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Autor: Ravindra, M.K. / Heaney, A.C. / Lind, N.C.

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II

Probabilistic Evaluation of Safety Factors

Evaluation probabilistique des coefficients de securité

Wahrscheinlichkeitstheoretische Auswertung der Sicherheitsfaktoren

M.K. RAVINDRA
Research Assistant
University of Waterloo
Canada

A.C. HEANEY
Research Assistant
University of Waterloo
Canada

N.C. LIND
Professor
University of Waterloo
Canada

INTRODUCTION

Over the last two decades, there has been an increased interest in the study of safety of structures from a probabilistic viewpoint. In these studies, two schools of thought can be identified: a classical probability analysis of the problem of safety exemplified in the works of Freudenthal [1], and an engineering approach to design codes, based on probabilistic concepts but aiming to maintain the simplicity of existing codes [2,3,4]. This paper pursues the latter approach.

It is recognized that the probability of failure of a structure is fundamental to a rational measure of the safety in view of the stochastic nature of resistance and load. The present state of knowledge permits ordinarily only an evaluation of the probability of failure of individual components (i.e. members) of a structure. The search for methods to calculate the probability of failure of structural systems remain an active field of research. In the spirit of the codes currently in use, this paper is concerned immediately with the design of individual components.

The load and resistance of a structure are functions of many stochastic variables. These variables are inter-related and their influence on the probability of failure is therefore very complex. Some design codes (e.g. the CEB Recommendations [5], attach partial safety factors on the effect of each specified variable. However, if the aim of a code is to achieve a constant probability of failure, it may not be valid <u>a priori</u> to assume that the effect of the stochastic variables can be separated; the partial safety factor would in general be mutually dependent. Therefore it would seem that a probabilistic design objective can only be achieved within a partial safety factor scheme at the expense of prohibitive complications in the expressions for the partial safety factors or, alternatively, by introducing coarse simplifications. Furthermore, the advantage of partial safety factors is partly lost [6] if they are selected arbitrarily.

Yet, the partial safety factor format remains attractive from a practical point of view, and it is worth the effort to examine how well it can be reconciled with the stochastic approach. The work reported in the following shows that it is always possible to derive a set of partial safety factors in such a way that consistency in the probability of failure is achieved with reasonable accuracy.

Following Cornell [2], the resistance R may be regarded as a product of three variables, M representing material strength, F representing fabrication and P representing the influence of professional assumptions, that is, the errors involved in the calculation of the resistance. For example, P includes variation within the limited discrete member sizes available, and accuracy of the formula for resistance used. The load S may be regarded [2] as a product of two variables: total load T and a factor E representing the uncertainty in engineering analysis of the evaluation of the load effect (for example; maximum moment) assuming that the actual loads were given.

Design then consists in the selection of 'characteristic values' of these five variables. The characteristic value of a load variable is the value at a specified number of standard deviations above the mean. This specified number may be called the 'characteristic coefficient' and is related to the probability of exceedance. Characteristic values of strength variables are defined in a corresponding manner, following established notions about strength and loads [5]. The ratio of the characteristic to the mean value of a variable is the corresponding central partial safety factor. Thus, it is seen that this partial safety factor for each variable depends only on its coefficient of variation and its characteristic coefficient.

This formulation permits selection of the coefficients of variation of the above variables, depending on experience and the particular design situation, in order to determine a set of partial safety factors.

DERIVATION OF PARTIAL SAFETY FACTORS

For the random variables, resistance R and load effect S (which may be an applied load, or applied moment, for example), with means $\bar{R}$ and $\bar{S}$, and coefficients of variation V_R and V_S, we may define the central safety factor θ as

$$\theta = \bar{R}/\bar{S} \tag{1}$$

Referring to Fig. 1, failure occurs when the resistance R is less than the applied load S, that is, when the stochastic variable (R - S), the safety margin, is less than zero.

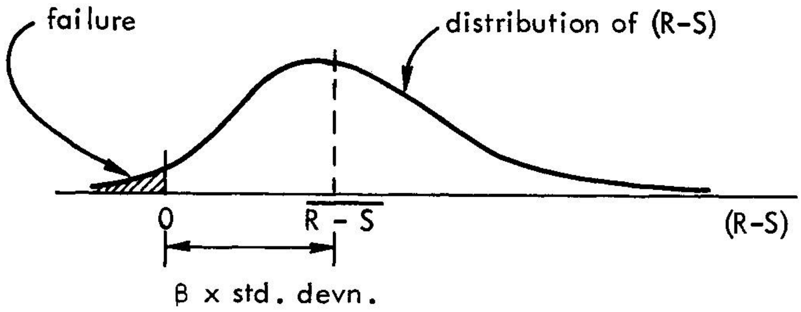

FIG 1
DEFINITION OF
SAFETY INDEX β

A measure of the degree of reliability β, called the 'safety index' is defined as the number of standard deviations of (R - S) between its mean value and zero. With a knowledge of the actual distributions of R and S, one can calculate the probability of failure of an element for any specified β. Thus:

$$\beta = \frac{\overline{R-S}}{\text{std. dev. }(R-S)} = \frac{\bar{R}-\bar{S}}{[(V_R\bar{R})^2 + (V_S\bar{S})^2]^{\frac{1}{2}}} = \frac{\theta - 1}{[\theta^2 V_R^2 + V_S^2]^{\frac{1}{2}}} \tag{2}$$

We now effect a linearization of the square root function, for any x and y, by introducing a function $\alpha = \alpha(x/y)$ defined by the relation:

$$(x^2 + y^2)^{\frac{1}{2}} = (x + y) \cdot \alpha(x/y) . \tag{3}$$

It is easily shown that α always lies between 0.707 and 1. Moreover, if x and y are roughly of the same magnitude, α is practically constant. For example, the assumption that α has a constant value of 0.75 would introduce a maximum error less than 10% for $0.25 < x/y < 4.0$.

By Eq. 2 and Eq. 3, the safety index is:

$$\beta = \frac{\theta - 1}{\alpha(\theta V_R + V_S)} \tag{4}$$

From Eq. 1 and 4, we get for the central safety factor:

$$\theta = \frac{1 + \alpha\beta V_S}{1 - \alpha\beta V_R} = \theta_R \theta_S \tag{5}$$

where $\theta_S \equiv 1 + \alpha\beta V_S$ is the partial safety factor on the load effect and $\theta_R \equiv (1 - \alpha\beta V_R)^{-1}$ is the partial safety factor on the resistance.

Now, θ_R can be separated into partial safety factors on the component variables M, F, and P as follows:

$$\theta_R = (1 - \alpha\beta V_R)^{-1}$$

$$= [1 - \alpha\beta(V_M^2 + V_F^2 + V_P^2)^{\frac{1}{2}}]^{-1} \tag{6}$$

By repeated use of Eq. 3, the partial safety factor on the resistance becomes:

$$\theta_R = [1 - \alpha\alpha_1\beta V_M - \alpha\alpha_1\alpha_2\beta V_F - \alpha\alpha_1\alpha_2\beta V_P]^{-1} \tag{7}$$

Factorizing, and keeping the term containing V_M independent of the other terms, we get:

$$\theta_R = [(1 - \alpha\alpha_1\beta V_M)(1 - \frac{\alpha\alpha_1\alpha_2\beta V_F}{1 - \alpha\alpha_1\beta V_M})(1 - \frac{\alpha\alpha_1\alpha_2\beta V_p}{1 - \alpha\alpha_1\beta V_M - \alpha\alpha_1\alpha_2\beta V_F})]^{-1}$$

$$= [(1 - \alpha\alpha_1\beta V_M)(1 - C_1\alpha\alpha_1\alpha_2\beta V_F)(1 - C_2\alpha\alpha_1\alpha_2\beta V_p)]^{-1}$$

$$= [(1 - K_M\beta V_M)(1 - K_F\beta V_F)(1 - K_p\beta V_p)]^{-1} = \theta_M\theta_F\theta_P, \tag{8}$$

where K_M, K_F, and K_p are functions of V_M, V_F, and V_p.

Each θ_i may be regarded as a partial safety factor on the variable i. It is shown below that the K_i are approximately constants, in the range of practical designs, so that each characteristic coefficient $(K_i\beta)$ varies predominantly with β only.

Similarly, the partial safety factor on the loads may be re-written:

$$\theta_S = (1 + K_T\beta V_T)(1 + K_E\beta V_E) = \theta_T\theta_E, \tag{9}$$

where K_T and K_E are functions of V_T and V_E and, as will be shown below, are practically constants. θ_T and θ_E are the partial safety factors on T and E respectively.

Furthermore, the effects of dead load and live load variations can be separated into individual partial safety factors, θ_D and θ_L respectively, which may be combined into the partial safety factor on total load, θ_T, by proportional addition as in the ACI 318-63 Code. Also, it can be shown that these additional partial safety factors depend only on the coefficients of variation of the loads.

Returning to Eq. 5, using Eq. 8 and 9, we get for the central safety factor

$$\theta = \theta_R \theta_S = \theta_M \theta_F \theta_P \theta_T \theta_E \tag{10}$$

CALIBRATION TO AN EXISTING CODE

The process of selecting appropriate values for the parameters in a code is called calibration [3]. A new code may be calibrated to an existing code so as to produce approximately the same member proportions as produced by current design, and, in the process, to produce approximately the same probability of failure, cost of failure, etc.

A convenient way to calibrate the proposed code format is first to calculate the implied value of β in the existing code by using a realistic set of $\{V\} = \{V_o\}$ of coefficients of variation of the variables M, F, P, T and E, and a calibration value of the central safety factor $\theta = \theta_o$.

With this value of β and for different combinations of the set $\{V\}$, the values of the set $\{K\} = \{K_M, K_F, K_P, K_T, K_E\}$ are calculated. The value of each K is approximately constant in the practical range of the set $\{V\}$ as shown (in the example for K_M) in Fig. 2. Accordingly, the uncertainty in the value of $\{V\}$ assumed in calibration to the existing code has very little influence on the resulting calibration, $\{K\} = \{K_o\}$

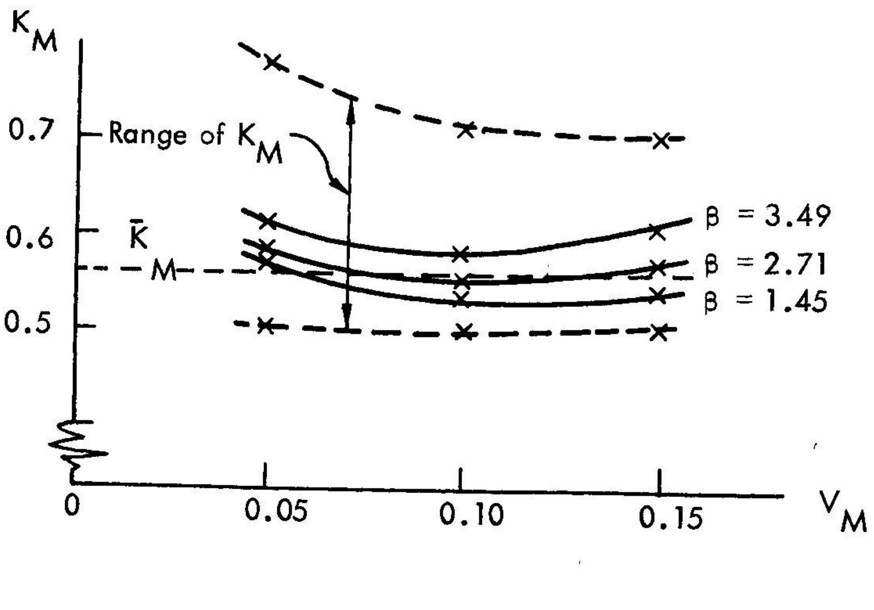

FIG 2.
VARIATION OF K_M WITH V_M.

Fig. 2 shows, furthermore, that the value of K_M as an approximation, can be replaced by a constant. In fact, the other functions in the set $\{K\}$ can similarly be assumed to be constant. Averaged over the domain of combinations of realistic values of the set $\{V\}$, we may put $\{K\} = \{0.56, 0.52, 0.58, 0.56, 0.50\}$. Moreover, we may simplify the results by inverting the expressions for θ_M, θ_F, and θ_P and neglecting terms of second and higher order. Finally we may even choose a global value, optimized over a realistic domain, of $K = 0.60$, say. Accordingly,

$$\theta_i = 1 + K\beta V_i; \quad i = M, F, P, T, E \tag{11}$$

can be used to calculate all partial safety factors for different conditions of materials, inspection etc.

The error in θ according to Eq. 10 arising from using Eq. 11, embodying all these approximations, rather than the correct expressions, Eq. 8 and Eq. 9, was determined using a digital computer over the unweighted practical ranges of the five coefficients of variations. Fig. 3 shows the distribution of this error.

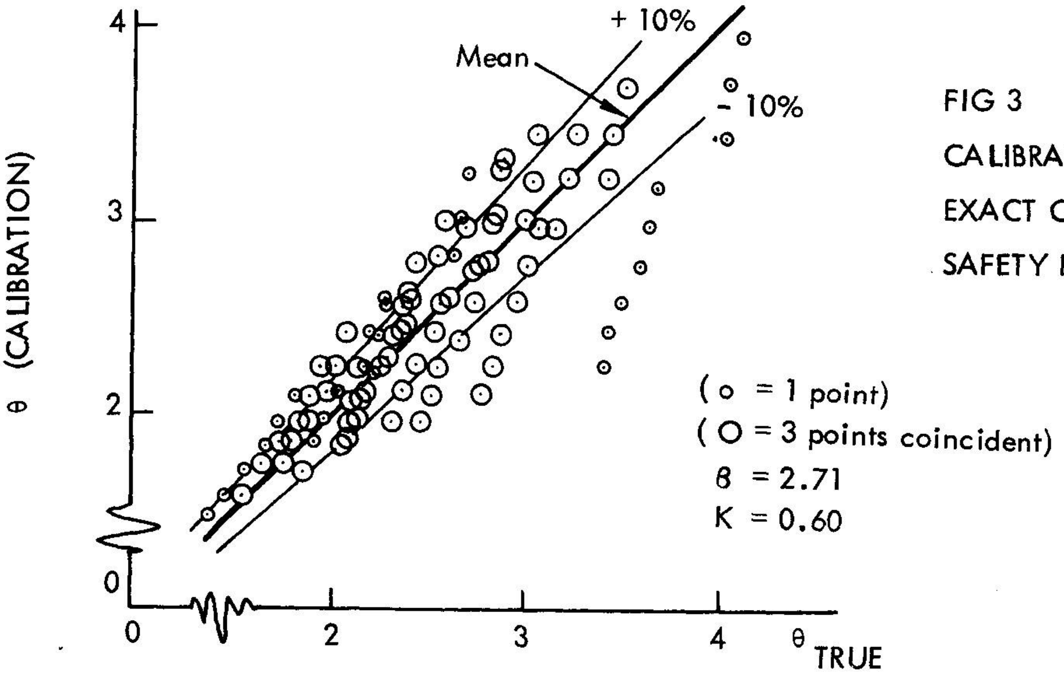

FIG 3
CALIBRATED VS
EXACT CENTRAL
SAFETY FACTOR

DESIGN PROCEDURE

In actual design the value of K as determined by the code authority could be given in the code and the designer might be free to select the set $\{V\}$ according to conditions. If the consequences of failure were particularly severe, a higher value for β would be specified. The central safety factor θ to be used would be calculated from Eq. 10 and 11.

Alternatively, the partial safety factors θ_i might be specified in the code in the manner similar to the C.E.B. Recommendations.

ILLUSTRATION

A partial safety factor code is to be calibrated to an existing code, (assumed to be National Building Code of Canada 1965 (7)).

As calibration point, we here select (somewhat arbitrarily, for the purpose of illustration only);

Office building	:	Nominal live load	= 50 psf.
Supported area	:	20 ft. span at 10 ft. c.c.	= 200 psf.
Dead load (6 in. slab, plus self weight, etc.)			= 80 psf.
Steel beams, simply supported $f_{all} = 0.6\, f_y$			= 21,900 psi.

Here, f_y is the mill test nominal minimum yield strength (for A36 steel). Actual yield strengths are assumed to have a mean of $\bar{f}_y = 36{,}000$ psi with a coefficient of variation for such beams equal to 12%, on the basis of tests (8) assumed to be relevant. The mean office live loading is assumed to be 25 psf.(9) and with a coefficient of variation equal to $c/A^{\frac{1}{2}} = 0.92$ for this particular area(10).

The central safety factor implied is therefore:

$$\theta = \bar{R}/\bar{S} = (80 + 50) \times \frac{20^2}{8} \times \frac{36{,}000}{21{,}900} / (80 + 25) \times \frac{20^2}{8} = 2.0$$

A realistic set of coefficients of variation is taken as:

$V_M = 0.12$, $V_F = 0.05$, (Good Control)

$V_P = 0.05$, (High accuracy), $V_L = 0.92$, $V_D = 0.05$, (Average)

$V_E = 0.10$ (Ordinary analysis).

Combining the loads, $T = L + D$, we get:

$$V_T = [(\bar{L}\ V_L)^2 + (\bar{D}\ V_D)^2]^{\frac{1}{2}} / (\bar{L} + \bar{D}) = [(25 \times 0.92)^2 + (80 \times 0.05)^2]^{\frac{1}{2}} / (25 + 80) = 0.22$$

Using these values, we calculate the coefficients of variation of the resistance and the load, respectively, as:

$$V_R = (V_M^2 + V_F^2 + V_P^2)^{\frac{1}{2}} = 0.14$$

$$V_S = (V_T^2 + V_E^2)^{\frac{1}{2}} = 0.24$$

By equation 2, the safety index β is equal to 2.71. The set $\{K\}$ in Eqs. 8 and 9 is found using this value of β. The result is:

$$\{K\} = \{K_M, K_F, K_p, K_T, K_E\} = \{0.52, 0.44, 0.47, 0.53, 0.41\} \tag{12}$$

The desired partial safety factor code should result in approximately the same safety level as in the existing code at the calibration point. Therefore, we select $\beta = 2.71$ for the new code. The code is to acknowledge the variability in all five variables as shown in Table 1, where the coefficient of variation of each condition is listed. For each of these conditions, the resulting partial safety factors from Eq. 8 and 9 range as shown in the Table. The values shown are the averages of the exact values for the entire domain of combinations of the coefficients of variation given in Table 1.

It can be seen that the proportioning of the safety margin between load and strength is quite different from that of the reference code.

TABLE 1

Partial safety factors derived for a safety index of 2.71.

RESISTANCE	Good Conditions	Average Conditions	Poor Conditions
Coefficient of variation	0.05	0.10	0.15
θ_M	1.09	1.17	1.29
θ_F	1.07	1.15	1.27
θ_P	1.08	1.18	1.33
LOAD	**low variability**	**average variability**	**high variability**
Coefficient of Variation	0.05	0.20	0.40
θ_T	1.08	1.31	1.65
STRUCTURAL ANALYSIS	**accurate**	**average**	**approximate**
Coefficient of Variation	0.05	0.10	0.15
θ_E	1.06	1.12	1.18

DISCUSSION

The performance of the partial safety factor code format suggested here, relative to the first order probabilistic code format can be judged from Fig. 3. Bearing in mind that the total cost of a structure near the optimum range is insensitive to the variations in the safety factor [11], most of the deviations are seen to be of no practical consequence. Moreover, practical limitations in feasible probabilistic codes, as reflected in the presence of the vague parameters V_F, V_P and V_E in Cornell's format [2], invalidate any attempts at increased accuracy at the expense of simplicity.

When the safety index β is reduced, the distribution narrows. For example, for β equal to 1.45 the ratio θ/θ_{true} is always between 0.97 and 1.10. Conversely, when it is attempted to raise the reliability level by increasing the safety index, the ratio θ/θ_{true} may be significantly below unity; but always for unreasonable combinations of the coefficients of variation.

The range of the ratio θ/θ_{true} can be reached considerably at several stages of the derivations, by optimization of the parameters; this is best done by an individual code committee after the operating range of the parameters and the calibration points have been carefully selected.

Figure 3 also reflects the variation in the actual central safety factor typically inherent in partial safety factor code formats. If fewer than five factors are used to represent the variation of design reality, greater error relative to the probabilistic ideal must occur.

It can be shown by partial differentiation of Eq. 2 that an error of 20% in either of the coefficients of variation of resistance or load, produces an error of approximately 10% in the calibrated value for β. Such an error in β would only alter the probability of failure a fraction of an order of magnitude [2]; this should be acceptable.

The value of the safety index β, that is, the ratio of the mean of safety margin to the standard deviation of $(R - S)$, is directly related to the probability of failure of the element. If the distributions for the variables M, F, P, T and E are given, the probability of failure is practically constant for all combinations of $\{V\}$, provided that the shape of the distribution of $(R - S)$ does not change significantly.

It is seen from Table I that in order to achieve a constant safety index under varying control conditions, a variable control safety factor is required; also from this table, it can be inferred -- and verified by calculation -- that the constant central safety factor computed using present deterministic procedures does not assure a constant level of safety.

The partial safety factors separate the effect of each stochastic factor, such that the individual influence of each variable can be directly appreciated as a valuable guide for decisions in design or research planning.

CONCLUSIONS

1. A first order probabilistic design, based on a consideration of the first and second moments of the stochastic variables in design can be made without introducing any new notions beyond that of the partial safety factor. In other words, a partial safety factor code can be derived, which may maintain the accepted concepts of deterministic design and which is also self-consistent in the probabilistic sense; that is, if achieves a sensibly constant probability of failure in all design situations.

2. It is possible effectively to separate the influence of the interdependent stochastic variables on the central safety factor, using a set of partial safety factors. These factors can be calculated by Eqs. 8 and 9. As in some present code formats, each of these partial safety factors is dependent on the coefficient of variation of the corresponding stochastic variable. However, the factors are not arbitrarily selected here and they are directly related to the safety index as defined in Eq. 2. A code committee can evaluate its code parameters and characteristic values from the derivation presented herein.

3. The results justify the common approach in code writing, whereby load criteria and strength criteria are separately prescribed -- often by separate code writing authorities. In contrast to present codes, the central safety factor can be evaluated explicitly even when the statistical data are limited.

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SUMMARY

A set of partial safety factors are derived from purely probabilistic concepts. In contrast to present codes, one may derive central safety factors for design which maintain a specified level of safety over a domain of the component variables. The analysis considers only the first and second moment of the distributions of the variables, thus not requiring the detail distribution to be specified.

Using these factors, one may evaluate, rationally, the 'characteristic values' and multiplicative, heretofore arbitrary, safety parameters.

RESUME

On dérive un ensemble de coefficients partiels de sécurité à l'aide de concepts probabilistiques. On peut aller plus loin que les normes actuelles et dériver des facteurs centraux de sécurité pour des calculs qui exigent un niveau donné de sécurité sur un domaine des variables. L'analyse ne considère que les premiers et seconds moments des distributions des variables stochastiques; ainsi il n'est pas nécessaire de spécifier la forme exacte de la distribution.

L'utilisation de ces facteurs permet d'évaluer d'une manière rationnelle les valeurs caractéristiques et multiplicatives des coefficients partiels de sécurité, qui étaient jusqu'à maintenant arbitraires.

ZUSAMMENFASSUNG

Ein Satz von Teilsicherheitsfaktoren wird aus der reinen Wahrscheinlichkeitslehre abgeleitet. Heutigen Vorschriften entgegen kann man zentrale Sicherheitsfaktoren für eine vorgeschriebene Sicherheitshöhe über einem Bereich der unabhängigen Zufallsvariablen auswerten. Die Berechnung zieht nur die ersten und zweiten Momente der Zufallsvariablen in Betracht, wobei die Verteilungsart unbekannt sein kann. Mit diesen Faktoren kann man auf einfache Weise die "charakteristischen Werte" und die multiplikativen, bisher beliebigen Sicherheitsbeiwerte schätzen.