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# VI

## DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

### Structural Safety Specifications Based on Second-Moment Reliability Analysis

Spécifications de la sécurité des structures basées sur l'analyse des moments de deuxième ordre

Bauwerkssicherheit mittels einer auf den zweiten Momenten beruhenden Wahrscheinlichkeitsrechnung

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The primary aim of the symposium and of Themes VI and VII in particular is to develop theoretically sound safety specification procedures that remain practical. Semi-probabilistic codes are admittedly only partially successful, because they lack the dependence on analytical probability theory necessary to promote consistency and interpretation of the means and the ends of structural codes. The purpose of this discussion is to demonstrate that a variety of simple code formats, including some identical to present proposals, can be developed in a manner rigorously consistent with a probability theory. To be sure, certain analytical and algebraic approximations are adopted in order to achieve the desired simplicity of final code form, but the approximations, first, are believed to be reasonable and, second, are made in a clear way that opens them to analytical, quantitative study and to informed criticism. Alternatively, since it is demonstrated what approximations in the theory are necessary to achieve the simple code forms in use today, these present codes can be evaluated with respect to these implicit approximations. The results presented here should prove useful in guiding the discussion, interpretation, and selection of numerical values of factors in presently used and proposed codes, as well as in pointing towards systematic improvements in codes.

Second-Moment Reliability; The probability theory upon which the developments here are based is in itself an approximate theory(1). It is a first-order theory only; it is based on only the mean values and second moments of random variables. The latter moment is a first-order measure of uncertainty. In functional relationships among random variables the theory retains only the first-order or linear terms in the random components, i.e., in the deviations from the mean.

For example, the force capacity, W, of a tensile bar is uncertain if the area, A, and yield stress, Y, are both uncertain. In this theory the uncertainty in W would be measured solely through its variance (not its entire probability distribution). The mean and variance of W would be found from those of A and Y using a first-order expansion about their means

$$W = AY \doteq m_A\, m_Y + m_A(Y-m_Y) + m_Y(A-m_A)$$

in which m denotes the mean and $\doteq$ denotes equality to first-order approximation. Then applying the expectation and variance operation

$$m_W \doteq m_A\, m_Y \tag{1}$$

$$\sigma_W^2 \doteq m_A^2\, \sigma_Y^2 + m_Y^2\, \sigma_A^2 + 2\, m_A\, m_Y\, \rho_{A,Y}\, \sigma_Y \sigma_A \tag{2}$$

in which $\sigma^2$ denotes variance and $\rho$ correlation coefficient. The "exact" result for the mean of W is

$$m_W = m_A\, m_Y (1 + \rho_{A,Y}\, V_A\, V_Y) \tag{3}$$

in which V denotes the coefficient of variation or $\sigma/m$. If A and Y are uncorrelated, the exact result for $V_W^2$ is $V_A^2 + V_Y^2 + V_A^2\, V_Y^2$, which should be compared to $V_A^2 + V_Y^2$, the result implicit in the equations above. Clearly the first-order approximations may not be accurate if the coefficients of variation are large. It is important to note that no assumptions (in particular, no Gaussian assumptions) have been made about the distributions of the variables.

The total is a self-consistent, distribution-free theory of uncertainty that is very easy to apply to practical engineering problems and that provides accuracy which may be sufficient for many real problems, in particular in normative (or prescriptive) engineering specifications and codes of practice.

Although the theory does not give a complete description of the uncertainty in any particular variable, it can be argued that it is as accurate as structural building applications can justify. Statistical data are, and perhaps always will be, insufficient to determine the distributions of material strengths, dimensions, and loads. In any case, the variables in conventional structural practice represent highly idealized strengths and loads (e.g., homogeneous material strengths, pseudo-static forces, and uniformly distributed floor loads). Adoption of a first-order probability theory as a basis of structural safety would perhaps be analogous to the long standing dependence of the profession on linear, elastic theory to predict forces and deformations in reinforced concrete structures; in both cases, the results are known to be approximations, but they are 1) simple, yet self-consistent, 2) an improvement upon the state-of-art prior to their adoption, and 3) capable of being systematically modified or replaced as the profession accumulates knowledge.

Characterization of Variables; Within this approach to structural safety and performance specification, a variable, X, is characterized by two numbers, a best estimate and an uncertainty measure. The former corresponds to an expected value or mean, $m_X$, the latter to a variance, $\sigma_X^2$, standard deviation, $\sigma_X$ or coefficient of variation, $V_X$. The expected value represents the profession's best prediction of a variable (e.g., material strength, member deflection, peak wind force). It is conducive to systematic progress of the profession that this best estimate, rather than a conservative estimate, be a product of any research investigation or committee report. The uncertainty value associated with a variable should be a measure of the various sources and kinds of uncertainty surrounding it. These include both "natural", inherent variablity (such as that observed in wind velocities and material strengths) and the uncertainty associated with the imperfect tools of the profession (simplifying assumptions, incomplete knowledge, human constructors, etc.).

The treatment of the latter sources of uncertainty probabilistically is a major distinction between this code proposal and most others. This procedure is, however, consistent with the most modern and most practical concept of probability[2,3,4], and it avoids difficult-to-reconcile distinctions among the interpretations, analyses, and code treatments of the various sources of uncertainty. For example, should the uncertainty in the initial shape of steel columns be considered as natural (or "random") variability and included in determining a nominal or "characteristic" strength for a member, or should it be accounted for in a multiplying strength reduction factor designed to account for fabrication variabilities? It is clearly in the latter factor in U.S. codes, whereas European tests(5) are aimed at placing it in the former value. When this is settled, what should be done with minor construction errors which cause accidental eccentricities in the applied load? Indeed, what virtually all codes accept as "inherent" or "random" variability in the strength of steel can be, on closer inspection, decomposed into a variety of identifiable sources which are in part systematic(6). Similarly, should that uncertainty in snow loads which is associated with its spacial disposition on a roof be considered as "random" or professional? Since the profession lacks methods of analyzing snow accumulation on irregular roof shapes, it has been proposed(7) to collect statistical data as if the problem were a "random" one. The distinctions in types of uncertainty are difficult, because they are not "basic" but dependent in part, it seems, on the data available at the moment and on the level of idealization in the conventional treatment of the phenomenon. In fact, these distinctions are unnecessary if all are treated as contributions to a total measure of uncertainty, denoted $\sigma$.

It is useful (but not technically necessary, as will be demonstrated) to define nominal or "characteristic values" of variables. For a material strength, the characteristic value in the CEB code(8) is one which a specified (large) fraction of all standard test results exceed. In a parallel way, in this first-order theory a characteristic strength, R*, would be

$$R^* = m_R - k_R\,\sigma_R = m_R(1 - k_R\,V_R) \qquad (4)$$

in which $k_R$ is a specified constant, the same for all materials, members, etc. A characteristic load or applied force, S*, is defined in an analogous way, $+k_S$ replacing $-k_R$. Because CEB code specifications present the formula for the characteristic strength in the same form as Eq. 4, it is important to point out the differences between the CEB and this proposal.

First, the CEB code (and others like it) set the characteristic value at a specified fractile of the distribution. Thus the factor $k_R$ must depend upon the shape of this distribution (and in some cases on m and $\sigma$ as well). For the levels of probabilities usually specified by present codes (1 to 10%), the value of $k_R$ is not too sensitive to the distribution, but, of course, the distributions of interest are not well established, and they probably change from place to place and time to time. This proposed code basis, being only first-order, does not attempt to distinguish between distribution shapes; $k_R$, not the probability level, is fixed by the code.

Second, the CEB code would base $\sigma_R$ solely on standard tests of standard materials specimens(8). In the proposed code, the interest is on strength in place. Therefore, $\sigma_R$ should include, in addition to the "inherent" dispersion observed in standard tests, the uncertainties associated with correlating these results to in-place strengths (e.g., construction versus laboratory practice, weather conditions, full-size member versus standard specimen, etc.). In short, in this proposal $\sigma_R$ should measure all the uncertainty that the engineer in fact faces when asked to predict the strength of the material in an actual beam to be built to his specifications.

Finally, the proposed code differs from the CEB in that it includes characteristic values for the strength of members (or assemblies, perhaps) and the force applied to members (in addition to the strength of materials and the environmental loads). It is member capacity and member force which ultimately determine safety. They depend, of course, on material strength and environmental loads, but only in part. The best prediction of and the uncertainty in the strength of a member depend upon the dimensions of the member and upon incompletely understood microscopic behavior of the material or materials of which it is constructed, as well as upon material strength. The first-order probability theory makes approximate analysis of these factors feasible.

For example, consider the moment resistance of the cross-section of a simple, rectangular reinforced concrete beam with width B, depth-to-steel D, concrete strength C, and total steel yield force T. Adopting conventional structural theory (nothing more can be justified in a code), the (under-reinforced) yield moment resistance, R, of the cross-section can be written

$$R = TD\left(1 - \eta \frac{T}{BDC}\right) \Delta \tag{5}$$

in which $\eta$ is a constant dependent upon the "theory" used, and $\Delta$ is a random factor introduced to describe the dispersion about the predicted resistance that is observed in laboratory test results in which the values of T, D, B, and C are known by relatively precise measurement; $\Delta$ is the (random) ratio of observed to predicted resistance. For an unbiased prediction formula, $m_\Delta = 1$. The uncertainty value $\sigma_\Delta^2$ is a measure of the accuracy of the prediction formula, or, in short, of the professional uncertainty inherent in the use of this theory.

Under first-order probability theory the mean and variance of the resistance become[8]

$$m_R \doteq m_T\, m_D \left(1 - \eta \frac{m_T}{m_B m_D m_C}\right) m_\Delta \tag{6}$$

$$\sigma_R^2 \doteq \sum_i \left(\frac{\partial R}{\partial X_i}\bigg|_{\underline{m}}\right)^2 \sigma_{X_i}^2 \tag{7}$$

in which the $X_i$ are T, D, B, C and $\Delta$, and the term in parentheses denotes the partial derivative of R with respect to a particular variable $X_i$, evaluated at the means of the variables. (It has been assumed that the $X_i$'s are uncorrelated.) Note that the uncertainty in each variable contributes to the uncertainty in R in a manner dependent both upon the uncertainty in that variable and upon the sensitivity of R to deviations in the variable.

The characteristic value of the member resistance is found by substituting into Eq. 4. Note that it is not simply the value of R obtained by substituting characteristic values of strengths, T* and C*, for T and C in Eq. 5, as is implicit in present codes.

The lack of sensitivity of resistance variables to certain factors can be exploited to simplify significantly the procedure above. It may be sufficient for most code purposes to assume in the computation of the uncertainty measure $V_R$ that the relationship between R and the other variables is of the form[10]

$$R = c\ M\ F\ P \tag{8}$$

in which c is a constant, M is a material variable, F is a fabrication-dependent member dimension variable, and P is a professional factor. (In the reinforced-concrete beam example above, T, D, and $\Delta$ can be associated with M, F, and P, respectively. The constant c is simply $1 - (\eta\, m_T)/m_B\, m_D\, m_C$.) In this case one obtains simply

$$V_R^2 \doteq V_M^2 + V_F^2 + V_P^2 \tag{9}$$

The uncertainty in R is made up of uncertainty in material strength, fabrication, and professional factors. If this simplification is adopted the characteristic resistance is simply

$$R^* = m_R\,(1 - k_R V_R) \doteq m_R\left(1 - k_R\sqrt{V_M^2 + V_F^2 + V_P^2}\right) \tag{10}$$

In the reinforced concrete example one obtains

$$R^* \doteq \left[ m_T m_D \left(1 - \eta \frac{m_T}{m_B m_D m_C}\right) m_\Delta \right] \left(1 - k_R \sqrt{V_M^2 + V_F^2 + V_P^2}\right) \qquad (11)$$

Since the coefficents of variation are probably rather insensitive to beam size and other factors, the last term might be the same for all beams throughout a design project, implying that the computation of theoretically consistent characteristic resistances of beams need be no more difficult than present computation of nominal resistances. Both involve calculation of the value of the first factor, that in square brackets. Note again, however, that in the proposed code mean values, not nominal values, are involved in these calculations. An implication is that a by-product of the calculation is the best professional prediction of the resistance of the cross-section, i.e., the first factor in Eq. 11; this best estimate is never obtained in present calculation procedures.

Similarly, characteristic loads and applied member forces can be determined as

$$S^* = m_S + k_S \sigma_S = m_S (1 + k_S V_S) \qquad (12)$$

The uncertainty measure in an applied member force should account for both the customary observed, inherent dispersion in environmental loads and the many professional uncertainties such as those involved in translating loads into member forces (i.e., in the structural analysis used), in approximating dynamic by static behavior, in idealizing spatial load variations, in predicting future changes in the loading environment, and in neglected (abnormal and unforeseen) loading combinations.

Again for many purposes it may be sufficient to assume in uncertainty calculations that the applied member force, S, is

$$S = c\,T\,E \qquad (13)$$

in which T is the environmental load or "field strength"(7) and E is a factor, perhaps with mean 1, reflecting professional engineering uncertainties. (The constant c is related to the structural analysis which translates load into applied force.) Then $V_S^2$ is simply $V_T^2 + V_E^2$, and the characteristic applied force is simply

$$S^* = m_S \left(1 + k_S \sqrt{V_T^2 + V_E^2}\right) \qquad (14)$$

$V_T$ can be obtained from load environment measurements and analysis, while $V_E$ must be judged, and/or obtained from calibration(10) of existing codes (although physical measurements of forces in full-scale structures subjected to known loads could provide partial information). It should be clear that how the uncertainty in S is proportioned between T and E will depend in part upon how the load is idealized (e.g., winds as pseudo-static gusts or as dynamic velocity time-histories), but that the net uncertainty in S may be unchanged. (It could be less, if the particular idealization is more accurate.)

Safety Specification Alternatives; Codes of practice must in some way cause the engineer to specify a structure which has a (best prediction of the) resistance sufficiently in excess of the (best prediction of the) applied force to insure adequate safety and performance without unduly penalizing the cost of the structure. This requirement can and has been effected in a variety of code "formats" (e.g., working stress, load factors, semi-probabilistic, etc.) In theoretical structural safety terms the purpose of the code is usually to promote a pre-determined level of reliability. In this section it will be demonstrated that this reliability requirement can be expressed in a variety of convenient code formats, all technically equivalent in that they will cause designers to specify the same mean resistance.

Satisfactory structural safety (or performance) will be achieved if the resistance, R, exceeds the applied force S, that is, if the safety margin M = R-S exceeds zero. The mean and variance of M are

$$m_M = m_R - m_S \tag{15}$$

$$\sigma^2{}_M = \sigma^2{}_R + \sigma^2{}_S \tag{16}$$

The reliability of the structure (or member) is defined as the probability that M exceeds zero. In terms of a first-order probability theory reliability is measured by the number of standard deviations $\sigma_M$ by which the mean $m_M$ exceeds zero[1,10]. Call this number $\beta$. The larger $\beta$, the more reliable the member. To impose a required reliability a code must require that

$$m_M \geqslant \beta\ \sigma_M \tag{17}$$

The appropriate value for $\beta$ is a matter of serious professional judgement. Values of about 4 have been found to be consistent with certain present codes[10,11]. The specified value of $\beta$ should be related to the consequences of the type of failure under consideration and to the marginal cost of increasing the resistance. Higher reliability, i.e., a higher value of $\beta$ should be required of sudden brittle shear failure modes than of ductile yielding, for example. If the mode of "failure" (or "limit state") under study is simply undesirable (but not unsafe) cracking or deflection, significantly lower values of $\beta$ (2 perhaps) are appropriate. Although it is not possible at higher values of $\beta$ to associate even approximate numerical probabilities to values of $\beta$, there is some justification[9] for assuming that at these levels an increase of $\beta$ by 1/2 implies about an order of magnitude decrease in the complement of the reliability, i.e., the probability of failure. The (desirable, but impractical) numerical evaluation of reliabilities discussed in theoretical reliability studies depends on precise knowledge of the probability distribution of M. This in turn depends on the distributions of R and S. These will probably never be known accurately, for they are affected by significant sources of professional uncertainty which are difficult to model and to measure.

This single technical safety requirement (17) can be expressed in a variety of alternate ways. It is valuable to display some of these formats and the approximations and assumptions necessary to achieve them. Direct substitution of Eq. 15 and 16 produces the safety requirement in the "safety margin form"

$$m_R \geqslant m_S + \beta\sqrt{\sigma_R^2 + \sigma_S^2} \tag{18}$$

Rearrangement leads to the "safety factor" form

$$m_R \geqslant \theta\ m_S \tag{19}$$

in which $\theta$ is a function of $V_R$, $V_S$, and $\beta$,

$$\theta \doteq (1 + \beta\sqrt{V_R^2 + V_S^2 - \beta^2\ V_R^2\ V_S^2}\ )/(1 - \beta^2 V_R{}^2) \tag{20}$$

It is shown in Figure 1 for $\beta$ = 4. Substitution of Eq. 10 and Eq. 12 for nominal values yields the "nominal safety factor" form

$$R^* \geqslant \theta^*\ S^* \tag{21}$$

in which $\theta^*$ is a function of $V_R$, $V_S$, $\beta$, $k_R$, and $k_S$:

$$\theta^* = \frac{1 - k_R\ V_R}{1 + k_S\ V_S}\ \theta \tag{22}$$

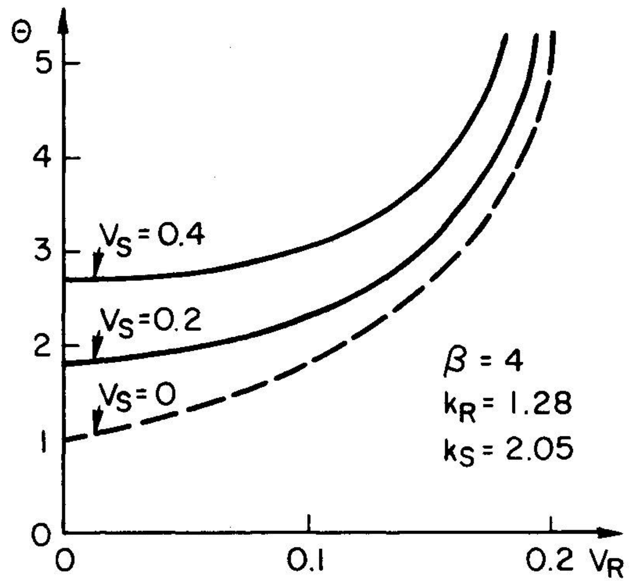


Figure 1

It is shown in Figure 2 for $\beta = 4$, $k_R = 1.28$, and $k_S = 2.05$. Notice that in the latter case $\theta^*$ is quite insensitive to $V_R$ and $V_S$ over the range of interest; this is an advantage of the nominal form. It is important to recognize that the values of $k_R$ and $k_S$ are, technically speaking, arbitrary. Increasing $k_R$ will reduce $R^*$ and reduce $\theta^*$ to compensate, leaving the required value of $m_R$ unchanged. The values of $k_R$ and $k_S$ can be chosen to satisfy legal problems surrounding quality assurance and liability, or to permit simplification of the code as will be discussed below.

The reliability theory adopted implies that you cannot obtain high reliability with highly uncertain resistances.

If $V_R \gg 1/\beta$, then it is not possible to obtain the desired level of reliability, since $\sigma_M$ (which is greater than or equal to $\sigma_R = V_R\, m_R$) will grow too fast with $m_R$ to permit $m_M$ ($= m_R - m_S$) to exceed $\beta\, \sigma_M$. At larger values of $m_R$, $V_R$ might very well be smaller, of course. On the other hand, a simplified practical code might simply overlook this problem by setting $\theta^*$ equal to an approximate, linearly increasing value for $V_R \gg 0.1$.

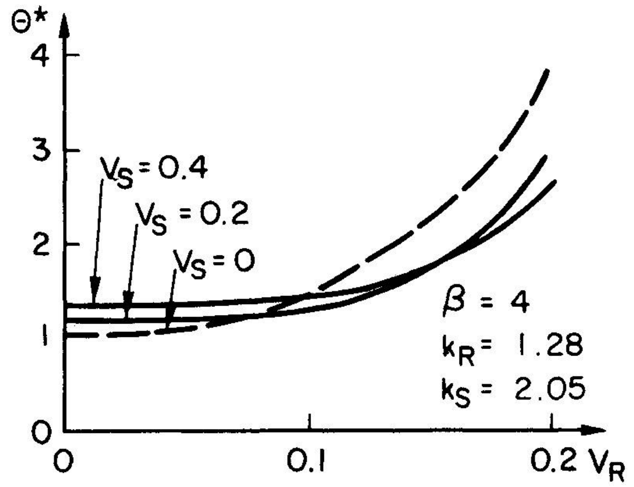


Figure 2

A variety of split factor code formats similar to the ACI or CEB forms are also possible. The possibility of decoupling these factors was recognized by N. C. Lind(11,12) who has defined and demonstrated the notion of "practical equivalence" of code formats. Lind showed that, with remarkable numerical accuracy,

$$\sqrt{\sigma_R^2 + \sigma_S^2} \cong \alpha\,(\sigma_R + \sigma_S) \qquad (23)$$

in which $\alpha$ is a constant chosen to fit the expected range of ratios of $\theta_R$ to $\sigma_S$. A value of $\alpha = 0.7$ will give errors of less than about 5% for $\sigma_R/\sigma_S$ in the entire range of 1/3 to 3. With this approximation Eq. 18 can be written in the "split form"

$$\phi\, m_R \gg \gamma_S\, m_S \qquad (24)$$

in which

$$\phi = 1 - \beta\alpha\, V_R \qquad \text{and} \qquad \gamma_S = 1 + \beta\alpha\, V_S \qquad (25)$$

Note that these strength reduction and load increase factors depend only on their corresponding uncertainty measures, $V_R$ and $V_S$, respectively, (and, of course, on the reliability level $\beta$). Note that, approximately, $\theta \cong \gamma_S/\phi$. A "nominal split form" can be written as

$$\phi^* R^* \geqslant \gamma_S^* S^* \tag{26}$$

in which

$$\phi^* = (1- \beta\alpha\, V_R)/(1- k_R V_R) \text{ and } \gamma_S^* = (1+ \beta\alpha\, V_S)/(1+ k_S V_S) \tag{27}$$

If combined loads are involved, then S is made up of the sum of two applied force effects, say, $S_D + S_L$, signifying dead and live loads. Assuming they are uncorrelated and using Eq. 23, $\sigma_S \cong \alpha(\sigma_{S_D} + \sigma_{S_L})$. Then, in Eq. 24, $\gamma_S\, m_S$ can be replaced by

$$\begin{aligned}\gamma_S m_S &= m_S\,(1+ \beta\alpha\, V_S) = m_S + \beta\alpha\, \sigma_S \cong m_{S_D} + m_{S_L} + \beta\alpha^2\sigma_{S_D} + \beta\alpha^2\sigma_{S_D} \\ &= m_{S_D}(1+ \beta\alpha^2\, V_{S_D}) + m_{S_L}(1+ \beta\alpha^2\, V_{S_L}) = m_{S_D}\,\gamma_D + m_{S_L}\,\gamma_L\end{aligned} \tag{28}$$

in which a dead load and a live load factor are defined, as indicated, in terms of $\beta$ and their respective uncertainty factors $V_{S_D}$ and $V_{S_L}$. This result is due to M. K. Ravindra of the University of Waterloo.

A "split load factor form" results,

$$\phi^* R^* \geqslant \gamma_D^* S_D^* + \gamma_L^* S_L^* \tag{29}$$

if characteristic live and dead loads are used and if nominal load factors are defined as, for example,

$$\gamma_D^* = (1+ \beta\alpha^2\, V_{S_D})/(1+ k_S\, V_{S_D}) \tag{30}$$

This factor will be relatively insensitive to $V_{S_D}$. Combined loads involving two or more transient loads(7) can be treated in a similar way provided the proper model is used. In coordination with the maximum wind load, for example, one should use the normal or "steady-state" live load moments, not those of the maximum (in time) live load.

Finally, it is useful to demonstrate that a consistent code format is possible which bases the characteristic resistances and applied forces on only certain portions of the total uncertainty. For example, if it is considered desirable to adopt a format analogous to the CEB code, the characteristic resistance, R', will be based on the uncertainty in material strength component only, i.e., on $V_M$,

$$R' = m_R\,(1- k_R\, V_M) \tag{31}$$

and the characteristic applied load on the load environment uncertainty, $V_T$, only

$$S' = m_S\,(1+ k_S\, V_T) \tag{32}$$

(Actually, as was discussed above, the CEB defines only characteristic material strengths, not characteristic member resistances.) Adopting the assumption in Eq. 9, we can define the "other" uncertainty, $V_O$, in the resistance as

$$V_O^2 = V_R^2 - V_M^2 = V_F^2 + V_P^2 \tag{33}$$

With the parallel assumption for applied force, Eq. 14, the code specification becomes the "separated form"

$$\phi' R' \geqslant \gamma_S' S' \tag{34}$$

in which

$$\phi' = (1- \beta\alpha\, V_R)/(1- k_R\, V_M) \text{ and } \gamma_S' = (1+ \beta\alpha\, V_S)/(1+ k_S\, V_T) \tag{35}$$

If it is assumed that $1-\beta\alpha V_R \cong 1- \beta\alpha^2 V_O - \beta\alpha^2 V_M \cong (1- \beta\alpha^2 V_O)(1- \beta\alpha^2 V_M)$, then

$$\phi' \cong (1- \beta\alpha^2 V_O)\,\frac{(1- \beta\alpha^2 V_M)}{(1- k_R V_M)} \text{ and, similarly, } \gamma_S' = (1+\beta\alpha^2 V_E)\,\frac{(1+\beta\alpha^2 V_T)}{(1+ k_S V_T)} \tag{36}$$

Note that since $k_R$ and $V_S$ are technically arbitrary, there is freedom in their choice. If the choices are $k_S=k_R=\beta\alpha^2$, then, simply

$$\phi' \cong 1-\beta\alpha^2 V_0 \quad \text{and} \quad \gamma_S' \cong 1+\beta\alpha^2 V_E \tag{37}$$

In addition, one could write, with additional approximation,

$$\phi' = 1-\beta\alpha^2 V_0 \cong 1-\beta\alpha^3 V_P - \beta\alpha^3 V_F = (1-\beta\alpha^3 V_P)(1-\beta\alpha^3 V_F) = \phi_P' \phi_F' \tag{38}$$

The advantage of making all of these approximations is that it further uncouples the problem, yielding strength reducing and applied force increasing factors that depend only on the "less tangible", fabrication and professional uncertainty, $V_F$, $V$ , and $V_E$. The "inherent" variation, $V_M$ and $V_T$, influences only the characteristic values R' and S'. This is very closely parallel to the ACI and CEB codes. The recent alterations to the basic CEB format involve factoring the strength reduction factor and load factor into several independent factors with identifiable "causes", similar to $\phi' = \phi_F' \phi_P'$ here. The process demonstrated here for $V_F$, $V_P$, and $\phi_F'$, $\phi_P'$ can, of course, be extended to the finer breakdown proposed by the CEB.

If there are combined loads, the "separated load factor" form is

$$\phi' R' \geqslant \gamma_D' S_D' + \gamma_L' S_L' \tag{39}$$

in which, for example, (with $k_S = \beta\alpha^3$),

$$\gamma_D' = 1 + \beta\alpha^3 V_{E_D} \tag{40}$$

where $V_{E_D}$ is the professional uncertainty in translating the dead load to applied force. The characteristic dead load effect in this case is

$$S'_D = m_{S_D}(1 + k_S V_D) \tag{41}$$

in which it is assumed $V_{S_D}^2 = V_D^2 + V_{E_D}^2$, $V_D$ being the uncertainty in the dead load "environment" itself.

Discussion; It is the author's hope that this discussion will aid professional committees who must choose reasonable and consistent values for the various factors in a code such as the CEB. The theoretical basis is in axiomatic probability theory, which does not require that all probabilities be defined as relative frequencies; this permits all uncertainties to be treated in a parallel and consistent manner. For example, this theory demonstrates that uncertainties in member dimensions should not be included in a load factor ($\gamma_{S_2}$ of the CEB) but in a strength reduction factor ($\phi_F'$ of this proposal); if the designer wants to obtain a safer structure by increasing the specified dimensions, uncertainty in the dimensions will influence the reliability actually achieved, a fact which is not properly reflected if the influence of this uncertainty is incorporated in the nominal load. Also, the factor $\alpha^3$ which permits $\phi$ and $\gamma$ factors to be less stringent, can be considered to be reflecting the theoretically small likelihood that one member will be simultaneously poorly fabricated, the recipient of low strength material, heavily loaded, etc. As another example of the benefit of the theory, it becomes clear that the influence of seriousness of failure should be reflected in the choice of the reliability level (here, $\beta$) independently of the uncertainty levels ($\sigma$ or V) in loads, materials, etc; the reliability value should, however, affect all factors $\phi$ and $\gamma$ (here through $\beta$) and not simply take the form of an additional multiplicative factor ($\gamma_C$ in the CEB). In any case, and at any time, code making is going to require professional judgement in selecting numerical values for the factors involved. Again it appears that this proposal will be helpful. It has been the author's experience that the easiest way to ponder the uncertainty in, say, the conventional professional procedure of translating live load to applied force is to ask oneself, "If I were given the value of the maximum total live load on the floor tributary to a column, what is the value $\varepsilon$ such that in 2/3 of all cases (or with probability 2/3) I would measure the maximum live load induced force within $1 \pm \varepsilon$ times the value predicted by my procedure (of load idealization, structural analysis, etc.)?" The value of $\varepsilon$ that answers this question is, in important part, an estimate of $V_{E_L}$ and hence gives $\gamma_L'(=1+\beta\alpha^2 V_{E_L}$, Eq. 40), once $\beta$ is selected.

In summary, the process of code development envisioned requires that the relevant professional committee prepare a report that gives the recommended procedure or formula for obtaining the best prediction of, say, the yield moment of a simple rectangular R.C. beam. In addition, they should report their quantitative assessment of the profession's uncertainty associated with the formula, namely, $V_P$. Means and standard deviations of ratios of predicted to observed resistances are commonly calculated by such committees and should serve as a basis for their value of $V_P$. In the reinforced concrete example, it may be of the order of 0.1. Still other appropriate committees might study reinforcing bar strength variability and conclude that, say, $V_M = 0.08$, while a committee on construction tolerances might estimate (or stipulate?) that $V_F = 0.03$, based on measurements or estimates of the depth of the steel in place. (This value may be smaller for deeper beams.) The implication is that $V_0 = \sqrt{V_F^2 + V_P^2} = \sqrt{0.03^2 + 0.1^2} = 0.10$. For $\beta=4$, $\alpha^2=0.5$, and $k_R = \beta\alpha^2 = 2$, one obtains $\phi' = 1-\beta\alpha^2 V_0 = 0.8$, and $R' = m_R(1-k_R V_M) = 0.84\ m_R$. Recall when comparing this with present procedures that the best estimate of the resistance, $m_R$, will be significantly greater than present nominal resistances.

The conclusions of parallel special committees on loads and on structural analysis and testing would yield predicted loads, load and structure idealizations, and analysis procedures, plus quantitative estimates or judgements of the measures of uncertainty in these phenomena and in these procedures. A major advantage of the code making process envisioned here over the present procedure (as understood by the author) is that the committees of specialists would have to judge and report on the uncertainties in their domain of interest. Their estimates would be quantitative inputs into a committee charged with selecting load factors and strength reduction factors. The proposed code basis provides an unambiguous means of communication and a formal framework within which this process can work in a rational and consistent manner.

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SUMMARY

The discussion demonstrates that a variety of practical formats for structural codes, including that of the ACI and CEB, can be developed directly from probability theory. A simplified, first-order probability theory based on first and second moments makes the development feasible . The theoretical basis for a code insures consistency and promotes objectivity in the discussion and specification of safety. All sources of uncertainty are treated uniformly, namely, by axiomatic probability theory, as modern interpretations of the notion of probability permit.

RESUME

La discussion précédente démontre qu'il y a une variété de formats, dérivés directement de la théorie des probabilités, qui peuvent être employés en pratique par les normes de calcul, comme celles de l'ACI ou du CEB. Une théorie simplifiée du premier ordre rend cette dérivation possible. En établissant les normes sur une base théorique, on garantit des spécifications consistentes et l'objectivité dans les discussions sur le degré de sécurité. Toutes les sources d'incertitude sont traitées d'une façon uniforme, au moyen des principes de la théorie des probabilités, pour autant que l'interprêtation moderne du concept de probabilité le permette.

ZUSAMMENFASSUNG

Die Diskussion zeigt, dass eine Vielzahl praktisch angewendeter Bauordnungen, unter anderem auch ACI und CEB, direkt aus der Wahrscheinlichkeitstheorie entwickelt werden können. Die Entwicklung wird durch eine vereinfachte Wahrscheinlichkeitstheorie ermöglicht, welche auf den ersten und zweiten Momenten beruht. Diese theoretische Grundlage für eine Bauordnung gewährleistet Folgerichtigkeit und Objektivität bei der Diskussion und Bestimmung der Sicherheit. Alle Unsicherheitsfaktoren werden gleichwertig behandelt, nämlich mittels axiomatischer Wahrscheinlichkeitstheorie, wie es durch die neuere Auslegung des Wahrscheinlichkeitsbegriffes möglich wurde.