

**Zeitschrift:** IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

**Band:** 5 (1970)

**Artikel:** Long-term deflections of cantilever prestressed concrete bridges

**Autor:** Keijer, Ulf

**DOI:** <https://doi.org/10.5169/seals-6904>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 23.02.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## Long-Term Deflections of Cantilever Prestressed Concrete Bridges

Des déformations verticales progressives des ponts en béton  
précontraint construit en encorbellement

Langzeitdurchbiegungen der Freivorbaubrücken

ULF KEIJER

Lic. Tech.

Royal Institute of Technology  
Stockholm, Sweden

During the last fifteen years a number of prestressed cantilever bridges have been constructed in Sweden with main spans up to 134 m. Fig.1 shows a bridge of this type with four spans. The bridges have rigid connections between the piers and the superstructure as shown in the figure, or they have a superstructure supported by bearings on the top of the piers. The bridges have boxsection, in general with two webs. Most of these bridges have shown larger timedependent vertical deflections than predicted at the time of construction. Further it does not seem as if these deflections would reach a stable final value even after a period of several years, e.g. one of these bridges has shown a total deflection in the middle of the 106,5 m main span of about 8 cm during a 12-year period, starting two years after the completion of the bridge.

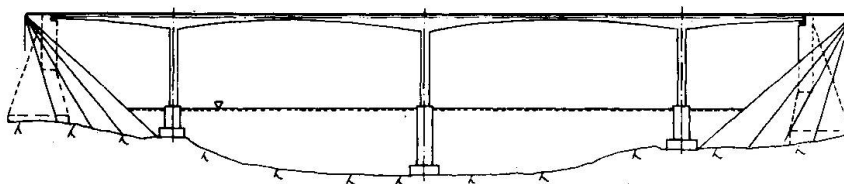


Fig.1

Four bridges of this type have been investigated. The purpose of the investigation has been to study a model which could give a satisfactory agreement between predicted and measured timedependent deflections. Values of the observed vertical deflections of the bridges during the years have been obtained from the National Swedish Road Board. The bridges have no transverse prestressing steel and the Dywidag

prestressing system was used.

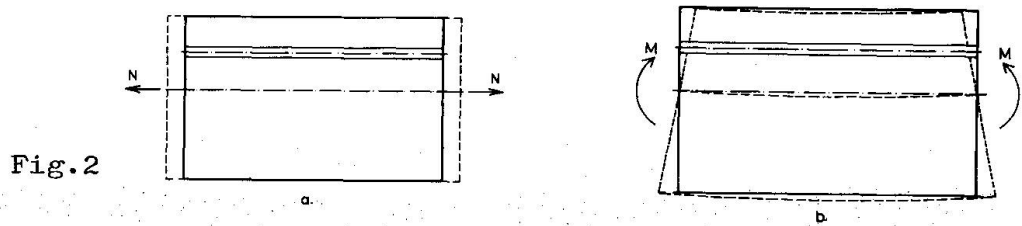
This type of bridge is built in the following way. After the construction of the foundations and the piers, the cantilevers are cast symmetrically out from the tops of the piers. The construction of a cantilever proceeds step by step using a trolley carrying the formwork. In each step the cantilever is prestressed by a number of prestressing bars. After completion of the construction the trolley is removed and two adjacent cantilevers are connected by a hinge in the midspan. Finally the wearing coat is laid on the entire bridge deck.

By this method the concrete elements in a cantilever will have different ages and will get their final loads in many steps. Besides, the loss of prestress due to creep and shrinkage of the concrete and the relaxation of the prestressing steel will change the forces on the different elements.

The principle of superposition, Mc Henry 1943, was adopted for concrete creep. As pointed out, Illston 1968 and others, this principle does not give a perfect agreement between calculated and measured values. However, at present it is the best available method to describe the influence on creep of concrete elements subjected to a varying force, CEB 1969.

Fig.2 shows an idealized prestressed concrete member, an element of a bridge cantilever, subjected to a) a normal force and b) a bending moment. Resulting stresses are obtained from Navier's formula. The influence of mild steel is neglected. Instant deformations are calculated according to Hooke's law as indicated in the figure. The concrete creep is assumed to be proportional to the applied stress and the shrinkage to be uniform over the entire concrete section. The creep and the shrinkage during a chosen time interval at the upper and the lower surfaces of the cross-section as well as at the level of the center of the prestressing steel are calculated. Adding the influence of the deformation of the concrete to the relaxation of the prestressing steel the magnitude of the total loss of prestressing during the actual time interval is obtained. The loss will affect the deformation of the member. Step by step during the time of construction this member will be subjected to varying forces and moments as the new elements of the cantilever are cast and prestressed and as the formwork trolley is moved forward. Even after the completion of the bridge the loss of prestress due to concrete creep continues. Each of these changes of the force on the member contributes to the long-term deformation according

to the principle of superposition. Finally, from the deformations of all the elements of the cantilever the vertical deflection of the latter is obtained.



In order to calculate the instant and the time-dependent deflections of a cantilever due to the different loading stages and to the rheological behavior of the constituent materials, i.e. concrete and prestressing steel, a computer programme was written. The programme analyses the applied forces and moments on the elements of which one is cast in each step during the construction of the cantilever. Instant stresses and deformations are evaluated. The concrete creep and shrinkage and the relaxation of the prestressed steel are calculated for the actual time interval. The calculation can proceed for the number of steps of time required. The programme was written in PL/I for a IBM 360/75.

Using the programme an extensive parameter variation was done. The influence of different creep and shrinkage coefficients and different values of the modulus of elasticity of concrete was investigated. So were different strain-time curves describing concrete creep and shrinkage. Similar calculations were done to study the influence of the modulus of elasticity and the relaxation of the prestressing steel. How different values of the density of concrete, the weight of the formwork trolley and the wearing coat as well as the initial prestressing force affect the long-term deflection of the cantilever was also investigated. Finally the influence of any changes of the time intervals between the start of construction of the cantilever, the removal of the formwork trolley and the laying of the wearing coat was studied.

The conclusions drawn from these calculations were that the creep coefficient and the shape of the concrete creep curve have major influence on the calculated long-term deflections from the time when the bridge is taken into traffic to an arbitrarily chosen point of time. The other parameters affect mainly the deformations during the period of construction.

It was also found that an ordinary creep curve with an upper bound

value, the creep coefficient, did not give values of the time dependent deflection that fitted the observations. Then the following creep function was assumed:

$$\varphi(t) = \varphi_0 [0.20 \cdot \ln(t) - 0.37] \quad (t \text{ in days})$$

The coefficient are chosen so that  $\varphi(1000) = \varphi_0$ . The function is assumed to be valid for  $t > 20$  days. Since the logarithm function has no upper bound value, no final creep coefficient will be obtained by this assumption. The value of  $\varphi_0$ , however, will determine the rate of creep at any moment, which is sufficient in this case.

The measured and calculated values of the vertical deflections in the outer ends of the cantilevers are given in fig:s 3-7. The influence of any vertical forces at the ends of the cantilevers has been neglected because in general the average of the deformations was taken of the two symmetrical cantilevers forming one complete bridge span. Further only the deformation of the very cantilever is regarded. The magnitude of the rotations at the supports is calculated on basis of levellings in the neighbourhood of the piers. The vertical deflections due to these rotations are then deducted from the total deflection, as well as the observed sinking of the tops of the piers. The resulting values are represented by small circles in the figures. The horizontal logarithmic time-axis starts from the date when the first observations of the deflection of the completed bridges were taken. The vertical axis represents the deflection. Every fifth cm is marked.

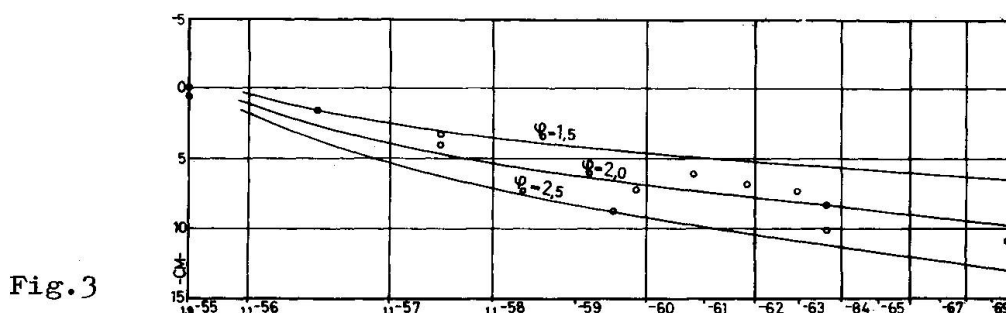


Fig.3

Fig.3 shows the results from the observations and calculations of the midspan of the Tunsta bridge. Its spans have the lengths of 41.1, 106.5 and 41.1 m. The observations form a rather irregular pattern but a value of  $\varphi_0 = 2.0$  gives a calculated deflection curve that on the average fits the observed values. Unfortunately no observations were taken during the years 1964-1968. The calculated deformations for  $\varphi_0 = 1.5$  and  $\varphi_0 = 2.5$  are shown as well.

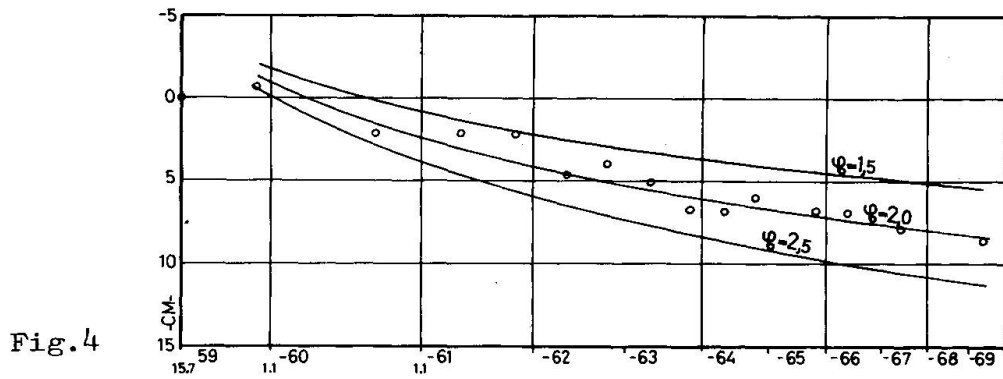


Fig. 4

Fig. 4 shows the corresponding results from the 55-94-55 m Stenungsund bridge. Also here  $\varphi_0 = 2.0$  leads to a good agreement between observed and calculated values. The observed deformation increases more slowly in the beginning than indicated by the calculations. This fact is probably due to the use of the principle of superposition. In the calculations the laying of the pavement influences the creep of concrete to a too large extent in the beginning, which is characteristic for the principle of superposition.

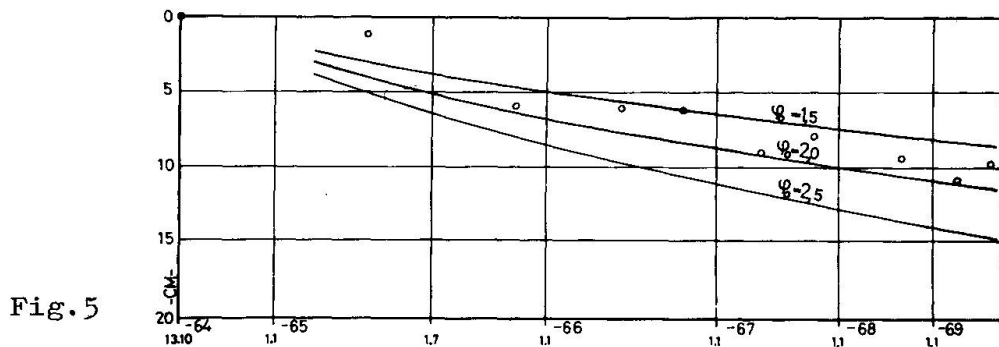


Fig. 5

Fig. 5 compares the results from the calculations and the observations of the 134 m midspan of the Alnö bridge. In this case a value of  $\varphi_0$  of about 1.8 gives a satisfactory agreement.

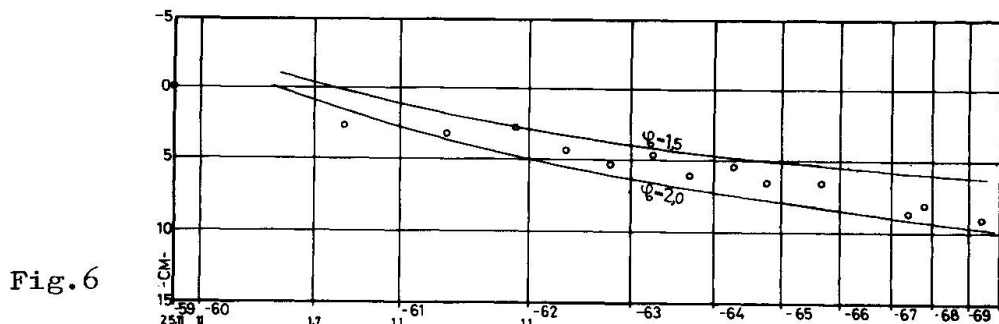
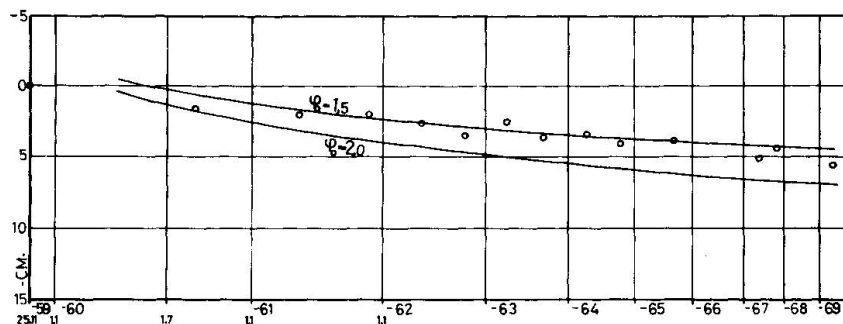


Fig. 6

Fig. 6 and fig. 7 show the results from the observations and the calculations of the two main spans of the Källösund bridge. The four spans have the following lengths: 50, 107, 107 and 50 m. Fig. 6 shows

the deformation of the two 57 m cantilevers built from the central pier. Fig.7 gives the corresponding values for the two adjacent 50 m cantilevers built from the lateral piers. The results indicate that a value  $\varphi_0 = 1.6$  à 1.7 gives satisfactory agreement between observed and calculated values for the investigated part of the bridge.

Fig.7



The differences between the obtained values of  $\varphi_0$  for the bridges studied above have not been fully interpreted, e.g. the influence of the temperature and other climatic conditions has not been considered. The irregularities of the observations of the deflection are probably partly due to different temperatures at the different times of observation. The Stenungsund and the Källösund bridges, are situated in the neighbourhood of each other. Therefore the difference of the obtained  $\varphi_0$ -values for these two bridges is hardly due to a different climate environment. The calculated concrete stress of the Stenungsund bridge is on the average about 20 percent higher than that of the Källösund bridge. If the stress-creep relation is not linear, which was assumed, the different  $\varphi_0$ -values of the two bridges might be partly explained.

### Conclusions

In the cantilever bridges of the investigated type with hinges in the centers of usually long spans, large vertical long-term deflections must be expected to occur. This investigation indicates that the concrete creep coefficient and the shape of the creep curve have dominating influence on the magnitude and development of the deflections. It was found that when a creep function containing a logarithm function is assumed good agreement between the observed and calculated values can be obtained.

### Acknowledgement

This work has been carried out at the Division of Structural Engineering and Bridge Building at the Royal Institute of Technology,

Stockholm. To the head of the division Professor Dr Tech. Georg Wästlund the author wishes to express his sincere gratitude for continuous support and interest during the course of the work. The work was sponsored by the Swedish National Road Board to which the author is very much obliged.

#### References

McHenry D, A New Aspect of Creep in Concrete and its Application to Design. ASTM Proceedings, Vol 43 pp.1069, 1943.  
Illston J M, Components of Creep in Mature Concrete. ACI Proceedings, Vol 65 pp.219, March 1968.  
CEB-FIP, Recommendations internationales pour le calcul et l'exécution des ouvrages en béton. Rédaction au 30 Novembre 1969, Paris 1969.

#### Summary

Four Bridges, cast in situ and prestressed stepwise by the cantilever method have been investigated. Long-term vertical deflections have been observed since the completion of the bridges. The structural system of the bridges was investigated including the rheological characteristics of the constituent materials. It was found from calculations by a computer that concrete creep have dominating influence on the deflections of this type of cantilever bridges. Satisfactory agreement between observed and calculated deflections is obtained when the concrete creep is assumed to follow a logarithm curve.

#### Résumé

Quatre ponts en béton précontraint construits en encorbellement sont étudiés. Des déformations verticales en fonction du temps sont observées depuis l'achèvement des ponts. Le système hyperstatique des ponts est étudié y compris les propriétés des matériaux du béton et de l'acier précontraint. Des calculs effectués sur ordinateur montraient que le fluage du béton a l'influence la plus prédominante sur la déformation des ponts de ce type. De bonnes concordances entre les calculs et les observations étaient obtenues en assimilant le fluage du béton à une courbe logarithmique.



### Zusammenfassung

Es wurden vier an Ort betonierte und stufenweise vorgespannte Freivorbaubrücken untersucht und nach ihrer Vollendung die Langzeitdurchbiegungen beobachtet. Ebenso ist das statische System der Brücken einschliesslich der rheologischen Eigenschaften des Betons und der Spannstähle untersucht worden. Berechnungen eines Elektronenrechners haben gezeigt, dass das Betonkriechen auf die Durchbiegungen dieser Freivorbaubrücken den Hauptanteil ausmacht. Zufriedenstellende Uebereinkunft ergab sich zwischen berechneten und beobachteten Durchbiegungen, sofern für den Kriechverlauf eine logarithmische Funktion angesetzt wurde.