of cracking were neglected in this study. Members subjected to axial compression may develop tension stresses due to shrinkage over a relatively small portion of the outer perifery of the cross-section. It is assumed that either these stresses are sufficiently small so that no cracking occurs, or if cracking occurs over a small area, then its overall effect is small.

The steel reinforcement was assumed to have a yield strength of 40 ksi and elastic modulus of 29,000 ksi. The concrete deformation properties based on experimental data regarding creep, were characterized by $C(t, \tau)$ defined by equation 6.

For the square columns used in the study the following variables were examined: (a) stresses in the concrete face, (b) stresses in the concrete core, (c) stresses in steel reinforcement, and (d) deformation (shortening). Typical results are shown in Table 2.

CONCLUDING REMARKS

Free shrinkage of concrete associated with moisture diffusion during drying occurs primarily in the outer periphery of the section. The non-uniform distribution of this free shrinkage and the requirement for plane strain deformation induce tensile and compressive stresses in concrete prisms. The uniform "apparent" shrinkage is the combined result of the free shrinkage and the instantaneous and creep deformations caused by the induced stresses.

Table 2. Stresses and Deformations in Reinforced Concrete Columns

(+) is tension, elongation (-) is compression, shortening

Loading	No External Load				0.5 Allowable Load			
Column Size, in.	6		20		6		20	
Reinforcement Ratio	0.02	0.08	0.02	0.02	0.02	0.08	0.02	0.08
Age*, days	Stresses (ksi) in concrete face (x = 0.1c, y = 0.9b)							
20	+ 1.64	+ 1.80	+ 0.41	+0.43	+ 1.00	+ 1.07	- 0.23	- 0.29
50	+ 0.85	+ 1.07	+ 0.71	+0.77	+ 0.29	+ 0.58	+ 0.15	+ 0.28
200	+ 0.54	+ 0.83	+ 0.75	+0.85	+ 0.03	+ 0.42	+ 0.24	+ 0.45
	Stresses (ksi) in concrete core (x = 0.1c, y = 0.1b)							
20	- 0.58	- 0.42	- 0.09	-0.07	- 1.22	- 1.15	- 0.73	- 0.79
50	- 0.44	- 0.21	- 0.16	-0.09	- 0.99	- 0.70	- 0.71	- 0.59
200	- 0.10	+ 0.19	- 0.21	-0.11	- 0.61	- 0.22	- 0.72	- 0.52
	Stresses (ksi) in steel reinforcement							
20	- 4.10	- 2.99	- 0.64	-0.47	- 8.64	- 8.14	- 5.18	- 5.62
50	- 8.17	- 4.86	- 2.29	-1.38	-16.96	-12.93	-11.08	- 9.45
200	-12.48	- 6.74	- 4.15	-2.27	-23.58	-15.90	-15.20	-11.40
	Unit strain (10^{-6}) in column							
20	-141.5	-103.0	- 22.0	-16.1	-297.9	-280.7	-178.7	-193.7
50	-281.8	-167.5	- 79.0	-47.7	-584.9	-445.9	-382.0	-326.0
200	-430.7	-232.7	-143.0	-78.3	-809.6	-548.4	-525.0	-394.0

*Loading and drying commences at age 15 days.

Neglecting stresses due to non-uniform shrinkage implies that plain concrete undergoing drying is stress-free, and does not indicate the magnitude of true stresses in concrete. Realistic prediction of crack initiation on the basis of tensile stress in concrete requires evaluation of the tension stresses due to non-uniform shrinkage as well as stresses due to the particular loading history.

Restrictions on minimum size and minimum and maximum amounts of reinforcement for concrete columns in the current codes should be reexamined with due consideration given to the effects of the duration of curing, environmental exposure, loading history, and properties of concrete. Based on the limited study reported here it appears that 6-in. square columns with 2 to 8 percent reinforcement exposed to drying would develop tension stresses which would cause the concrete to crack. While the compression load bearing capacity of such sections might not be significantly impaired, their stiffness in bending would be significantly reduced. If these members were used in a frame, the distribution of bending moments would be significantly influenced by the cracking of the small size members. Increasing the size of the column restricts tension stresses to the exterior portion of the concrete and for the 20-inch columns, cracking may not develop at all, or be limited to the exterior 1 or 2 inches of concrete and thus have a less significant influence on stresses in and stiffness of the members.

In prisms with least dimension greater than 10 inches the inner portion of the concrete acts as a restraint on the shrinkage of the exterior portion in a manner similar to steel reinforcement. Assumption of uniform shrinkage distribution overemphasizes the role of reinforcing steel as a shrinkage restraining device. Thus, the limit on the maximum amount of reinforcement in large columns may not be realistic, as the inner portion of the concrete seems to play a dominant role in restraining the shrinkage of the exterior. It appears that large columns using high strength steel reinforcement in excess of 8 per cent may be used economically without significant adverse effects due to shrinkage.

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SUMMARY

An analysis to determine the stresses and deformations in symmetrically reinforced concrete columns is presented which takes into account non-uniform distribution of shrinkage throughout the section, creep in concrete, and axial loading history. Comparison with experimental results indicates reasonable agreement.

RESUME

Il est décrit un procédé pour déterminer les tensions et les déformations dans les piliers en béton armé dont il est pris en valeur le retrait inégal sur la section, le fluage dans le béton et les caractéristiques de la charge centrale. Une comparaison avec les résultats expérimentaux indique une concordance vague.

ZUSAMMENFASSUNG

Es wird ein Verfahren zur Bestimmung der Spannungen und Verformungen in symmetrischen Stahlbetonstützen gezeigt, welches ungleichmässiges Schwinden über dem Querschnitt, Kriechen im Beton und zentrische Belastungsgeschichte berücksichtigt. Vergleiche mit den Experimenten sind zufriedenstellend.

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