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Autor: Okada, Kiyoshi / Koyanagi, Wataru / Yoshioka, Yasuhiko

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Study on the Differential Shrinkage of Composite Prestressed Concrete Beam

Etude du retrait différentiel d'une poutre composite de béton précontraint
Untersuchung über das differentielle Schwinden von vorgespannten Betonverbundträgern

KIYOSHI OKADA Professor Kyoto University WATARU KOYANAGI
Associate Professor
Kyoto University

YASUHIKO YOSHIOKA Research Assistant Kyoto University

I. Introduction

Composite prestressed concrete beam which consists of precast prestressed concrete girder and cast-in-place slab is a favourite structure from both economical and technologycal points of view. Many more advantages could be expected by using for the cast-in-place slab the artificial lightweight aggregate concrete having enough strength for structural use.

However, the secondary stress as well as the rotation of section arise in the beam due to the relative differences of shrinkage and creep characteristics between the two composed elements of the beam. The differential shrinkage may in some cases greatly affect the cracking strength and the warping of the composite structure. Further, the interface between the slab and the girder might form possible structural weakness.

This study was carried out especially to clarify the effects of differential shrinkage of the composite prestressed concrete beam having a rectangular or T-shaped cross section of which the upper or the flange portion was cast with the artificial light-weight aggregate concrete on the precast beam.

II. Analytical Method on Differential Shrinkage Stress
Internal stress induced by the differential shrinkage varies
dependently upon the differences of mechanical properties between

slab and girder concretes (modulus of elasticity, shrinkage and creep), the characteristics of cross section, the age of slab casting and so on. Many investigations have been made on the differential shrinkage effect and the analytical methods have been proposed using various assumptions, for example by Mörsh, Birkeland, Evans, Parker, Ozell, Branson, one of the authors and so on.

One of the authors analysed approximately the differential shrinkage stress considering the relaxation due to creep and the rotation of section of the precast prestressed girder caused by the eccentrical prestressing. Normal force N_{ℓ} and moment M_{ℓ} in the slab portion and N_{2} , M_{2} in the precast girder are induced by the differential shrinkage as shown in Fig. 1.

Assuming that shrinkage develops with time similarly to creep factor, shrinkage strains are given as follows.

$$S_{i} = \frac{S_{in}}{\varphi_{in}} \cdot \varphi_{it} \qquad S_{2} = \frac{S_{2n}}{\varphi_{2n}} \cdot \varphi_{2t} \quad , \quad \Delta S_{2} = \frac{\Delta S_{2n}}{\varphi_{2n}} \cdot \varphi_{2t}$$
 (1)

Where ΔS_2 shows the relative difference of contraction between the upper and the lower fibers of precast girder due to the eccentrical prestress.

Equilibrium of normal forces and moments are held because no external force is applied, so we can obtain the following equations.

$$N_1 = N_2 \tag{2}$$

 $\mathcal{M}_{l} + \mathcal{M}_{2} = \mathcal{N}_{l} \cdot \alpha$ (3) Assuming that the creep develops with time according to the principles of Whitney and Davis-Granville

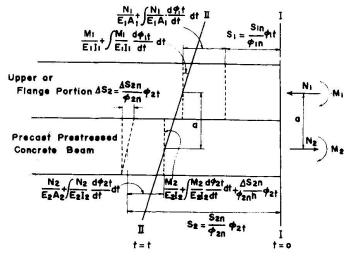


Fig. 1 Strains over Cross Section

and the plane of cross section remains plane also after the interaction between the slab and the girder is completed, the strains over the cross section consist of shrinkage, elastic and creep strains due to the normal forces and moments are given as shown in Fig. 1. Thus the next two equations are obtained.

$$\frac{M_i}{E_i I_i} + \int_0^t \frac{M_i}{E_i I_i} \frac{d\mathcal{G}_{it}}{dt} dt = \frac{M_2}{E_2 I_2} + \int_0^t \frac{M_2}{E_2 I_2} \frac{d\mathcal{G}_{2t}}{dt} dt + \frac{\Delta S_{2n}}{\mathcal{G}_{2n} h} \mathcal{G}_{2t}$$
(4)

$$\frac{S_{2h}}{\varphi_{2h}}\varphi_{2t} = \frac{S_{in}}{\varphi_{ih}}\varphi_{it} + \frac{N_{i}}{E_{i}A_{i}} + \int_{o}^{t} \frac{N_{i}}{E_{i}A_{i}} \frac{d\varphi_{it}}{dt}dt + a\left(\frac{M_{i}}{E_{i}I_{i}} + \int_{o}^{t} \frac{M_{i}}{E_{i}I_{i}} \frac{d\varphi_{it}}{dt}dt\right) + \frac{N_{2}}{E_{2}A_{2}} + \int_{o}^{t} \frac{N_{2}}{E_{2}A_{2}} \frac{d\varphi_{2t}}{dt}dt$$

$$(5)$$

Solutions of the above simultaneous integral equations are too complex for practical use, so the following approximate equation on creep strain can be utilized.

$$X_{t} + \int_{1}^{t} X_{t} \frac{d\varphi_{t}}{dt} dt = X_{t} \left(1 + \frac{1}{2} \varphi_{t} \right) \tag{6}$$

Where X_t is elastic strain varying under creep phenomenon and \mathcal{G}_t is creep factor.

By substituting eq.(6) into eqs.(4) and (5), the unknown normal forces and moments N_1 , N_2 , M_1 and M_2 are obtained as follows.

$$M_1 = \frac{\Delta_1}{\Delta}$$
 , $M_2 = \frac{\Delta_2}{\Delta}$, $N_1 = N_2 = \frac{1}{a}(M_1 + M_2)$ (7)

where

$$\Delta = A_{1}B_{2} - A_{2}B_{1} , \quad \Delta_{2} = F_{1}B_{2} - B_{1}F_{2} , \quad \Delta_{1} = A_{1}F_{2} - F_{1}A_{2}$$

$$A_{1} = M(1 + \frac{1}{2}\mathcal{G}_{2t}) + (1 + \frac{1}{2}\mathcal{G}_{1t})$$

$$B_{1} = M(1 + \frac{1}{2}\mathcal{G}_{2t}) + (1 + \frac{D_{1}\alpha^{2}}{K_{1}})(1 + \frac{1}{2}\mathcal{G}_{1t})$$

$$A_{2} = \lambda(1 + \frac{1}{2}\mathcal{G}_{2t}) , \quad B_{2} = -(1 + \frac{1}{2}\mathcal{G}_{1t})$$

$$F_{1} = (S_{2} - S_{1})\alpha D_{1} , \quad F_{2} = -(\frac{\Delta S_{2}}{h})K_{1}$$

$$\mu = \frac{D_{1}}{D_{2}} = \frac{E_{1}A_{1}}{E_{2}A_{2}} , \quad \lambda = \frac{K_{1}}{K_{2}} = \frac{E_{1}I_{1}}{E_{2}I_{2}}$$

Thus, the stress over cross section due to differential shrinkage can be easily computed with the above \mathcal{N}_{l} , \mathcal{N}_{2} , \mathcal{M}_{l} and \mathcal{M}_{2} .

Further investigation should be made to get more exact solution which takes into consideration the change of modulus of elasticity and creep factor with time, especially of the cast-in-place concrete. The analytical method proposed by the others are also based on a similar procedure, but the above method has characteristics of dealing with the rotation of section of precast prestressed concrete beam due to eccentric prestress just as considered by Evans⁵⁾ the influence of reinforcement in cast-in-situ slab on the

differential shrinkage stress.

III. Test on the Composite Beams

(1) Materials used

The artificial lightweight aggregates of pelletized type were used in the concrete of upper or flange portion cast on the precast prestressed beam made with river sand and gravel concrete. The mechanical properties of concretes are listed in Table 1. The 14 mm and 16 mm prestressing bars are used in the precast portion of the rectangular and T-shaped composite beams, recpectively.

Properties Kind of Concrete	Compressive Strength (kg/cm²)	Splitting Strength (kg/cm ²)	Modulus of Rupture (kg/cm²)	Modulus of Elasticity (kg/cm²)	Age of the Concrete (weeks)
Lightweight	211	14.6	28.2	14.1×10 ⁴	4
Concrete	196	14.0	26.1	15.0×104	5
Normal Concrete	374	25.8	48.4	35.6×10 ⁴	9
Normal Concrete	402	24.8	47.8	37.0×10 ⁴	19

Table 1 Mechanical Properties of Concretes (at the Age of Loading Test)

(2) Beam specimens

Two kinds of beam specimens were fabricated, one has rectangular cross section and the other T-shaped cross section, both as shown in Fig. 2.

Precast concrete
beams were prestressed
by the prestressing bars
at the age of 3 weeks to
about 100 kg/cm² at the
lower fiber and zero at
the upper fiber, and represtressed and grouted
at the age of 4 weeks.
Concrete of flange or
upper portion was cast
on the prestressed beam
at 5 or 14 weeks.
These specimens were

cured in the laboratory

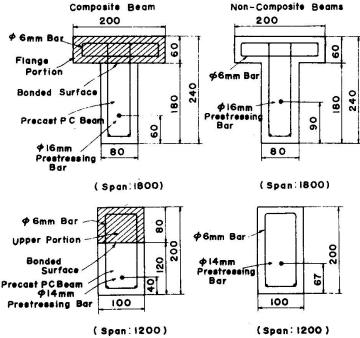


Fig. 2 Cross Section of Test Beams

till they were tested statically at the age of 9 or 19 weeks.

Specimens used in this test have three variables, these are

the treatment of the bonded surface, the ratio of shear span length to beam height and the age of flange casting. The former two variables are accommodated to investigate the strength of bonded surface and the contribution of shear connector to shear resistance, and the last one is concerned with the effect of differential shrinkage. Details of specimens are given in Table 2 and Fig. 2.

		Kind of Beam	Treatment of * Bonded Surface	Age of Flange Casting (weeks)	Age of Test (weeks)	a/h Ratio	Notes
Rectangular Beams	Group I	R-HL-1-5-2.0	1	5	9	2.0	
		R-HL-11-5-2.0	. 11	5	9	2.0	
		R-HH-II-2.0	(II)		9	2.0	non-Compos.
		R-HL-II-5-2.5	II	5	9	2.5	
	Group II	R-HH-II-2.5	(II)	AL 1 (AL 1010ACC)	9	2.5	non-Compos.
		R-HL-II-14-2.0	11	14	19	2.0	
		R-HL-II-14-2.5	11	14	19	2.5	
T-shaped Beams		T-HL-I-5-2.0	1	5	9	2.0	
		T-HL-11-5-2.0	II	5	9	2.0	
	P I	T-HL-111-5-2.0	III	5	9	2.0	American so was a
	Group	Т-НН-11-2.0	(II)		9	2.0	non-Compos.
		T-HL-11-5-3.0	11	5	9	3.0	
		Т-НН-11-3.0	(11)		9	3.0	non-Compos.
	II d	T-HL-II-14-2.0	11	14	19	2.0	
	Group II	T-HL-II-14-3.0	II	14	19	3.0	<u> </u>

Table 2 Test Beams

Control specimens were also made to measure the free shrinkage and creep strains of the concretes used. Ordinary prestressed concrete beams (non-composite beams) were made for comparison.

To make clear the effect of differential shrinkage on the cracking strength, cracking load of the beam was measured by using wire strain gages attached on the bottom of the beam.

- (3) Test results and discussion
- (a) Calculated differential shrinkage stresses in test beams

Differential shrinkage strains expected to occur in the test beams after flange casting were estimated from both control specimens as the difference between the drying shrinkage of the light-weight concrete beam and the total contraction induced by shrinkage and creep in the precast prestressed concrete beam. Thus, the differential shrinkage strain at the time of loading test was ex-

^{* 1:} rough without shear connector

II: rough with shear connector (20 cm spacing)

III: rough with shear connector (10 cm spacing)

pected to be about $17x10^{-5}$ for the composite beams of which the flange portion were cast at 5 weeks and about $28x10^{-5}$ for the beams of 14 weeks flange casting.

By substituting the above differential shrinkage strain and other measured data into the analytical equations, the differential shrinkage stresses over the cross section of test beams can be calculated.

Fig. 3 shows these differential shrinkage stresses calculated in two ways, that is, (i) neglecting the relaxation due to creep and (ii) taking into account the relaxation due to creep as well as the rotation of the section of precast beam.

It is seen from Fig. 3 that considerably large stresses set up due to differential shrinkage especially in group II beams composited at 14 weeks.

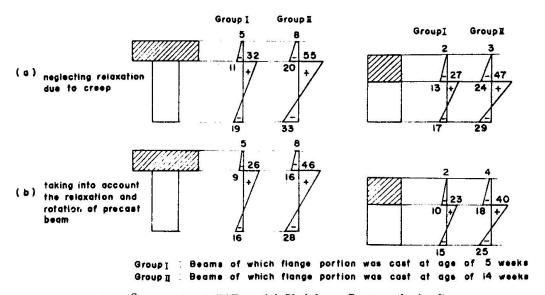


Fig. 3 Calculated Differential Shrinkage Stresses (kg/cm²)

These stresses are thought to be rather overestimated because the variations of modulus of elasticity and of initial creep development with time, especially of the cast-in-situ lightweight concrete are neglected in the above calculation.

(b) Cracking strength

In Table 3 the measured and calculated cracking moments (\mathcal{M}_{cr}) of each beam are given. The cracking moment is computed by the following formula.

$$M_{cr} = W_c \cdot (O_{cb} + O_{ce})$$
 (8)

The effective prestress Occof the composite beam is estimated

	1	W 1 0 D	Measured	Calculated Cracking Moment			Measured	Calculated	
i		Kind of Beam	Cracking Moment <i>Mer</i>	$M_1 (M_{cr}/M_1)$ $M_2 (M_{cr}/M_2)$ $M_3 (M_{cr}/M_3)$		$M_3 (M_{cr}/M_3)$	Moment_	Ultimate Moment	
		RH-HL-1-5-2.0	85.0	80.4 (1.06)	71.1 (1.20)	72.3 (1.18)	170.0	164.0	(1.04
			80.0	80.4 (1.00)	71.1 (1.13)	72.3 (1.11)	160.0	164.0	(0.98
		R-HL-II-5-2.0	90.0	84.7 (1.06)	75.6 (1.19)	76.8 (1.17)	170.0	147.0	(1.15
İ.			85.0	84.7 (1.00)	75.6 (1.12)	76.8 (1.10)	168.0	147.0	(1.14
. 1 .	d d	R-HH-11-2.0	85.0	98.6 (0.86)		_	173.0	180.2	(0.95
	Group		80.0	98.6 (0.81)		_	159.0	180.2	88.0
9		R-HL-II-5-2.5	75.0	81.1 (0.92)	72.0 (1.04)	73.2 (1.02)	160.0	160.6	(1.00
0			68.0	81.1 (0.84)	72.0 (0.94)	73.2 (0.93)	150.0	160.6	(0.94
		R-HH-II-2.5	93.8	98.1 (0.96)	_		166.3	174.3	(0.9
			87.5	98.1 (0.89)			175.0	174.3	(1.0
		R-HL-II-14-2.0	60.0	78.5 (0.76)	62.6 (0.96)	65.0 (0.92)	130.0	139.2	(0.9
1	D I		55.0	78.5 (0.70)	62.6 (0.88)	65.0 (0.86)	135.0	139.2	(0.9
١,	Group	R-HL-II-14-2.5	62.5	78.5 (0.80)	62.6 (1.00)	65.0 (0.96)	161.3	156.2	(1.0
`	۱ ٔ		56.3	78.5 (0.71)	62.6 (0.90)	65.0 (0.87)	180.0	156.2	(1.1
j		T-HL-I-5-2.0	114.0	120.6 (0.95)	105.0 (1.09)	107.5 (1.06)	296.4	338.0	(0.8
			108.0	120.6 (0.90)	105.0 (1.03)	107.5 (1.00)	300.0	338.0	8.0)
		T-HL-II-5-2.0	108.0	127.5 (0.85)	112.7 (0.96)	114.4 (0.94)	164.0	309.8	(0.8
			108.0	127.5 (0.85)	112.7 (0.96)	114.4 (0.94)	285.6	309.8	(0.9
Group I		m	120.0	126.5 (0.95)	110.9 (1.08)	113.5 (1.06)	322.8	313.8	(1.0
	T-HL-III-5-2.0	114.0	126.5 (0.90)	110.9 (1.03)	113.5 (1.00)	336.0	313.8	1.0	
	20	Т-НН-11-2.0	132.0	142.6 (0.93)		_	333.6*	341.7	(0.9
`			138.0	142.6 (0.97)		_	350.4*	341.7	(1.0
	ļ	T-HL-II-5-3,0	126.0	122.1 (1.03)	106.4 (1.18)	109.0 (1.15)	318.6	330.0	(0.9
ł			108.0	122.1 (0.88)	106.4 (1.02)	109.0 (0.99)	324.0	330.0	(0.9
		Т-НН-11-3.0	117.0	141.8 (0.83)		_	324.0	318.0	(1.0
			117.0	141.8 (0.83)	-	_	309.6	318.0	(0.9
		T-HL-II-14-2.0	96.0	119,1 (0,81)	92.1 (1.04)	96.1 (1.00)	289.2	295.9	(0.9
1	- I		84.0	119.1 (0.79)	92.1 (0.91)	96.1 (0.87)	278.4	295.9	(0.9
	Group	T-HL-II-14-3.0	99.0	119.1 (0.84)	92.1 (1.08)	96.1 (1.03)	345.6	324.8	(1.0
	-		90.0	119.1 (0.76)	92.1 (0.98)	96.1 (0.94)	324.0	324.8	(1.0

Table 3. Cracking and Ultimate Moment of Test Beams (unit: t*cm)

in the following three ways:

- (i) (); Considering only the loss of prestress due to shrinkage and creep in the precast prestressed beam and neglecting the effects of differential shrinkage between the slab and girder concretes.
- (ii) (O_{ce}, O_{ce}) ; In addition to (O_{ce}) , considering the differential shrinkage stresses in two ways as described before and corresponding to Fig. 3(a) and (b), respectively.

Thus, three kinds of cracking moment \mathcal{M}_1 , \mathcal{M}_2 and \mathcal{M}_3 are calculated each corresponding to the effective prestress $\widetilde{\mathcal{O}_{ce}}$, $\widetilde{\mathcal{O}_{ce}}$ and $\widetilde{\mathcal{O}_{ce}}$.

Table 3 shows that the measured cracking moments \mathcal{M}_{cr} of the beams composited at the age of 14 weeks are about 0.7 times as large as those of the similar beams composited at 5 weeks. Comparison \mathcal{M}_{cr} with \mathcal{M}_{i} gives that \mathcal{M}_{cr} is much smaller than \mathcal{M}_{i} on the beams composited at 14 weeks. This fact shows that the differential shrinkage stress, which is neglected in calculating \mathcal{M}_{i} , has a large influence on the cracking moment of the composite beam, especially when the flange portion was cast at later age. The calculated moments \mathcal{M}_{2} and \mathcal{M}_{3} taking into account the differential shrinkage show fairly good agreement with the measured moments \mathcal{M}_{cr} . (c) Ultimate strength

All the test beams failed in flexure except the non-composite T-shaped beams which were loaded with 2.0 a/h ratio. The ultimate flexural moments of test beams are shown in Table 3 as well as the calculated ones. The calculated ultimate flexural moments of the composite beams are computed by the same method as is used for ordinary prestressed concrete beams.

In Table 3 the measured ultimate moments show good agreement with the calculated ones. Standing on another view point, this means that the ultimate flexural moments are little affected by the differential shrinkage stresses.

(d) Other results

Other results concerning the differential shrinkage are as follows.

- (i) No harmful slip was found in the flexural test even of the beams which have small a/h ratio and even no shear connector.

 And all the composite beams used in this test failed in flexure, neither in slip of bonded surface nor in shear.
- (ii) As far as differential shrinkage was concerned, lightweight concrete appears to be more advantageous because of its small modulus of elasticity.

IV. Conclusions

The results obtained from this test are summarized as follows.

(i) Differential shrinkage may considerably affect the cracking strength and the warping of the composite prestressed concrete beams especially when the flange portion is cast at later age. The ultimate flexural strength, however, is little affected by differential shrinkage.

- (ii) The cracking moment calculated by the method taking into account the differential shrinkage stress as described in this paper shows good agreement with the measurement.
- (iii) It appears more favourable to use the lightweight concrete in flange portion of the composite beam in respect to reducing the differential shrinkage effects because of its small modulus of elasticity.

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Notation (suffix 1,2 indicate slab portion and prestressed concrete beam, respectively)

 M_1 , M_2 : internal moment induced by differential shrinkage

 N_1 , N_2 : internal normal force induced by differential shrinkage

 S_1 , S_2 : free shrinkage strain after flange casting

 ΔS_2 : relative difference of shrinkage after flange casting between the upper fiber and the lower fiber of precast girder induced by eccentrical prestressing

 \mathcal{G}_{n} , \mathcal{G}_{n} : creep factor at the age "t" after flange casting

 φ_{in} , φ_{2n} : final creep factor

 E_1 , E_2 : modulus of elasticity

 A_1 , A_2 : area

 I_1 , I_2 : moment of inertia

a : distance from the centroid of slab portion to that of precast concrete girder

h : height of precast concrete girder

Mcr: cracking moment

Wc : tranceformed modulus of section of composite beam

Och : flexural strength of precast concrete

Oce : effective prestress in lower fiber of the beam at the

age of loading test

Summary

Experimental studies were made on the effect of differential shrinkage in composite concrete beam of rectangular or T-shaped section, the flange or the upper half portion of which was placed with lightweight concrete on the precast prestressed beam made with normal weight concrete.

The effect of differential shrinkage set up in the composite beam on its cracking and ultimate strength were investigated and compared with the proposed analytical method.

Résumé

Une série d'études expérimentales a été effectuée sur les effets du retrait différentiel dans une poutre composite de forme rectangulaire ou de forme en T. Le hourdis ou la moitié supérieure de la poutre en T était placé, au moyen de béton léger, sur la poutre préfabriquée faite de béton précontraint de poids normal.

Les effets du retrait différentiel constatés par l'apparition de fissures et la résistance limite de rupture ont été étudiés et comparés avec ceux de la méthode analytique proposée.

Zusammenfassung

Es sind experimentelle Untersuchungen angestellt worden über die Wirkung des differentiellen Schwindens in Verbund-trägern rechteckigen oder T-förmigen Querschnittes. Der Flansch oder die obere Hälfte des Balkens wurden aus Leichtbeton hergestellt, die auf dem vorgespannten, normalgewichtigen Betonbalken ruhten.

Die Wirkung des differentiellen Schwindens im Verbundträger auf Riss- und Bruchfestigkeit ist untersucht und mit den vorgeschlagenen theoretischen Methoden verglichen worden.