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Study on the Differential Shrinkage of Composite Prestressed Concrete Beam

Etude du retrait différentiel d'une poutre composite de béton précontraint

Untersuchung über das differentielle Schwinden von vorgespannten Betonverbundträgern

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I. Introduction

Composite prestressed concrete beam which consists of precast prestressed concrete girder and cast-in-place slab is a favourite structure from both economical and technologycal points of view. Many more advantages could be expected by using for the cast-in-place slab the artificial lightweight aggregate concrete having enough strength for structural use.

However, the secondary stress as well as the rotation of section arise in the beam due to the relative differences of shrinkage and creep characteristics between the two composed elements of the beam. The differential shrinkage may in some cases greatly affect the cracking strength and the warping of the composite structure. Further, the interface between the slab and the girder might form possible structural weakness.

This study was carried out especially to clarify the effects of differential shrinkage of the composite prestressed concrete beam having a rectangular or T-shaped cross section of which the upper or the flange portion was cast with the artificial lightweight aggregate concrete on the precast beam.

II. Analytical Method on Differential Shrinkage Stress

Internal stress induced by the differential shrinkage varies dependently upon the differences of mechanical properties between slab and girder concretes (modulus of elasticity, shrinkage and creep), the characteristics of cross section, the age of slab casting and so on. Many investigations have been made on the differential shrinkage effect and the analytical methods have been proposed using various assumptions, for example by Mörsh, Birkeland[4], Evans[5], Parker, Ozell[6], Branson[6], one of the authors[1) 2) 3)] and so on.

One of the authors analysed approximately the differential shrinkage stress considering the relaxation due to creep and the rotation of section of the precast prestressed girder caused by the eccentrical prestressing. Normal force N_1 and moment M_1 in the slab portion and N_2, M_2 in the precast girder are induced by the differential shrinkage as shown in Fig. 1.

Assuming that shrinkage develops with time similarly to creep factor, shrinkage strains are given as follows.

$$S_1 = \frac{S_{1n}}{\varphi_{1n}} \cdot \varphi_{1t} \qquad S_2 = \frac{S_{2n}}{\varphi_{2n}} \cdot \varphi_{2t} \; , \quad \Delta S_2 = \frac{\Delta S_{2n}}{\varphi_{2n}} \cdot \varphi_{2t} \qquad (1)$$

Where ΔS_2 shows the relative difference of contraction between the upper and the lower fibers of precast girder due to the eccentrical prestress.

Equilibrium of normal forces and moments are held because no external force is applied, so we can obtain the following equations.

$$N_1 = N_2 \qquad (2)$$

$$M_1 + M_2 = N_1 \cdot a \qquad (3)$$

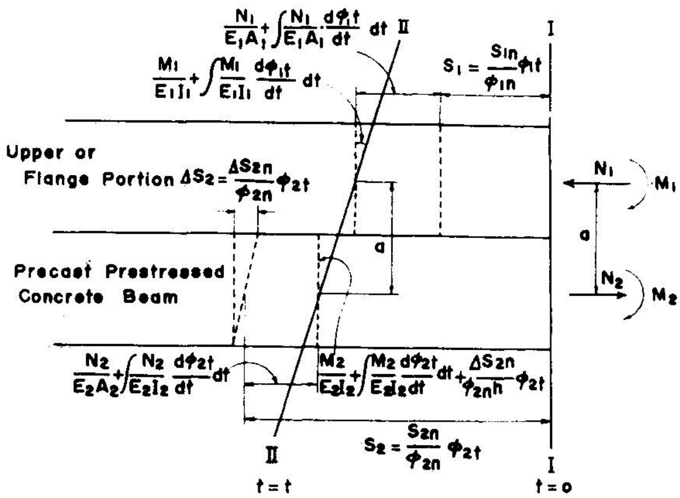

Fig. 1 Strains over Cross Section

Assuming that the creep develops with time according to the principles of Whitney and Davis-Granville and the plane of cross section remains plane also after the interaction between the slab and the girder is completed, the strains over the cross section consist of shrinkage, elastic and creep strains due to the normal forces and moments are given as shown in Fig. 1. Thus the next two equations are obtained.

$$\frac{M_1}{E_1 I_1} + \int_0^t \frac{M_1}{E_1 I_1} \frac{d\varphi_{1t}}{dt} dt = \frac{M_2}{E_2 I_2} + \int_0^t \frac{M_2}{E_2 I_2} \frac{d\varphi_{2t}}{dt} dt + \frac{\Delta S_{2n}}{\varphi_{2n} \cdot h} \cdot \varphi_{2t} \qquad (4)$$

$$\frac{S_{2n}}{\varphi_{2n}}\varphi_{2t}=\frac{S_{1n}}{\varphi_{1n}}\varphi_{1t}+\frac{N_1}{E_1A_1}+\int_0^t\frac{N_1}{E_1A_1}\frac{d\varphi_{1t}}{dt}dt+a\left(\frac{M_1}{E_1I_1}+\int_0^t\frac{M_1}{E_1I_1}\frac{d\varphi_{1t}}{dt}dt\right)$$
$$+\frac{N_2}{E_2A_2}+\int_0^t\frac{N_2}{E_2A_2}\frac{d\varphi_{2t}}{dt}dt \qquad (5)$$

Solutions of the above simultaneous integral equations are too complex for practical use, so the following approximate equation on creep strain can be utilized.

$$X_t+\int_0^t X_t\frac{d\varphi_t}{dt}dt \doteqdot X_t\left(1+\frac{1}{2}\varphi_t\right) \qquad (6)$$

Where X_t is elastic strain varying under creep phenomenon and φ_t is creep factor.

By substituting eq.(6) into eqs.(4) and (5), the unknown normal forces and moments N_1, N_2, M_1 and M_2 are obtained as follows.

$$M_1=\frac{\Delta_1}{\Delta}\ ,\quad M_2=\frac{\Delta_2}{\Delta}\ ,\quad N_1=N_2=\frac{1}{a}(M_1+M_2) \qquad (7)$$

where

$$\Delta=A_1B_2-A_2B_1\ ,\quad \Delta_2=F_1B_2-B_1F_2\ ,\quad \Delta_1=A_1F_2-F_1A_2$$
$$A_1=\mu\left(1+\frac{1}{2}\varphi_{2t}\right)+\left(1+\frac{1}{2}\varphi_{1t}\right)$$
$$B_1=\mu\left(1+\frac{1}{2}\varphi_{2t}\right)+\left(1+\frac{D_1a^2}{K_1}\right)\left(1+\frac{1}{2}\varphi_{1t}\right)$$
$$A_2=\lambda\left(1+\frac{1}{2}\varphi_{2t}\right)\ ,\quad B_2=-\left(1+\frac{1}{2}\varphi_{1t}\right)$$
$$F_1=(S_2-S_1)\,aD_1\ ,\quad F_2=-\left(\frac{\Delta S_2}{h}\right)K_1$$
$$\mu=\frac{D_1}{D_2}=\frac{E_1A_1}{E_2A_2}\ ,\quad \lambda=\frac{K_1}{K_2}=\frac{E_1I_1}{E_2I_2}$$

Thus, the stress over cross section due to differential shrinkage can be easily computed with the above N_1, N_2, M_1 and M_2.

Further investigation should be made to get more exact solution which takes into consideration the change of modulus of elasticity and creep factor with time, especially of the cast-in-place concrete. The analytical method proposed by the others are also based on a similar procedure, but the above method has characteristics of dealing with the rotation of section of precast prestressed concrete beam due to eccentric prestress just as considered by Evans[5] the influence of reinforcement in cast-in-situ slab on the differential shrinkage stress.

III. Test on the Composite Beams

(1) Materials used

The artificial lightweight aggregates of pelletized type were used in the concrete of upper or flange portion cast on the precast prestressed beam made with river sand and gravel concrete. The mechanical properties of concretes are listed in Table 1. The 14 mm and 16 mm prestressing bars are used in the precast portion of the rectangular and T-shaped composite beams, recpectively.

Table 1 Mechanical Properties of Concretes (at the Age of Loading Test)

Kind of Concrete \ Properties	Compressive Strength (kg/cm^2)	Splitting Strength (kg/cm^2)	Modulus of Rupture (kg/cm^2)	Modulus of Elasticity (kg/cm^2)	Age of the Concrete (weeks)
Lightweight Concrete	211	14.6	28.2	14.1×10^4	4
	196	14.0	26.1	15.0×10^4	5
Normal Concrete	374	25.8	48.4	35.6×10^4	9
	402	24.8	47.8	37.0×10^4	19

(2) Beam specimens

Two kinds of beam specimens were fabricated, one has rectangular cross section and the other T-shaped cross section, both as shown in Fig. 2.

Precast concrete beams were prestressed by the prestressing bars at the age of 3 weeks to about 100 kg/cm^2 at the lower fiber and zero at the upper fiber, and re-prestressed and grouted at the age of 4 weeks. Concrete of flange or upper portion was cast on the prestressed beam at 5 or 14 weeks. These specimens were cured in the laboratory till they were tested statically at the age of 9 or 19 weeks.

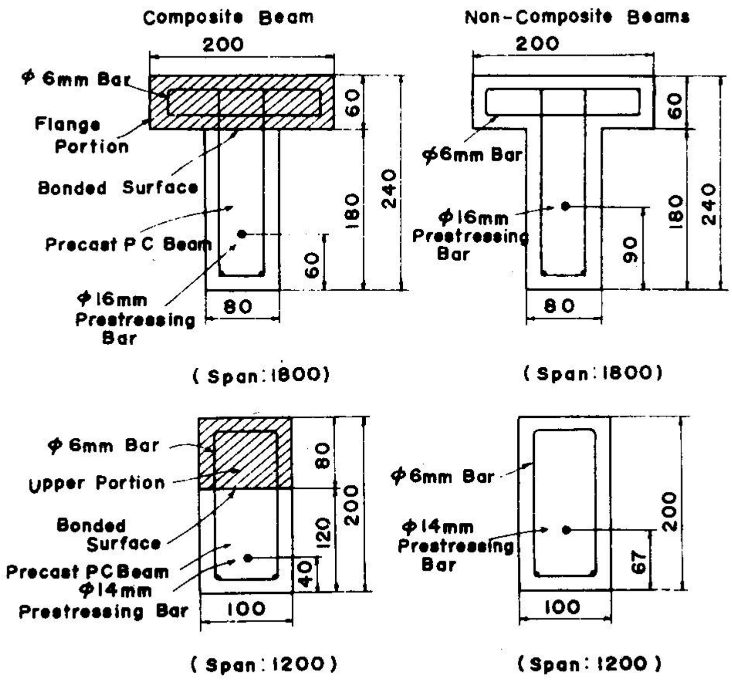

Fig. 2 Cross Section of Test Beams

Specimens used in this test have three variables, these are the treatment of the bonded surface, the ratio of shear span length to beam height and the age of flange casting. The former two variables are accommodated to investigate the strength of bonded surface and the contribution of shear connector to shear resistance, and the last one is concerned with the effect of differential shrinkage. Details of specimens are given in Table 2 and Fig. 2.

Table 2. Test Beams

		Kind of Beam	Treatment of* Bonded Surface	Age of Flange Casting (weeks)	Age of Test (weeks)	a/h Ratio	Notes
Rectangular Beams	Group I	R-HL-I-5-2.0	I	5	9	2.0	
		R-HL-II-5-2.0	II	5	9	2.0	
		R-HH-II-2.0	(II)		9	2.0	non-Compos.
		R-HL-II-5-2.5	II	5	9	2.5	
		R-HH-II-2.5	(II)		9	2.5	non-Compos.
	Group II	R-HL-II-14-2.0	II	14	19	2.0	
		R-HL-II-14-2.5	II	14	19	2.5	
T-shaped Beams	Group I	T-HL-I-5-2.0	I	5	9	2.0	
		T-HL-II-5-2.0	II	5	9	2.0	
		T-HL-III-5-2.0	III	5	9	2.0	
		T-HH-II-2.0	(II)		9	2.0	non-Compos.
		T-HL-II-5-3.0	II	5	9	3.0	
		T-HH-II-3.0	(II)		9	3.0	non-Compos.
	Group II	T-HL-II-14-2.0	II	14	19	2.0	
		T-HL-II-14-3.0	II	14	19	3.0	

* I: rough without shear connector
II: rough with shear connector (20 cm spacing)
III: rough with shear connector (10 cm spacing)

Control specimens were also made to measure the free shrinkage and creep strains of the concretes used. Ordinary prestressed concrete beams (non-composite beams) were made for comparison.

To make clear the effect of differential shrinkage on the cracking strength, cracking load of the beam was measured by using wire strain gages attached on the bottom of the beam.

(3) Test results and discussion

(a) Calculated differential shrinkage stresses in test beams

Differential shrinkage strains expected to occur in the test beams after flange casting were estimated from both control specimens as the difference between the drying shrinkage of the lightweight concrete beam and the total contraction induced by shrinkage and creep in the precast prestressed concrete beam. Thus, the differential shrinkage strain at the time of loading test was expected to be about 17×10^{-5} for the composite beams of which the flange portion were cast at 5 weeks and about 28×10^{-5} for the beams of 14 weeks flange casting.

By substituting the above differential shrinkage strain and other measured data into the analytical equations, the differential shrinkage stresses over the cross section of test beams can be calculated.

Fig. 3 shows these differential shrinkage stresses calculated in two ways, that is, (i) neglecting the relaxation due to creep and (ii) taking into account the relaxation due to creep as well as the rotation of the section of precast beam.

It is seen from Fig. 3 that considerably large stresses set up due to differential shrinkage especially in group II beams composited at 14 weeks.

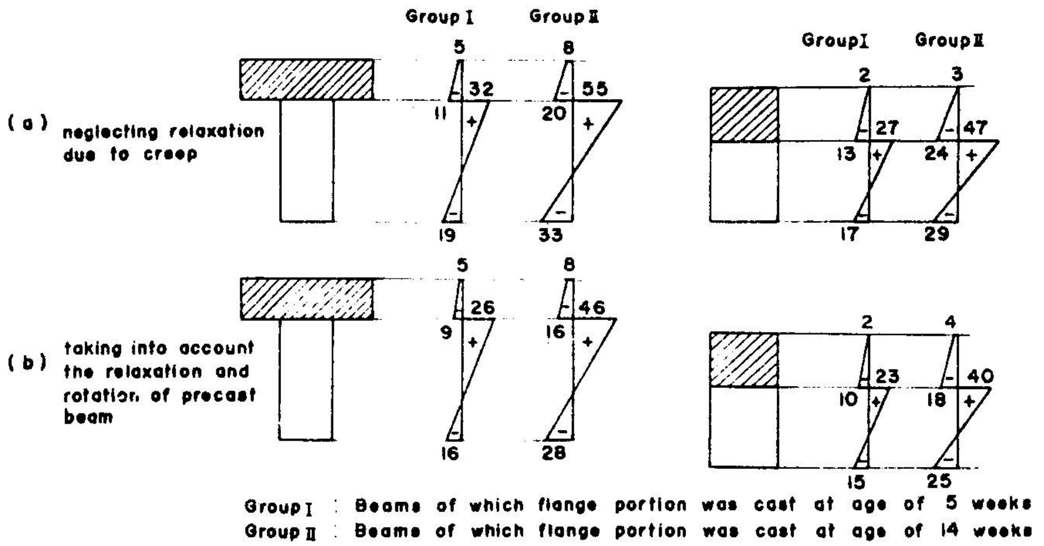

Fig. 3 Calculated Differential Shrinkage Stresses (kg/cm²)

These stresses are thought to be rather overestimated because the variations of modulus of elasticity and of initial creep development with time, especially of the cast-in-situ lightweight concrete are neglected in the above calculation.

(b) Cracking strength

In Table 3 the measured and calculated cracking moments (M_{cr}) of each beam are given. The cracking moment is computed by the following formula.

$$M_{cr} = W_c \cdot (\sigma_{cb} + \sigma_{ce}) \qquad (8)$$

The effective prestress σ_{ce} of the composite beam is estimated

Table 3. Cracking and Ultimate Moment of Test Beams
(unit: t·cm)

		Kind of Beam	Measured Cracking Moment M_{cr}	Calculated Cracking Moment M_1 (M_{cr}/M_1)	M_2 (M_{cr}/M_2)	M_3 (M_{cr}/M_3)	Measured Ultimate Moment	Calculated Ultimate Moment
Rectangular Beams	Group I	RH-HL-I-5-2.0	85.0	80.4 (1.06)	71.1 (1.20)	72.3 (1.18)	170.0	164.0 (1.04)
			80.0	80.4 (1.00)	71.1 (1.13)	72.3 (1.11)	160.0	164.0 (0.98)
		R-HL-II-5-2.0	90.0	84.7 (1.06)	75.6 (1.19)	76.8 (1.17)	170.0	147.0 (1.15)
			85.0	84.7 (1.00)	75.6 (1.12)	76.8 (1.10)	168.0	147.0 (1.14)
		R-HH-II-2.0	85.0	98.6 (0.86)	—	—	173.0	180.2 (0.95)
			80.0	98.6 (0.81)	—	—	159.0	180.2 (0.88)
		R-HL-II-5-2.5	75.0	81.1 (0.92)	72.0 (1.04)	73.2 (1.02)	160.0	160.6 (1.00)
			68.0	81.1 (0.84)	72.0 (0.94)	73.2 (0.93)	150.0	160.6 (0.94)
		R-HH-II-2.5	93.8	98.1 (0.96)	—	—	166.3	174.3 (0.95)
			87.5	98.1 (0.89)	—	—	175.0	174.3 (1.00)
	Group II	R-HL-II-14-2.0	60.0	78.5 (0.76)	62.6 (0.96)	65.0 (0.92)	130.0	139.2 (0.94)
			55.0	78.5 (0.70)	62.6 (0.88)	65.0 (0.86)	135.0	139.2 (0.97)
		R-HL-II-14-2.5	62.5	78.5 (0.80)	62.6 (1.00)	65.0 (0.96)	161.3	156.2 (1.03)
			56.3	78.5 (0.71)	62.6 (0.90)	65.0 (0.87)	180.0	156.2 (1.15)
T-shaped Beams	Group I	T-HL-I-5-2.0	114.0	120.6 (0.95)	105.0 (1.09)	107.5 (1.06)	296.4	338.0 (0.88)
			108.0	120.6 (0.90)	105.0 (1.03)	107.5 (1.00)	300.0	338.0 (0.89)
		T-HL-II-5-2.0	108.0	127.5 (0.85)	112.7 (0.96)	114.4 (0.94)	164.0	309.8 (0.85)
			108.0	127.5 (0.85)	112.7 (0.96)	114.4 (0.94)	285.6	309.8 (0.92)
		T-HL-III-5-2.0	120.0	126.5 (0.95)	110.9 (1.08)	113.5 (1.06)	322.8	313.8 (1.03)
			114.0	126.5 (0.90)	110.9 (1.03)	113.5 (1.00)	336.0	313.8 1.07)
		T-HH-II-2.0	132.0	142.6 (0.93)	—	—	333.6*	341.7 (0.98)
			138.0	142.6 (0.97)	—	—	350.4*	341.7 (1.03)
		T-HL-II-5-3.0	126.0	122.1 (1.03)	106.4 (1.18)	109.0 (1.15)	318.6	330.0 (0.97)
			108.0	122.1 (0.88)	106.4 (1.02)	109.0 (0.99)	324.0	330.0 (0.98)
		T-HH-II-3.0	117.0	141.8 (0.83)	—	—	324.0	318.0 (1.02)
			117.0	141.8 (0.83)	—	—	309.6	318.0 (0.97)
	Group II	T-HL-II-14-2.0	96.0	119.1 (0.81)	92.1 (1.04)	96.1 (1.00)	289.2	295.9 (0.98)
			84.0	119.1 (0.79)	92.1 (0.91)	96.1 (0.87)	278.4	295.9 (0.94)
		T-HL-II-14-3.0	99.0	119.1 (0.84)	92.1 (1.08)	96.1 (1.03)	345.6	324.8 (1.06)
			90.0	119.1 (0.76)	92.1 (0.98)	96.1 (0.94)	324.0	324.8 (1.00)

in the following three ways:

(i) (σ_{ce}'); Considering only the loss of prestress due to shrinkage and creep in the precast prestressed beam and neglecting the effects of differential shrinkage between the slab and girder concretes.

(ii) (σ_{ce}'', σ_{ce}'''); In addition to (σ_{ce}'), considering the differential shrinkage stresses in two ways as described before and corresponding to Fig. 3(a) and (b), respectively.

Thus, three kinds of cracking moment M_1, M_2 and M_3 are calculated each corresponding to the effective prestress σ_{ce}', σ_{ce}'' and σ_{ce}'''.

Table 3 shows that the measured cracking moments M_{cr} of the beams composited at the age of 14 weeks are about 0.7 times as large as those of the similar beams composited at 5 weeks. Comparison M_{cr} with M_1 gives that M_{cr} is much smaller than M_1 on the beams composited at 14 weeks. This fact shows that the differential shrinkage stress, which is neglected in calculating M_1, has a large influence on the cracking moment of the composite beam, especially when the flange portion was cast at later age. The calculated moments M_2 and M_3 taking into account the differential shrinkage show fairly good agreement with the measured moments M_{cr}.

(c) Ultimate strength

All the test beams failed in flexure except the non-composite T-shaped beams which were loaded with 2.0 a/h ratio. The ultimate flexural moments of test beams are shown in Table 3 as well as the calculated ones. The calculated ultimate flexural moments of the composite beams are computed by the same method as is used for ordinary prestressed concrete beams.

In Table 3 the measured ultimate moments show good agreement with the calculated ones. Standing on another view point, this means that the ultimate flexural moments are little affected by the differential shrinkage stresses.

(d) Other results

Other results concerning the differential shrinkage are as follows.

(i) No harmful slip was found in the flexural test even of the beams which have small a/h ratio and even no shear connector. And all the composite beams used in this test failed in flexure, neither in slip of bonded surface nor in shear.

(ii) As far as differential shrinkage was concerned, lightweight concrete appears to be more advantageous because of its small modulus of elasticity.

IV. Conclusions

The results obtained from this test are summarized as follows.

(i) Differential shrinkage may considerably affect the cracking strength and the warping of the composite prestressed concrete beams especially when the flange portion is cast at later age. The ultimate flexural strength, however, is little affected by differential shrinkage.

(ii) The cracking moment calculated by the method taking into account the differential shrinkage stress as described in this paper shows good agreement with the measurement.

(iii) It appears more favourable to use the lightweight concrete in flange portion of the composite beam in respect to reducing the differential shrinkage effects because of its small modulus of elasticity.

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Notation (suffix 1,2 indicate slab portion and prestressed concrete beam, respectively)

M_1, M_2 : internal moment induced by differential shrinkage

N_1, N_2 : internal normal force induced by differential shrinkage

S_1, S_2 : free shrinkage strain after flange casting

ΔS_2 : relative difference of shrinkage after flange casting between the upper fiber and the lower fiber of precast girder induced by eccentrical prestressing

$\varphi_{1t}, \varphi_{2t}$: creep factor at the age "t" after flange casting

$\varphi_{1n}, \varphi_{2n}$: final creep factor

E_1, E_2 : modulus of elasticity

A_1, A_2 : area

I_1, I_2 : moment of inertia

a : distance from the centroid of slab portion to that of precast concrete girder

h : height of precast concrete girder

M_{cr} : cracking moment

W_c : tranceformed modulus of section of composite beam

σ_{cb} : flexural strength of precast concrete

σ_{ce} : effective prestress in lower fiber of the beam at the age of loading test

Summary

Experimental studies were made on the effect of differential shrinkage in composite concrete beam of rectangular or T-shaped section, the flange or the upper half portion of which was placed with lightweight concrete on the precast prestressed beam made with normal weight concrete.

The effect of differential shrinkage set up in the composite beam on its cracking and ultimate strength were investigated and compared with the proposed analytical method.

Résumé

Une série d'études expérimentales a été effectuée sur les effets du retrait différentiel dans une poutre composite de forme rectangulaire ou de forme en T. Le hourdis ou la moitié supérieure de la poutre en T était placé, au moyen de béton léger, sur la poutre préfabriquée faite de béton précontraint de poids normal.

Les effets du retrait différentiel constatés par l'apparition de fissures et la résistance limite de rupture ont été étudiés et comparés avec ceux de la méthode analytique proposée.

Zusammenfassung

Es sind experimentelle Untersuchungen angestellt worden über die Wirkung des differentiellen Schwindens in Verbundträgern rechteckigen oder T-förmigen Querschnittes. Der Flansch oder die obere Hälfte des Balkens wurden aus Leichtbeton hergestellt, die auf dem vorgespannten, normalgewichtigen Betonbalken ruhten.

Die Wirkung des differentiellen Schwindens im Verbundträger auf Riss- und Bruchfestigkeit ist untersucht und mit den vorgeschlagenen theoretischen Methoden verglichen worden.