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VI

DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

Structural Safety Specifications Based on Second-Moment Reliability Analysis

Spécifications de la sécurité des structures basées sur l'analyse des moments de deuxième ordre

Bauwerkssicherheit mittels einer auf den zweiten Momenten beruhenden Wahrscheinlichkeitsrechnung

C. ALLIN CORNELL
Associate Professor of Civil Engineering
Massachusetts Institute of Technology
Cambridge, Mass., U.S.A.

The primary aim of the symposium and of Themes VI and VII in particular is to develop theoretically sound safety specification procedures that remain practical. Semi-probabilistic codes are admittedly only partially successful, because they lack the dependence on analytical probability theory necessary to promote consistency and interpretation of the means and the ends of structural codes. The purpose of this discussion is to demonstrate that a variety of simple code formats, including some identical to present proposals, can be developed in a manner rigorously consistent with a probability theory. To be sure, certain analytical and algebraic approximations are adopted in order to achieve the desired simplicity of final code form, but the approximations, first, are believed to be reasonable and, second, are made in a clear way that opens them to analytical, quantitative study and to informed criticism. Alternatively, since it is demonstrated what approximations in the theory are necessary to achieve the simple code forms in use today, these present codes can be evaluated with respect to these implicit approximations. The results presented here should prove useful in guiding the discussion, interpretation, and selection of numerical values of factors in presently used and proposed codes, as well as in pointing towards systematic improvements in codes.

Second-Moment Reliability; The probability theory upon which the developments here are based is in itself an approximate theory(1). It is a first-order theory only; it is based on only the mean values and second moments of random variables. The latter moment is a first-order measure of uncertainty. In functional relationships among random variables the theory retains only the first-order or linear terms in the random components, i.e., in the deviations from the mean.

For example, the force capacity, W, of a tensile bar is uncertain if the area, A, and yield stress, Y, are both uncertain. In this theory the uncertainty in W would be measured solely through its variance (not its entire probability distribution). The mean and variance of W would be found from those of A and Y using a first-order expansion about their means

$$W = AY \doteq m_A\, m_Y + m_A(Y-m_Y) + m_Y(A-m_A)$$

in which m denotes the mean and $\doteq$ denotes equality to first-order approximation. Then applying the expectation and variance operation

$$m_W \doteq m_A\, m_Y \tag{1}$$

$$\sigma_W^2 \doteq m_A^2\, \sigma_Y^2 + m_Y^2\, \sigma_A^2 + 2\, m_A\, m_Y\, \rho_{A,Y}\, \sigma_Y \sigma_A \tag{2}$$

in which σ^2 denotes variance and ρ correlation coefficient. The "exact" result for the mean of W is

$$m_W = m_A\, m_Y (1 + \rho_{A,Y}\, V_A\, V_Y) \tag{3}$$

in which V denotes the coefficient of variation or σ/m. If A and Y are uncorrelated, the exact result for V_W^2 is $V_A^2 + V_Y^2 + V_A^2 V_Y^2$, which should be compared to $V_A^2 + V_Y^2$, the result implicit in the equations above. Clearly the first-order approximations may not be accurate if the coefficients of variation are large. It is important to note that no assumptions (in particular, no Gaussian assumptions) have been made about the distributions of the variables.

The total is a self-consistent, distribution-free theory of uncertainty that is very easy to apply to practical engineering problems and that provides accuracy which may be sufficient for many real problems, in particular in normative (or prescriptive) engineering specifications and codes of practice.

Although the theory does not give a complete description of the uncertainty in any particular variable, it can be argued that it is as accurate as structural building applications can justify. Statistical data are, and perhaps always will be, insufficient to determine the distributions of material strengths, dimensions, and loads. In any case, the variables in conventional structural practice represent highly idealized strengths and loads (e.g., homogeneous material strengths, pseudo-static forces, and uniformly distributed floor loads). Adoption of a first-order probability theory as a basis of structural safety would perhaps be analogous to the long standing dependence of the profession on linear, elastic theory to predict forces and deformations in reinforced concrete structures; in both cases, the results are known to be approximations, but they are 1) simple, yet self-consistent, 2) an improvement upon the state-of-art prior to their adoption, and 3) capable of being systematically modified or replaced as the profession accumulates knowledge.

Characterization of Variables; Within this approach to structural safety and performance specification, a variable, X, is characterized by two numbers, a best estimate and an uncertainty measure. The former corresponds to an expected value or mean, m_X, the latter to a variance, σ_X^2, standard deviation, σ_X or coefficient of variation, V_X. The expected value represents the profession's best prediction of a variable (e.g., material strength, member deflection, peak wind force). It is conducive to systematic progress of the profession that this best estimate, rather than a conservative estimate, be a product of any research investigation or committee report. The uncertainty value associated with a variable should be a measure of the various sources and kinds of uncertainty surrounding it. These include both "natural", inherent variablity (such as that observed in wind velocities and material strengths) and the uncertainty associated with the imperfect tools of the profession (simplifying assumptions, incomplete knowledge, human constructors, etc.).

The treatment of the latter sources of uncertainty probabilistically is a major distinction between this code proposal and most others. This procedure is, however, consistent with the most modern and most practical concept of probability[2,3,4], and it avoids difficult-to-reconcile distinctions among the interpretations, analyses, and code treatments of the various sources of uncertainty. For example, should the uncertainty in the initial shape of steel columns be considered as natural (or "random") variability and included in determining a nominal or "characteristic" strength for a member, or should it be accounted for in a multiplying strength reduction factor designed to account for fabrication variabilities? It is clearly in the latter factor in U.S. codes, whereas European tests(5) are aimed at placing it in the former value. When this is settled, what should be done with minor construction errors which cause accidental eccentricities in the applied load? Indeed, what virtually all codes accept as "inherent" or "random" variability in the strength of steel can be, on closer inspection, decomposed into a variety of identifiable sources which are in part systematic(6). Similarly, should that uncertainty in snow loads which is associated with its spacial disposition on a roof be considered as "random" or professional? Since the profession lacks methods of analyzing snow accumulation on irregular roof shapes, it has been proposed(7) to collect statistical data as if the problem were a "random" one. The distinctions in types of uncertainty are difficult, because they are not "basic" but dependent in part, it seems, on the data available at the moment and on the level of idealization in the conventional treatment of the phenomenon. In fact, these distinctions are unnecessary if all are treated as contributions to a total measure of uncertainty, denoted σ.

It is useful (but not technically necessary, as will be demonstrated) to define nominal or "characteristic values" of variables. For a material strength, the characteristic value in the CEB code(8) is one which a specified (large) fraction of all standard test results exceed. In a parallel way, in this first-order theory a characteristic strength, R*, would be

$$R^* = m_R - k_R\,\sigma_R = m_R(1 - k_R\,V_R) \qquad (4)$$

in which k_R is a specified constant, the same for all materials, members, etc. A characteristic load or applied force, S*, is defined in an analogous way, $+k_S$ replacing $-k_R$. Because CEB code specifications present the formula for the characteristic strength in the same form as Eq. 4, it is important to point out the differences between the CEB and this proposal.

First, the CEB code (and others like it) set the characteristic value at a specified fractile of the distribution. Thus the factor k_R must depend upon the shape of this distribution (and in some cases on m and σ as well). For the levels of probabilities usually specified by present codes (1 to 10%), the value of k_R is not too sensitive to the distribution, but, of course, the distributions of interest are not well established, and they probably change from place to place and time to time. This proposed code basis, being only first-order, does not attempt to distinguish between distribution shapes; k_R, <u>not</u> the probability level, is fixed by the code.

Second, the CEB code would base σ_R solely on standard tests of standard materials specimens(8). In the proposed code, the interest is on strength in place. Therefore, σ_R should include, in addition to the "inherent" dispersion observed in standard tests, the uncertainties associated with correlating these results to in-place strengths (e.g., construction versus laboratory practice, weather conditions, full-size member versus standard specimen, etc.). In short, in this proposal σ_R should measure all the uncertainty that the engineer in fact faces when asked to predict the strength of the material in an actual beam to be built to his specifications.

Finally, the proposed code differs from the CEB in that it includes characteristic values for the <u>strength of members</u> (or assemblies, perhaps) and the <u>force applied to members</u> (in addition to the strength of materials and the environmental loads). It is member capacity and member force which ultimately determine safety. They depend, of course, on material strength and environmental loads, but only in part. The best prediction of and the uncertainty in the strength of a member depend upon the dimensions of the member and upon incompletely understood microscopic behavior of the material or materials of which it is constructed, as well as upon material strength. The first-order probability theory makes approximate analysis of these factors feasible.

For example, consider the moment resistance of the cross-section of a simple, rectangular reinforced concrete beam with width B, depth-to-steel D, concrete strength C, and total steel yield force T. Adopting conventional structural theory (nothing more can be justified in a code), the (under-reinforced) yield moment resistance, R, of the cross-section can be written

$$R = TD\left(1 - \eta \frac{T}{BDC}\right) \Delta \tag{5}$$

in which η is a constant dependent upon the "theory" used, and Δ is a random factor introduced to describe the dispersion about the predicted resistance that is observed in laboratory test results in which the values of T, D, B, and C are known by relatively precise measurement; Δ is the (random) ratio of observed to predicted resistance. For an unbiased prediction formula, $m_\Delta = 1$. The uncertainty value σ_Δ^2 is a measure of the accuracy of the prediction formula, or, in short, of the professional uncertainty inherent in the use of this theory.

Under first-order probability theory the mean and variance of the resistance become[8]

$$m_R \stackrel{\prime}{=} m_T\, m_D \left(1 - \eta \frac{m_T}{m_B m_D m_C}\right) m_\Delta \tag{6}$$

$$\sigma_R^2 \stackrel{\prime}{=} \sum_i \left(\frac{\partial R}{\partial X_i}\bigg|_{\underline{m}}\right)^2 \sigma_{X_i}^2 \tag{7}$$

in which the X_i are T, D, B, C and Δ, and the term in parentheses denotes the partial derivative of R with respect to a particular variable X_i, evaluated at the means of the variables. (It has been assumed that the X_i's are uncorrelated.) Note that the uncertainty in each variable contributes to the uncertainty in R in a manner dependent both upon the uncertainty in that variable and upon the sensitivity of R to deviations in the variable.

The characteristic value of the member resistance is found by substituting into Eq. 4. Note that it is not simply the value of R obtained by substituting characteristic values of strengths, T* and C*, for T and C in Eq. 5, as is implicit in present codes.

The lack of sensitivity of resistance variables to certain factors can be exploited to simplify significantly the procedure above. It may be sufficient for most code purposes to assume in the computation of the uncertainty measure V_R that the relationship between R and the other variables is of the form[10]

$$R = c\,M\,F\,P \tag{8}$$

in which c is a constant, M is a material variable, F is a fabrication-dependent member dimension variable, and P is a professional factor. (In the reinforced-concrete beam example above, T, D, and Δ can be associated with M, F, and P, respectively. The constant c is simply $1 - (\eta\, m_T)/m_B\, m_D\, m_C$.) In this case one obtains simply

$$V_R^2 \stackrel{\prime}{=} V_M^2 + V_F^2 + V_P^2 \tag{9}$$

The uncertainty in R is made up of uncertainty in material strength, fabrication, and professional factors. If this simplification is adopted the characteristic resistance is simply

$$R^* = m_R\,(1 - k_R V_R) \stackrel{\prime}{=} m_R\left(1 - k_R \sqrt{V_M^2 + V_F^2 + V_P^2}\right) \tag{10}$$

In the reinforced concrete example one obtains

$$R^* \stackrel{1}{=} \left[m_T m_D \left(1 - \eta \frac{m_T}{m_B m_D m_C}\right) m_\Delta \right] \left(1 - k_R \sqrt{V_M^2 + V_F^2 + V_P^2}\right) \qquad (11)$$

Since the coefficents of variation are probably rather insensitive to beam size and other factors, the last term might be the same for all beams throughout a design project, implying that the computation of theoretically consistent characteristic resistances of beams need be no more difficult than present computation of nominal resistances. Both involve calculation of the value of the first factor, that in square brackets. Note again, however, that in the proposed code mean values, not nominal values, are involved in these calculations. An implication is that a by-product of the calculation is the best professional prediction of the resistance of the cross-section, i.e., the first factor in Eq. 11; this best estimate is never obtained in present calculation procedures.

Similarly, characteristic loads and applied member forces can be determined as

$$S^* = m_S + k_S \sigma_S = m_S (1 + k_S V_S) \qquad (12)$$

The uncertainty measure in an applied member force should account for both the customary observed, inherent dispersion in environmental loads and the many professional uncertainties such as those involved in translating loads into member forces (i.e., in the structural analysis used), in approximating dynamic by static behavior, in idealizing spatial load variations, in predicting future changes in the loading environment, and in neglected (abnormal and unforeseen) loading combinations.

Again for many purposes it may be sufficient to assume in uncertainty calculations that the applied member force, S, is

$$S = c\, T\, E \qquad (13)$$

in which T is the environmental load or "field strength"(7) and E is a factor, perhaps with mean 1, reflecting professional engineering uncertainties. (The constant c is related to the structural analysis which translates load into applied force.) Then V_S^2 is simply $V_T^2 + V_E^2$, and the characteristic applied force is simply

$$S^* = m_S \left(1 + k_S \sqrt{V_T^2 + V_E^2}\right) \qquad (14)$$

V_T can be obtained from load environment measurements and analysis, while V_E must be judged, and/or obtained from calibration(10) of existing codes (although physical measurements of forces in full-scale structures subjected to known loads could provide partial information). It should be clear that how the uncertainty in S is proportioned between T and E will depend in part upon how the load is idealized (e.g., winds as pseudo-static gusts or as dynamic velocity time-histories), but that the net uncertainty in S may be unchanged. (It could be less, if the particular idealization is more accurate.)

Safety Specification Alternatives; Codes of practice must in some way cause the engineer to specify a structure which has a (best prediction of the) resistance sufficiently in excess of the (best prediction of the) applied force to insure adequate safety and performance without unduly penalizing the cost of the structure. This requirement can and has been effected in a variety of code "formats" (e.g., working stress, load factors, semi-probabilistic, etc.) In theoretical structural safety terms the purpose of the code is usually to promote a pre-determined level of reliability. In this section it will be demonstrated that this reliability requirement can be expressed in a variety of convenient code formats, all technically equivalent in that they will cause designers to specify the same mean resistance.

Satisfactory structural safety (or performance) will be achieved if the resistance, R, exceeds the applied force S, that is, if the safety margin $M = R-S$ exceeds zero. The mean and variance of M are

$$m_M = m_R - m_S \tag{15}$$

$$\sigma^2{}_M = \sigma^2{}_R + \sigma^2{}_S \tag{16}$$

The reliability of the structure (or member) is defined as the probability that M exceeds zero. In terms of a first-order probability theory reliability is measured by the number of standard deviations σ_M by which the mean m_M exceeds zero[1,10]. Call this number β. The larger β, the more reliable the member. To impose a required reliability a code must require that

$$m_M \geqslant \beta\ \sigma_M \tag{17}$$

The appropriate value for β is a matter of serious professional judgement. Values of about 4 have been found to be consistent with certain present codes[10,11]. The specified value of β should be related to the consequences of the type of failure under consideration and to the marginal cost of increasing the resistance. Higher reliability, i.e., a higher value of β should be required of sudden brittle shear failure modes than of ductile yielding, for example. If the mode of "failure" (or "limit state") under study is simply undesirable (but not unsafe) cracking or deflection, significantly lower values of β (2 perhaps) are appropriate. Although it is not possible at higher values of β to associate even approximate numerical probabilities to values of β, there is some justification[9] for assuming that at these levels an increase of β by 1/2 implies about an order of magnitude decrease in the complement of the reliability, i.e., the probability of failure. The (desirable, but impractical) numerical evaluation of reliabilities discussed in theoretical reliability studies depends on precise knowledge of the probability distribution of M. This in turn depends on the distributions of R and S. These will probably never be known accurately, for they are affected by significant sources of professional uncertainty which are difficult to model and to measure.

This single technical safety requirement (17) can be expressed in a variety of alternate ways. It is valuable to display some of these formats and the approximations and assumptions necessary to achieve them. Direct substitution of Eq. 15 and 16 produces the safety requirement in the "safety margin form"

$$m_R \geqslant m_S + \beta\sqrt{\sigma_R^2 + \sigma_S^2} \tag{18}$$

Rearrangement leads to the "safety factor" form

$$m_R \geqslant \theta\ m_S \tag{19}$$

in which θ is a function of V_R, V_S, and β,

$$\theta \doteq (1 + \beta\sqrt{V_R^2 + V_S^2 - \beta^2\ V_R^2\ V_S^2}\ )/(1 - \beta^2 V_R{}^2) \tag{20}$$

It is shown in Figure 1 for $\beta = 4$. Substitution of Eq. 10 and Eq. 12 for nominal values yields the "nominal safety factor" form

$$R^* \geqslant \theta^*\ S^* \tag{21}$$

in which θ^* is a function of V_R, V_S, β, k_R, and k_S:

$$\theta^* = \frac{1 - k_R\ V_R}{1 + k_S\ V_S}\ \theta \tag{22}$$

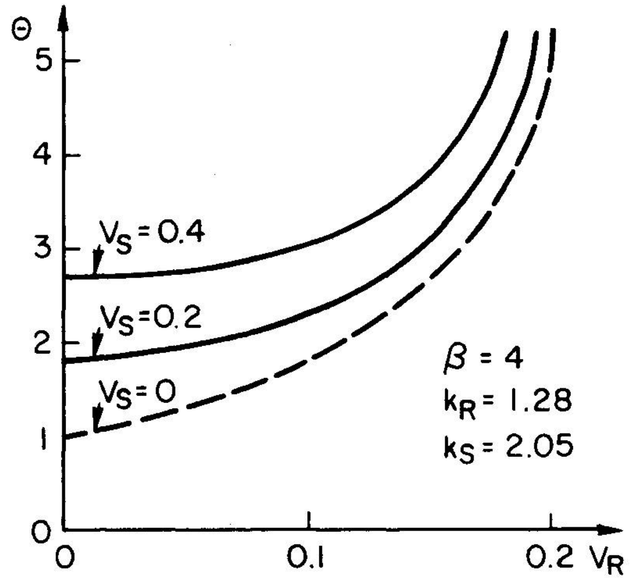

Figure 1

It is shown in Figure 2 for $\beta = 4$, $k_R = 1.28$, and $k_S = 2.05$. Notice that in the latter case θ^* is quite insensitive to V_R and V_S over the range of interest; this is an advantage of the nominal form. It is important to recognize that the values of k_R and k_S are, technically speaking, arbitrary. Increasing k_R will reduce R^* and reduce θ^* to compensate, leaving the required value of m_R unchanged. The values of k_R and k_S can be chosen to satisfy legal problems surrounding quality assurance and liability, or to permit simplification of the code as will be discussed below.

The reliability theory adopted implies that you cannot obtain high reliability with highly uncertain resistances. If $V_R \gg 1/\beta$, then it is not possible to obtain the desired level of reliability, since σ_M (which is greater than or equal to $\sigma_R = V_R\, m_R$) will grow too fast with m_R to permit m_M $(=m_R - m_S)$ to exceed $\beta\, \sigma_M$. At larger values of m_R, V_R might very well be smaller, of course. On the other hand, a simplified practical code might simply overlook this problem by setting θ^* equal to an approximate, linearly increasing value for $V_R \gg 0.1$.

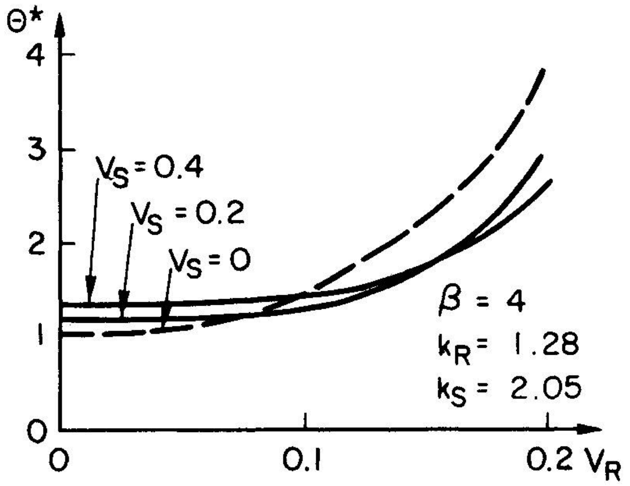

Figure 2

A variety of split factor code formats similar to the ACI or CEB forms are also possible. The possibility of decoupling these factors was recognized by N. C. Lind[11,12] who has defined and demonstrated the notion of "practical equivalence" of code formats. Lind showed that, with remarkable numerical accuracy,

$$\sqrt{\sigma_R^2 + \sigma_S^2} \cong \alpha\,(\sigma_R + \sigma_S) \qquad (23)$$

in which α is a constant chosen to fit the expected range of ratios of θ_R to σ_S. A value of $\alpha = 0.7$ will give errors of less than about 5% for σ_R/σ_S in the entire range of 1/3 to 3. With this approximation Eq. 18 can be written in the "split form"

$$\phi\, m_R \gg \gamma_S\, m_S \qquad (24)$$

in which

$$\phi = 1 - \beta\alpha\, V_R \qquad \text{and} \qquad \gamma_S = 1 + \beta\alpha\, V_S \qquad (25)$$

Note that these strength reduction and load increase factors depend only on their corresponding uncertainty measures, V_R and V_S, respectively, (and, of course, on the reliability level β). Note that, approximately, $\theta \cong \gamma_S/\phi$. A "nominal split form" can be written as

$$\phi^* R^* \geqslant \gamma_S^* S^* \tag{26}$$

in which

$$\phi^* = (1-\beta\alpha V_R)/(1-k_R V_R) \text{ and } \gamma_S^* = (1+\beta\alpha V_S)/(1+k_S V_S) \tag{27}$$

If combined loads are involved, then S is made up of the sum of two applied force effects, say, $S_D + S_L$, signifying dead and live loads. Assuming they are uncorrelated and using Eq. 23, $\sigma_S \cong \alpha(\sigma_{S_D} + \sigma_{S_L})$. Then, in Eq. 24, $\gamma_S\, m_S$ can be replaced by

$$\begin{aligned}\gamma_S m_S &= m_S\,(1+\beta\alpha V_S) = m_S + \beta\alpha\,\sigma_S \cong m_{S_D} + m_{S_L} + \beta\alpha^2\sigma_{S_D} + \beta\alpha^2\sigma_{S_D} \\ &= m_{S_D}(1+\beta\alpha^2 V_{S_D}) + m_{S_L}(1+\beta\alpha^2 V_{S_L}) = m_{S_D}\,\gamma_D + m_{S_L}\,\gamma_L\end{aligned} \tag{28}$$

in which a dead load and a live load factor are defined, as indicated, in terms of β and their respective uncertainty factors V_{S_D} and V_{S_L}. This result is due to M. K. Ravindra of the University of Waterloo.

A "split load factor form" results,

$$\phi^* R^* \geqslant \gamma_D^* S_D^* + \gamma_L^* S_L^* \tag{29}$$

if characteristic live and dead loads are used and if nominal load factors are defined as, for example,

$$\gamma_D^* = (1+\beta\alpha^2 V_{S_D})/(1+k_S V_{S_D}) \tag{30}$$

This factor will be relatively insensitive to V_{S_D}. Combined loads involving two or more transient loads(7) can be treated in a similar way provided the proper model is used. In coordination with the maximum wind load, for example, one should use the normal or "steady-state" live load moments, not those of the maximum (in time) live load.

Finally, it is useful to demonstrate that a consistent code format is possible which bases the characteristic resistances and applied forces on only certain portions of the total uncertainty. For example, if it is considered desirable to adopt a format analogous to the CEB code, the characteristic resistance, R', will be based on the uncertainty in material strength component only, i.e., on V_M,

$$R' = m_R\,(1-k_R\,V_M) \tag{31}$$

and the characteristic applied load on the load environment uncertainty, V_T, only

$$S' = m_S\,(1+k_S\,V_T) \tag{32}$$

(Actually, as was discussed above, the CEB defines only characteristic material strengths, not characteristic member resistances.) Adopting the assumption in Eq. 9, we can define the "other" uncertainty, V_O, in the resistance as

$$V_O^2 = V_R^2 - V_M^2 = V_F^2 + V_P^2 \tag{33}$$

With the parallel assumption for applied force, Eq. 14, the code specification becomes the "separated form"

$$\phi' R' \geqslant \gamma_S' S' \tag{34}$$

in which

$$\phi' = (1-\beta\alpha V_R)/(1-k_R V_M) \text{ and } \gamma_S' = (1+\beta\alpha V_S)/(1+k_S V_T) \tag{35}$$

If it is assumed that $1-\beta\alpha V_R \cong 1-\beta\alpha^2 V_O - \beta\alpha^2 V_M \cong (1-\beta\alpha^2 V_O)(1-\beta\alpha^2 V_M)$, then

$$\phi' \cong (1-\beta\alpha^2 V_O)\,\frac{(1-\beta\alpha^2 V_M)}{(1-k_R V_M)} \text{ and, similarly, } \gamma_S' = (1+\beta\alpha^2 V_E)\,\frac{(1+\beta\alpha^2 V_T)}{(1+k_S V_T)} \tag{36}$$

Note that since k_R and V_S are technically arbitrary, there is freedom in their choice. If the choices are $k_S = k_R = \beta\alpha^2$, then, simply

$$\phi' \cong 1- \beta\alpha^2 V_0 \quad \text{and} \quad \gamma'_S \cong 1+ \beta\alpha^2 V_E \tag{37}$$

In addition, one could write, with additional approximation,

$$\phi' = 1- \beta\alpha^2 V_0 \cong 1- \beta\alpha^3 V_P - \beta\alpha^3 V_F = (1- \beta\alpha^3 V_P)(1- \beta\alpha^3 V_F) = \phi'_P \, \phi'_F \tag{38}$$

The advantage of making all of these approximations is that it further uncouples the problem, yielding strength reducing and applied force increasing factors that depend only on the "less tangible", fabrication and professional uncertainty, V_F, V , and V_E. The "inherent" variation, V_M and V_T, influences only the characteristic values R' and S'. This is very closely parallel to the ACI and CEB codes. The recent alterations to the basic CEB format involve factoring the strength reduction factor and load factor into several independent factors with identifiable "causes", similar to $\phi' = \phi'_F \, \phi'_P$ here. The process demonstrated here for V_F, V_P, and ϕ'_F, ϕ'_P can, of course, be extended to the finer breakdown proposed by the CEB.

If there are combined loads, the "separated load factor" form is

$$\phi' R' \geqslant \gamma'_D S'_D + \gamma'_L S'_L \tag{39}$$

in which, for example, (with $k_S = \beta\alpha^3$),

$$\gamma'_D = 1 + \beta\alpha^3 V_{E_D} \tag{40}$$

where V_{E_D} is the professional uncertainty in translating the dead load to applied force. The characteristic dead load effect in this case is

$$S'_D = m_{S_D}(1 + k_S V_D) \tag{41}$$

in which it is assumed $V_{S_D}^2 = V_D^2 + V_{E_D}^2$, V_D being the uncertainty in the dead load "environment" itself.

Discussion; It is the author's hope that this discussion will aid professional committees who must choose reasonable and consistent values for the various factors in a code such as the CEB. The theoretical basis is in axiomatic probability theory, which does not require that all probabilities be defined as relative frequencies; this permits all uncertainties to be treated in a parallel and consistent manner. For example, this theory demonstrates that uncertainties in member dimensions should not be included in a load factor (γ_{S_2} of the CEB) but in a strength reduction factor (ϕ'_F of this proposal); if the designer wants to obtain a safer structure by increasing the specified dimensions, uncertainty in the dimensions will influence the reliability actually achieved, a fact which is not properly reflected if the influence of this uncertainty is incorporated in the nominal load. Also, the factor α^3 which permits ϕ and γ factors to be less stringent, can be considered to be reflecting the theoretically small likelihood that one member will be simultaneously poorly fabricated, the recipient of low strength material, heavily loaded, etc. As another example of the benefit of the theory, it becomes clear that the influence of seriousness of failure should be reflected in the choice of the reliability level (here, β) independently of the uncertainty levels (σ or V) in loads, materials, etc; the reliability value should, however, affect all factors ϕ and γ (here through β) and not simply take the form of an additional multiplicative factor (γ_C in the CEB). In any case, and at any time, code making is going to require professional judgement in selecting numerical values for the factors involved. Again it appears that this proposal will be helpful. It has been the author's experience that the easiest way to ponder the uncertainty in, say, the conventional professional procedure of translating live load to applied force is to ask oneself, "If I were given the value of the maximum total live load on the floor tributary to a column, what is the value ε such that in 2/3 of all cases (or with probability 2/3) I would measure the maximum live load induced force within $1 \pm \varepsilon$ times the value predicted by my procedure (of load idealization, structural analysis, etc.)?" The value of ε that answers this question is, in important part, an estimate of V_{E_L} and hence gives $\gamma'_L(=1+ \beta\alpha^2 V_{E_L}$, Eq. 40), once β is selected.

In summary, the process of code development envisioned requires that the relevant professional committee prepare a report that gives the recommended procedure or formula for obtaining the best prediction of, say, the yield moment of a simple rectangular R.C. beam. In addition, they should report their quantitative assessment of the profession's uncertainty associated with the formula, namely, V_P. Means and standard deviations of ratios of predicted to observed resistances are commonly calculated by such committees and should serve as a basis for their value of V_P. In the reinforced concrete example, it may be of the order of 0.1. Still other appropriate committees might study reinforcing bar strength variability and conclude that, say, $V_M = 0.08$, while a committee on construction tolerances might estimate (or stipulate?) that $V_F = 0.03$, based on measurements or estimates of the depth of the steel in place. (This value may be smaller for deeper beams.) The implication is that $V_0 = \sqrt{V_F^2 + V_P^2} = \sqrt{0.03^2 + 0.1^2} = 0.10$. For $\beta=4$, $\alpha^2=0.5$, and $k_R = \beta\alpha^2 = 2$, one obtains $\phi' = 1-\beta\alpha^2 V_0 = 0.8$, and $R' = m_R(1-k_R V_M) = 0.84\ m_R$. Recall when comparing this with present procedures that the best estimate of the resistance, m_R, will be significantly greater than present nominal resistances.

The conclusions of parallel special committees on loads and on structural analysis and testing would yield predicted loads, load and structure idealizations, and analysis procedures, plus quantitative estimates or judgements of the measures of uncertainty in these phenomena and in these procedures. A major advantage of the code making process envisioned here over the present procedure (as understood by the author) is that the committees of specialists would have to judge and report on the uncertainties in their domain of interest. Their estimates would be quantitative inputs into a committee charged with selecting load factors and strength reduction factors. The proposed code basis provides an unambiguous means of communication and a formal framework within which this process can work in a rational and consistent manner.

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SUMMARY

The discussion demonstrates that a variety of practical formats for structural codes, including that of the ACI and CEB, can be developed directly from probability theory. A simplified, first-order probability theory based on first and second moments makes the development feasible . The theoretical basis for a code insures consistency and promotes objectivity in the discussion and specification of safety. All sources of uncertainty are treated uniformly, namely, by axiomatic probability theory, as modern interpretations of the notion of probability permit.

RESUME

La discussion précédente démontre qu'il y a une variété de formats, dérivés directement de la théorie des probabilités, qui peuvent être employés en pratique par les normes de calcul, comme celles de l'ACI ou du CEB. Une théorie simplifiée du premier ordre rend cette dérivation possible. En établissant les normes sur une base théorique, on garantit des spécifications consistentes et l'objectivité dans les discussions sur le degré de sécurité. Toutes les sources d'incertitude sont traitées d'une façon uniforme, au moyen des principes de la théorie des probabilités, pour autant que l'interprêtation moderne du concept de probabilité le permette.

ZUSAMMENFASSUNG

Die Diskussion zeigt, dass eine Vielzahl praktisch angewendeter Bauordnungen, unter anderem auch ACI und CEB, direkt aus der Wahrscheinlichkeitstheorie entwickelt werden können. Die Entwicklung wird durch eine vereinfachte Wahrscheinlichkeitstheorie ermöglicht, welche auf den ersten und zweiten Momenten beruht. Diese theoretische Grundlage für eine Bauordnung gewährleistet Folgerichtigkeit und Objektivität bei der Diskussion und Bestimmung der Sicherheit. Alle Unsicherheitsfaktoren werden gleichwertig behandelt, nämlich mittels axiomatischer Wahrscheinlichkeitstheorie, wie es durch die neuere Auslegung des Wahrscheinlichkeitsbegriffes möglich wurde.

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VI

The probability of failure when the characteristic values are used as a design method

La probabilité de ruine quand la méthode des valeurs caractéristiques est utilisée

Die Versagenswahrscheinlichkeit, wenn die charakteristischen Werte als Bemessungsmethode verwendet werden

EERO PALOHEIMO
Dr. Ing.
Helsinki

The only method to determine the dimensions of structures which seems to have a logical justification, would be a form of calculation giving an equal reliability (or equal probability of failure) in different parts of the structure.

Another, and purely practical, requirement for this calculation method is simplicity, as the method should be available for the average engineer in his everyday work.

It seems possible to determine by computers the probability of failure for different types of structures. The question is, can we find a general and relatively simple method of calculation, which gives automatically a given and similar reliability to the different parts of the structure under consideration? If this is not possible, what method would best fulfil the previous conditions?

Four different design methods will be studied in the following, and for simplicity called methods 1,2,3 and 4.

A simple and rather general model of the reliability can be presented as follows:

The condition for failure will be given by

(1) $g(x_1 \ldots x_n) \leqq 1$

where $x_1 \ldots x_n$ represent the various quantities of the structural element or the external forces and moments loading this element.

We assume that the distribution functions of $x_1 \ldots x_n$ are known, and denote the mean-values of these quantities by $m_1 \ldots m_n$

and the standard deviations by $\sigma_1 \ldots \sigma_n$.

For the probability of failure we have

(2) $P(g(x_1 \ldots x_n) \leqq 1)$

The four different design methods which will be compared are as follows:

<u>Method 1.</u> We choose the mean values of the r first quantities $x_1 \ldots x_r$ (the internal properties of the structural element) and the n-r quantities $x_{r+1} \ldots x_n$ (the external forces and moments) so that

$$g(m_1 \ldots m_r,\ k \cdot m_{r+1} \ldots k \cdot m_n) = 1$$

We always use the same "total safety factor" k and try to determine k so that in some common cases

$$P(g(x_1 \ldots x_n) \leqq 1) = c$$

where we denote the probability of failure considered as suitable by c.

<u>Method 2.</u> We choose the mean values of the various quantities so that

$$g(m_1 \ldots m_r, k \cdot m_{r+1} \ldots k \cdot m_n) = 1$$

and use, depending on the values of $\rho_1 = \sigma_1/m_1 \ldots \rho_n = \sigma_n/m_n$ and different functions g, various "total safety factors" k, so that in all cases

$$P(g(x_1 \ldots x_n) \leqq 1) = c$$

<u>Method 3.</u> We choose the various mean values so that

$$g(m_1 \pm \alpha \cdot \sigma_1 \ldots m_n \pm \alpha \cdot \sigma_n) = 1 \text{ (+ or - chosen unfavourably)}$$

We always use the same "characteristic coefficient" α and try to determine α so that in some common cases

$$P(g(x_1 \ldots x_n) \leqq 1) = c$$

<u>Method 4.</u> We choose the mean values so that

$$g(m_1 \pm \alpha \cdot \sigma_1 \ldots m_n \pm \alpha \cdot \sigma_n) = 1 \text{ (+ or - chosen unfavourably)}$$

and use various "characteristic coefficients" α depending on the values of $\rho_1 \ldots \rho_n$ and g, so that in all cases

$P(g(x_1 \ldots x_n) \leqq 1) = c$

We see immediately that methods 2 and 4 strive for mathematical exactness and methods 1 and 3 aim at simplicity in everyday use.

We shall now study and compare these methods in four different cases.

1. The simplest model of reliability is the case when both the capacity of the structure x_1, and the external load x_2, are normal and independent with mean values m_1, m_2 and s.d. σ_1, σ_2.

The probability of failure is then

$$(3) \qquad P({}^{x_1}/x_2 \leqq 1) = P((x_1 - x_2) \leqq 0$$

As we know, the distribution of $(x_1 - x_2)$ is also normal with

$$(4) \qquad \begin{cases} m = m_1 - m_2 \\ \sigma = \sqrt{\sigma_1^2 + \sigma_2^2} \end{cases}$$

and we have

$$(5) \qquad P({}^{x_1}/x_2 \leqq 1) = \Phi\left(\frac{m_2 - m_1}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)$$

where

$$(6) \qquad \Phi(\beta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\beta} e^{-t^2/2} \cdot dt$$

now writing

$$\begin{cases} \sigma_1 = \varrho_1 \cdot m_1 \\ \sigma_2 = \varrho_2 \cdot m_2 \end{cases}$$

and ${}^{m_1}/m_2 = k$

we get

$$(7) \qquad P(({}^{x_1}/x_2) \leqq 1) = \Phi\left(\frac{1 - k}{\sqrt{\varrho_1^2 \cdot k^2 + \varrho_2^2}}\right)$$

which gives the probability of failure when different "total safety factors" are used.

In the same way we get

$$(8) \qquad k = \frac{1 + \sqrt{1 - (1 - \varrho_1^2 \cdot \beta^2) \cdot (1 - \varrho_2^2 \cdot \beta^2)}}{1 - \varrho_1^2 \cdot \beta^2}$$

to calculate "the total safety factors" corresponding to certain $\Phi(\beta) = c$

Using the "characteristic values" we write

$$(9)\quad \begin{cases} x_1^* = m_1 - \alpha \cdot \sigma_1 \\ x_2^* = m_2 + \alpha \cdot \sigma_2 \end{cases}$$

Through $x_1^* = x_2^*$ we get for k'

$$(10)\quad k' = m_1/m_2 = \frac{1 + \alpha \cdot \rho_2}{1 - \alpha \cdot \rho_1}$$

and β

$$(11)\quad \beta = \frac{-\alpha(\sigma_1 + \sigma_2)}{\sqrt{\sigma_1^2 + \sigma_2^2}} = \frac{-\alpha(\rho_1 + \rho_2)}{\sqrt{\rho_1^2 + \rho_2^2 + 2 \cdot \alpha^2 \cdot \rho_1^2 \rho_2^2 + 2\alpha\rho_1 \rho_2(\rho_1 - \rho_2)}}$$

to define the dependence between α, β and k'. By k' we denote "the total safety factor", which gives as result the same β as we get using the corresponding α from (10).

These relations are illustrated in Fig.1 and Fig.2. The equalities (7), (8), (10) and (11) have been solved for some special cases of ρ_1 and ρ_2, which are usual in practice and the results are given in Table 1.

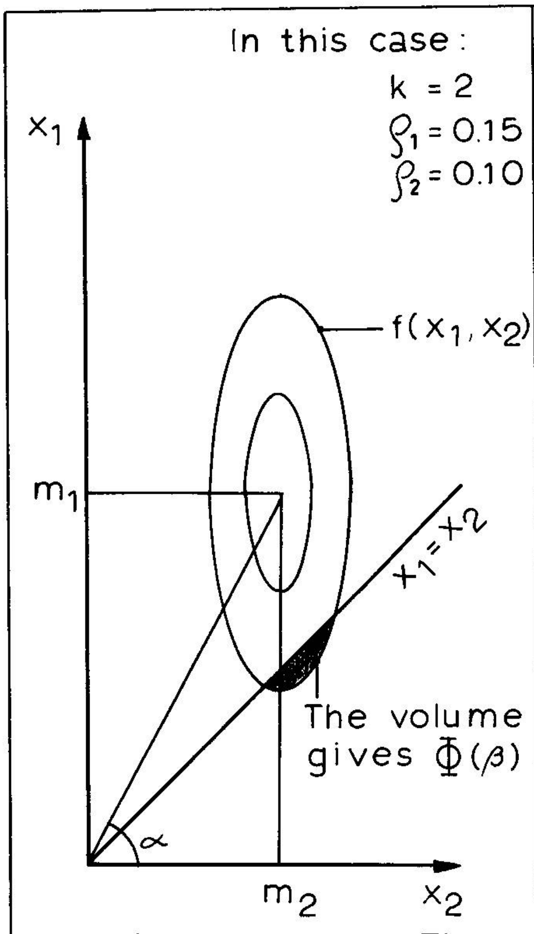

Fig 1

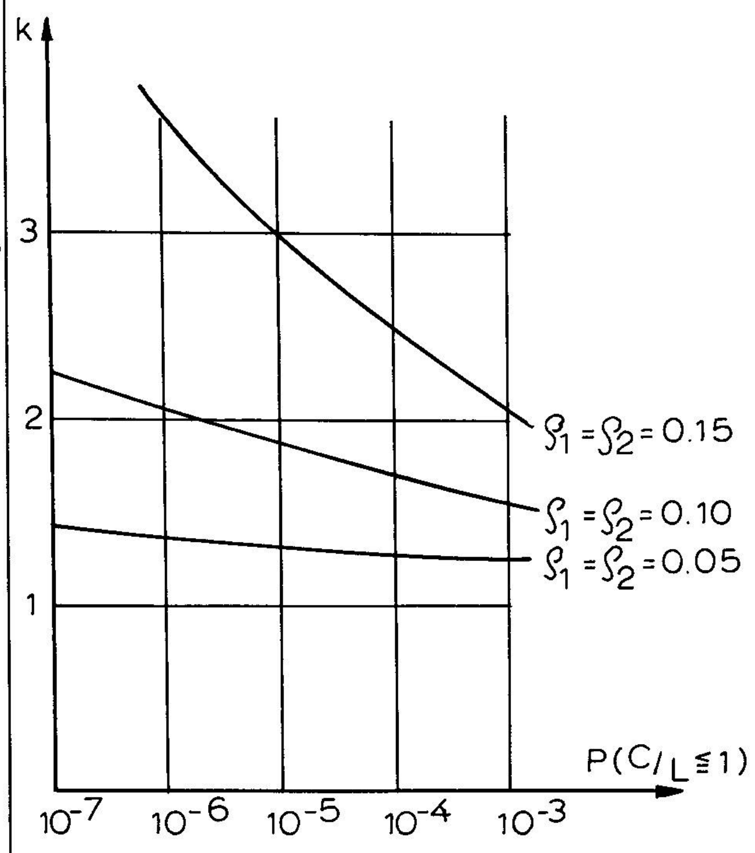

Fig 2

Table 1

σ/m			$k = 2$		$-\beta = 4{,}65$	$\alpha = 3{,}30$				$\alpha = 4{,}25$	
	x_1	x_2	$-\beta$	$\Phi(\beta)$	k	$-\beta$	Φ/β	k'	k/k'	k'	k/k'
1	0,05/1	0,05/1	8,95	$\sim 10^{-17}$	1,40	4,60		1,40	1,00		
2	0,10/1	0,05/1	4,85		1,92	4,09		1,74	1,10		
3	0,15/1	0,05/1	3,29		3,35	3,75	$\sim 10^{-5}$	2,31	1,45	3,35	1,00
4	0,05/1	0,10/1	7,07		1,59	4,65	$\sim 10^{-6}$	1,59	1,00		
5	0,10/1	0,10/1	4,47		2,07	4,43		1,99	1,04		
6	0,15/1	0,10/1	3,16		3,46	4,01		2,64	1,31		
7	0,05/1	0,15/1	5,55		1,81	4,53		1,79	1,01		
8	0,10/1	0,15/1	4,00		2,26	4,58		2,23	1,01		
9	0,15/1	0,15/1	2,98	$\sim 10^{-3}$	3,62	4,17		2,96	1,22		

The complete analysis of these results will be given later, but we can now note that

<u>Method 1</u> with k=2 gives $-8{,}95 \leqq \beta \leqq -2{,}98$, which shows that the method is mathematically not justified. $(10^{-17} < \Phi(\beta) < 0{,}14 \cdot 10^{-2})$

<u>Method 2</u> with $c \approx 10^{-6}$; $\beta = 4{,}65$ gives $1{,}40 \leqq k \leqq 3{,}62$. The method is mathematically justified but the definition of k is too complicated

<u>Method 3</u> with $\alpha = 3{,}3$ gives $-4{,}65 \leqq \beta \leqq -3{,}75$, which shows that the method is mathematically more correct than 1, but a little more complicated. $(0{,}16\ 10^{-5} < \Phi(\beta) < 0{,}9 \cdot 10^{-4})$

<u>Method 4</u> with $c \approx 10^{-6}$; $\beta = 4{,}65$ gives $3{,}3 \leqq \alpha \leqq 4{,}25$. The method is mathematically justified but the definition of α is too complicated.

2. A more developed model for determining the reliability is when both the capacity of the structural element and the external load are linear functions

$$
(12) \quad \begin{cases} \sum_{i=1}^{m} a_i \cdot x_i & \text{(capacity = C)} \\ \sum_{i=m+1}^{n} a_i \cdot x_i & \text{(load = L)} \end{cases}
$$

Assuming that x_i are all independent and normal with m_i and σ_i we then have the mean and s.d. of C - L

$$
(13) \quad m = \sum_{i=1}^{m} a_i \cdot m_i - \sum_{i=m+1}^{n} a_i \cdot m_i
$$

$$(13)\quad \sigma = \sqrt{\sum_{i=1}^{n} (a_i \cdot \sigma_i)^2}$$

As the probability of failure we obtain

$$(14)\quad P(C/L \leqq 1) = \Phi\left(\frac{\sum_{i=m+1}^{n} a_i \cdot m_i - \sum_{i=1}^{m} a_i \cdot m_i}{\sqrt{\sum_{i=1}^{n} (a_i \cdot \sigma_i)^2}}\right)$$

We assume now that values ϱ_i are independent of m_i, and write

$$(15)\quad \begin{cases} \sum_{i=m+1}^{n} a_i \cdot m_i = \sum_{i=1}^{m} a_i \cdot m_i' \\ \sigma_i' = \varrho_i \cdot m_i' \end{cases}$$

We then have

$$(16)\quad P(C/L \leqq 1) = \Phi\left(\frac{(1-k) \sum_{i=m+1}^{n} a_i \cdot m_i}{\sqrt{k^2 \cdot \sum_{i=1}^{m} (a_i \cdot \sigma_i')^2 + \sum_{i=m+1}^{n} (a_i \cdot \sigma_i)^2}}\right)$$

and

$$(17)\quad k = \frac{1 + \sqrt{1 - (1 - C_1 \cdot \beta^2)(1 - C_2 \cdot \beta^2)}}{1 - C_1 \cdot \beta^2}$$

where

$$(18)\quad C_1 = \frac{\sum_{i=1}^{m} (a_i \cdot \sigma_i')^2}{\left(\sum_{i=m+1}^{n} a_i \cdot m_i\right)^2} \quad ; \quad C_2 = \frac{\sum_{i=m+1}^{n} (a_i \cdot \sigma_i)^2}{\left(\sum_{i=m+1}^{n} a_i \cdot m_i\right)^2}$$

We can see that equations (16) and (17) correspond to the earlier equations (7) and (8) for that special case of (12) which was treated before.

Using now the "characteristic values" we get

$$(19)\quad k' = \frac{\sum_{i=m+1}^{n} a_i (m_i + \alpha \cdot \sigma_i)}{\sum_{i=1}^{m} a_i (m_i' - \alpha \cdot \sigma_i')}$$

and we can calculate the corresponding β-values from (16). Some cases with $n=3$ and $m=1$ have been treated, and the results are given in Table 2. The values for m_1', σ_1', m_2, σ_2, m_3, σ_3 have been chosen so that x_1 could represent the capacity of an element, while x_2 and x_3 could represent dead and live load in a practical case. The analysis of this case follows later.

Table 2

	σ/m			k=2	$-\beta=4{,}65$	$\alpha = 2{,}80$		$\alpha = 3{,}60$	
	x_1	x_2	x_3	$-\beta$	k	k'	k/k'	k'	k/k'
1	0,1 /1,0	0,02/0,4	0,06/0,6	4,77	1,95	1,70	1,15		
2	0,1 /1,0	0,02/0,4	0,09/0,6	4,38	2,04	1,82	1,12		
3	0,1 /1,0	0,02/0,4	0,12/0,6	4,27	2,15	1,93	1,12		
4	0,1 /1,0	0,04/0,8	0,02/0,2	4,88	1,91	1,63	1,17	1,91	1,00
5	0,1 /1,0	0,04/0,8	0,03/0,2	4,85	1,92	1,66	1,16		
6	0,1 /1,0	0,04/0,8	0,04/0,2	4,81	1,94	1,70	1,14		
7	0,05/1,0	0,02/0,4	0,06/0,6	8,45	1,44	1,42	1,02		
8	0,05/1,0	0,02/0,4	0,09/0,6	7,35	1,56	1,52	1,03		
9	0,05/1,0	0,02/0,4	0,12/0,6	6,35	1,69	1,62	1,04		
10	0,05/1,0	0,04/0,8	0,02/0,2	9,15	1,38	1,36	1,02		
11	0,05/1,0	0,04/0,8	0,03/0,2	8,95	1,40	1,39	1,01		
12	0,05/1,0	0,04/0,8	0,04/0,2	8,58	1,42	1,42	1,00		

What has been said earlier of case 1 holds good here. In addition it can be seen that the α-values giving $\beta = -4{,}65$ are considerably smaller than in case 1. In case 1 we had $3{,}3 \leqq \alpha \leqq 4{,}25$ and here $2{,}8 \leqq \alpha \leqq 3{,}60$.

This will be explained. From (16) and (19) we obtain

$$\alpha = -\beta \frac{\sqrt{k'^2 \sum_{i=1}^{m} (a_i \cdot \sigma_i')^2 + \sum_{i=m+1}^{n} (a_i \cdot \sigma_i)^2}}{k' \sum_{i=1}^{m} a_i \cdot \sigma_i' + \sum_{i=m+1}^{n} a_i \cdot \sigma_i} \tag{20}$$

Replacing the variables we have

$$\alpha = -\beta \frac{\sqrt{\sum_{i=1}^{n} (u_i)^2}}{\sum_{i=1}^{n} u_i} \tag{21}$$

At least two conclusions can be made from (21):

- α decreases with increasing n and constant
- α decreases with u_i values of the same size and increases with u_i values of variable sizes.

It can also be seen from Fig. 1 and Table 2 that both these conclusions hold good.

Case 2 is the most general case which the author has been able to treat in an exact mathematical form. More complicated cases, such as cases 3 and 4, have been treated approximately by computer.

3. As the third example we shall study the breakage of a rectangular prestressed or reinforced concrete element, when the section is partly cracked and the tension of the reinforcement has reached the yield-limit. The section is loaded with a moment.

The condition of failure is, on the basis of elementary statics:

$$(22) \quad 1 - \frac{S_{1g} + S_{1p}}{x_1 \left(x_2 - \dfrac{x_6 \cdot x_1}{x_4 \cdot x_5 \cdot x_3}\right)} \leq 0$$

where S_{1_g} = the external moment caused by the invariable load.

S_{1_p} = the external moment caused by the extremal value of the variable load.

x_1 = the tensile-load of the reinforcement, when the tension has reached the yield-limit.

x_2 = the distance of the reinforcement from the compressed edge of the element.

x_3 = the fullness of the compressed section of the element, (usually called β)

x_4 = the ultimate compressive strength of the concrete.

x_5 = the breadth of the compressed zone.

x_6 = the distance of the centre of gravity of the compressed zone from the compressed edge in relation to the height of the compressed zone.

All distributions are assumed to be normal. The means and standard deviations are given in Table 3. The values of k corresponding to the probability of failure 10^{-6} have been calculated by computer and the values k′ have been found with (23) using $\alpha = 1,90$.

$$(23) \quad k' = \frac{S^*_{1_g} + S^*_{1_p}}{x^*_1 \left(x^*_2 - \dfrac{x^*_1 \; x^*_3}{x^*_4 \; x^*_5 \; x^*_6}\right)} : \frac{m_{S_{1g}} + m_{S_{1p}}}{m_{x1} \left(m_{x2} - \dfrac{m_{x1} \; m_{x3}}{m_{x4} \; m_{x5} \; m_{x6}}\right)}$$

It can be seen that the greatest α-value among these cases (giving $k/k' = 1,0$ in case 10) is 2,20.

The analysis of the results follows later.

4. As the last example we again study a similar reinforced concrete section, but the section is now loaded with a moment and a normal force. We get the condition of failure [9]

$$(24) \quad 1 - \frac{S_1 + S_2(x_2 - c)}{x_2(x_1 + S_2)\left(1 - \dfrac{x_6(x_1 + S_2)}{x_2 \; x_3 \; x_4 \; x_5}\right)} \leq 0$$

The loads and forces are dependent in the following ways:

Cases 1 and 2: $\begin{cases} S_{1g} = 0{,}5\ g \\ S_{2g} = 1{,}0\ S_{1g} \end{cases}$ $\begin{cases} S_{1p} = 0{,}08\ p_2 + 0{,}12\ p_3 \\ S_{2p} = 0{,}10\ p_1 + 0{,}20\ p_2 \end{cases}$

Cases 3 and 4: $\begin{cases} S_{1g} = 0{,}2\ g \\ S_{2g} = 1{,}5\ S_{1g} \end{cases}$ $\begin{cases} S_{1p} = 0{,}20\ p_1 + 0{,}30\ p_2 \\ S_{2p} = 0{,}40\ p_1 + 0{,}10\ p_2 \end{cases}$

g = the invariable load

p_1, p_2, p_3 = different independent variable loads

x_1, x_2, x_3, x_4, x_5, x_6 = as in example 3.

All distributions are assumed to be normal. The number N, which gives the relation between the life time of the construction and the interval used to define the d.f. of the variable loads, is here 10. The means and s.d.: s are given in Table 4. The values of k and k' corresponding to $\alpha = 1{,}8$ have been calculated as before.

Table 3

Quantity / Case	S_{1g}	S_{1p}	x_1	x_2	x_3	x_4
1	0,025/0,5	0,1/1,0	0,02/1,0	0,1/5,0	0,08/1,0	0,15/1,0
2	0,05 /1,0	0,1/1,0	0,02/1,0	0,1/5,0	0,08/1,0	0,15/1,0
3	0,25 /5,0	0,1/1,0	0,02/1,0	0,1/5,0	0,08/1,0	0,15/1,0
4	0,025/0,5	0,2/1,0	0,05/1,0	0,25/5,0	0,08/1,0	0,15/1,0
5	0,05 /1,0	0,2/1,0	0,05/1,0	0,25/5,0	0,08/1,0	0,15/1,0
6	0,25/5,0	0,2/1,0	0,05/1,0	0,25/5,0	0,08/1,0	0,15/1,0
7	0,025/0,5	0,1/1,0	0,02/1,0	0,4/20,0	0,08/1,0	0,15/1,0
8	0,05 /1,0	0,1/1,0	0,02/1,0	0,4/20,0	0,08/1,0	0,15/1,0
9	0,25 /5,0	0,1/1,0	0,02/1,0	0,4/20,0	0,08/1,0	0,15/1,0
10	0,025/0,5	0,2/1,0	0,05/1,0	1,0/20,0	0,08/1,0	0,15/1,0
11	0,05 /1,0	0,2/1,0	0,05/1,0	1,0/20,0	0,08/1,0	0,15/1,0
12	0,25 /5,0	0,2/1,0	0,05/1,0	1,0/20,0	0,08/1,0	0,15/1,0

Quantity / Case	x_5	x_6	k (10^{-6})	k' $\alpha = 1.90$	k/k'
1	0,0/1,0	0,08/1,0	1,76	1,61	1,09
2	0,0/1,0	0,08/1,0	1,75	1,59	1,10
3	0,0/1,0	0,08/1,0	1,70	1,55	1,10
4	0,0/1,0	0,08/1,0	2,18	2,02	1,08
5	0,0/1,0	0,08/1,0	2,09	1,95	1,07
6	0,0/1,0	0,08/1,0	1,92	1,80	1,05
7	0,0/1,0	0,08/1,0	1,38	1,32	1,05
8	0,0/1,0	0,08/1,0	1,32	1,30	1,02
9	0,0/1,0	0,08/1,0	1,27	1,27	1,00
10	0,0/1,0	0,08/1,0	1,90	1,57	1,21
11	0,0/1,0	0,08/1,0	1,76	1,51	1,17
12	0,0/1,0	0,08/1,0	1,56	1,40	1,11

Analysis of the results.

From the preceding examples we can see that it is possible to define the total safety factors, which correspond to some probability of failure, here $\sim 10^{-6}$. We have also seen that even in the simplest cases this definition is rather complicated and leads to a number of different values. Method 1

Table 4

Quantity / Case	g	p_1	p_2	p_3	x_1	x_2
1	0,05/1,0	0,10/1,0	0,10/1,0	0,10/1,0	0,05/1,0	0,05/1,0
2	0,05/1,0	0,10/1,0	0,10/1,0	0,10/1,0	0,02/1,0	0,02/1,0
3	0,05/1,0	0,10/1,0	0,10/1,0		0,05/1,0	0,05/1,0
4	0,05/1,0	0,10/1,0	0,10/1,0		0,02/1,0	0,02/1,0

Quantity / Case	x_3	x_4	x_5	x_6	k	k' $\alpha = 1.80$	k/k'
1	0,08/1,0	0,50/5,0	0,00/1,0	0,04/0,55	1,39	1,41	0,99
2	0,08/1,0	1,5/10	0,00/1,0	0,04/0,55	1,40	1,22	1,15
3	0,08/1,0	0,50/5,0	0,00/1,0	0,04/0,55	1,55	1,45	1,07
4	0,08/1,0	1,5/10	0,00/1,0	0,04/0,55	1,47	1,24	1,18

seems to have no mathematical justification and Method 2 seems to be much too complicated for practical purposes.

We are now going to compare methods 3 and 4. From (21) it can be seen that, assuming the various u_i values to be equal, we get for $-\beta = 4,65$ the relation in Table 5 between α and n. The relation holds good with the conditions given in example 2. If the u_i-values are not equal, the α-value tries to increase.

Table 5

n	1	2	3	4	5	6	7	8
α	4,65	3,29	2,69	2,33	2,08	1,90	1,76	1,65

Tables 3 and 4 show that method 3 gives quite satisfactory results even when the conditions of example 2 do not hold good. However, we can see that with increasing n we get smaller α-values, and also that with very different standard deviations for some essential quantities, the α-values corresponding to $\beta = -4,65$ begin to increase.

It does not seem mathematically justified, to use always the same α-values, independent of the structure and other circumstances. It is also impossible to define the α-values separately for all cases.

A compromiss between methods 3 and 4 could perhaps lead to results satisfying the conditions given at the beginning of this paper. Using a computer we could find different α-values for different types of structures, corresponding to e.g.,

- a timber column with normal force
- a prestressed rectangular beam with moment
- a steel column with normal force and moment.

The α -values should be given in standards, and would form a basis for the design of structures. The standard deviations of different factors should also be given in the standards.

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SUMMARY

Four different design methods are compared, two based on the use of a "total safety factor" and two on the use of "characteristic values". Four examples are treated and it is seen that in these cases the method using "characteristic values" is more reasonable than the other. Some conclusions on the way of determining the characteristic values have also been made.

RESUME

Quatre méthodes d'étude différentes sont comparées, deux méthodes se basent sur l'emploi d'un "facteur total de sécurité" et les deux autres sur l'emploi des "valeurs caractéristiques". Quatre exemples sont traités et l'on y voit que dans ces cas la méthode qui emploie les "valeurs caractéristiques" est plus raisonnable que l'autre. On a tiré aussi quelques conclusions de la façon déterminée des valeurs caractéristiques.

ZUSAMMENFASSUNG

Vier verschiedene Bemessungsmethoden sind verglichen worden, zwei von ihnen gründen sich auf die Verwendung von einem "totalen Sicherheitsfaktor" und zwei auf die Verwendung von "charakteristischen Werten". Vier Beispiele sind behandelt worden und als Ergebnis hat man festgestellt, dass die Methode mit den "charakteristischen Werten" in diesen Fällen zweckmässiger als die anderen sind. Auch einige Schlussfolgerungen über die Art dieser Werte sind gemacht worden.

Artificial Equation Errors

Erreurs d'équations artificielles

Künstliche Gleichungsfehler

MILÍK TICHÝ
Building Research Institute
Technical University
Prague, Czechoslovakia

Definition of the equation error

In structural analysis and design various prediction formulas are used to describe behaviour of structures, structural elements and sections under different conditions. In most cases, these formulas do not exactly predict the response of the structure, so that the resulting quantities, y^*, deviate less or more from the assumed reality, y, even if exact values or input quantities, g_i, are introduced into the formulas. These deviations are called equation errors; they can be defined either as the differences

$$\Delta = y^* - y$$

or the ratios

$$\lambda = \frac{y^*}{y} .$$

The second definition will be kept in the following text.

Character of the equation errors

Essentially, the equation errors can have two distinctly different characters.

First of all, the equation errors can be caused by unsufficient knowledge of the predicted phenomenon. In most formulas only a certain part of primary quantities influencing the phenomenon (say, g_1 through g_k) is involved, whereas the remaining quantities (g_{k+1} through g_n) are not considered for various reasons. Sometimes even their effect is not known at all.

It is evident that neglecting of only one influence would make the equation error systematical. However, the number of influences and, thus, of primary quantities which are not considered in the formula is frenquently large. Due to this fact and due also to the complexity of phenomena the equation errors acquire random character and, consequently, can be treated by statistical methods.

The equation errors of this type are a necessary but unintentional consequence of our limited possibilities of predicting the phenomena. Therefore, they can be considered as natural.

On the other hand, however, the mathematical model of the phenomena is well known in many cases, but the resulting formulas are too complex. The practical designers often demand their simplification in order to facilitate the design procedures. If, then, an approximate formula giving y^* is found it must always lead to an equation error, λ. It is clear, that this equation error is an intentional, though unwanted consequence of the approximation, and it can be considered as artificial.

Natural equation errors have been already discussed by some authors (Zsutty /1/, Tichý-Vorlíček /2/, Murzewski /3/). Therefore, no special attention will be paid to them in this contribution which, on the other hand, will concentrate on a particular type of the artificial equation error, resulting from simplification of prediction formulas.

Problem formulation

Assume that a function

$$y = f(g_1, g_2, \ldots, g_n) \tag{1}$$

is perfectly defined in the given range, i.e. its magnitude is known for any set of primary quantities g_1 through g_n.

Evidently, function (1) can be replaced by different functions, e.g. by polynomials, Fourier series, etc. However, if a simplification of the original function (1) is desired, the choice of substitutes is rather limited. It appears that for practical purposes an important simplification can be achieved by substituting for (1) the following exponential function

$$y^* = g_1^{x_1} \cdot g_2^{x_2} \cdot g_3^{x_3} \cdots g_n^{x_n} \tag{2}$$

where x_1 through x_n are constant exponents which are to be found for each separate function and for each set of ranges of primary quantities g_i. By taking the logarithm of Eq. (2) it follows that

$$\log y^* = x_1 \log g_1 + x_2 \log g_2 + \ldots + x_n \log g_n \tag{3}$$

It is clear that the computation of the investigated quantity is reduced to the summing of logarithms of primary quantities and taking the antilogarithm of the sum. This reduction has a considerable importance by itself, since in most cases the number of mathematical operations involved will be limited to a few. Thus the errors due possibly to the imprecision of the computation itself are to a large extent eliminated and, what is probably often more important, a source of human errors (e.g. the omission of some operation) is lessened. The establishing of exponents x_i is, as it will be shown, in general simple, even if some practical problems must be solved.

Function (2) has already been successfully used by Zsutty /1/ for multiple regression analysis of ultimate strength tests of reinforced concrete sections. The intention of his work was to fit a function to populations of experimental results and to values of primary quantities applying in the ultimate strength which were ascertained in the tests. Equation errors resulting from this are of the natural type.

Method of Solution

The problem of finding the unknown exponents x_i can be solved by using the least squares method for logarithms of y and y^*. The sum of squares of differences between $\log y^*$ and $\log y$ should be minimum, i.e. symbolically.

$$\sum_{j=1}^{M} (\log y_j^* - \log y_j)^2 = \text{minimum} \tag{4}$$

where M is the number of points for which the difference is found.

Considering the decimal logarithms, it is convenient to put

$$g_1 = 10.0 \tag{5}$$

thus the first factor of the right-hand side of Eq.(2) will be a constant.

Substitute for y^* from Eq. (2) into Eq. (4).

$$\sum_{j=1}^{M}(x_1 \log g_{1j} + x_2 \log g_{2j} + \dots + x_n \log g_{nj} - \log y_j)^2 = min$$

and differentiate successively by unknowns x_i. In order to minimize the left-hand side of the Eq. (4) the first derivative must be put equal to zero. Hence

$$\sum_{j=1}^{M}(x_1 \log g_{1j} + x_2 \log g_{2j} + \dots + x_n \log g_{nj} - \log y_j) \log g_{ij} = 0$$

for all $i = 1, 2, \dots n$.

After rearrangement and taking into account Eq. (5) a system of $n+1$ simultaneous linear equations is obtained:

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1$$

$$a_{12} x_1 + a_{22} x_2 + \dots + a_{2n} x_n = b_2$$

$$\dots$$

$$a_{1n} x_1 + a_{2n} x_2 + \dots + a_{nn} x_n = b_n$$

where

$$a_{ik} = \sum_{j=1}^{M} \log g_{ij} \cdot \log g_{kj}$$
$$b_i = \sum_{j=1}^{M} \log g_{ij} \cdot \log y_j$$

It is clear that this system can be easily solved for the unknowns x_i .

If the least squares method is used for substituting a set of experimentally obtained values y_j by a function depending on parameters g_i, the magnitudes of y_j's and g_i's are known from the tests; they form a discrete population of points. This is schematically shown for y depending on one parameter only, g_2 in Fig. 1. However, if applying the method to the substitution of

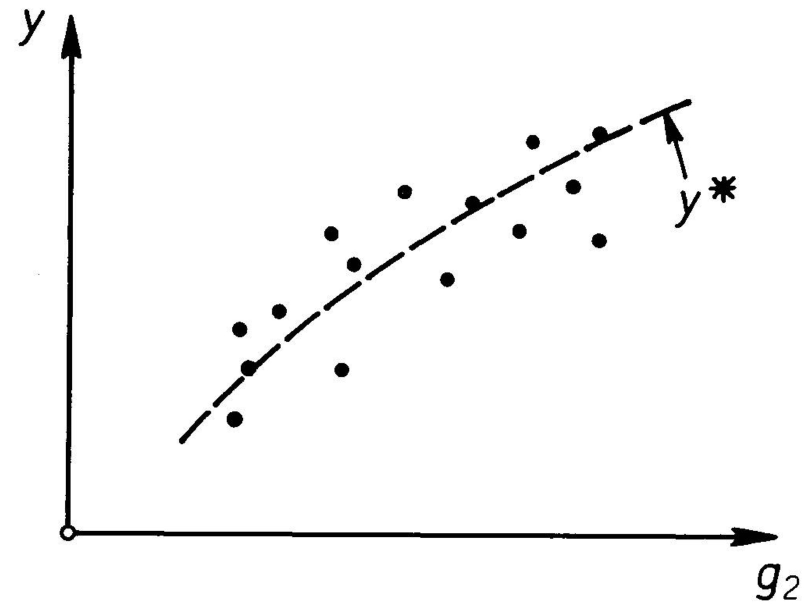

Fig. 1. - Substitute function derived from experimental results.

a defined continuous function, values of y_j and g_i must be artificially generated (Fig. 2). The larger will be the number of generated points, M , the better will be the fit.

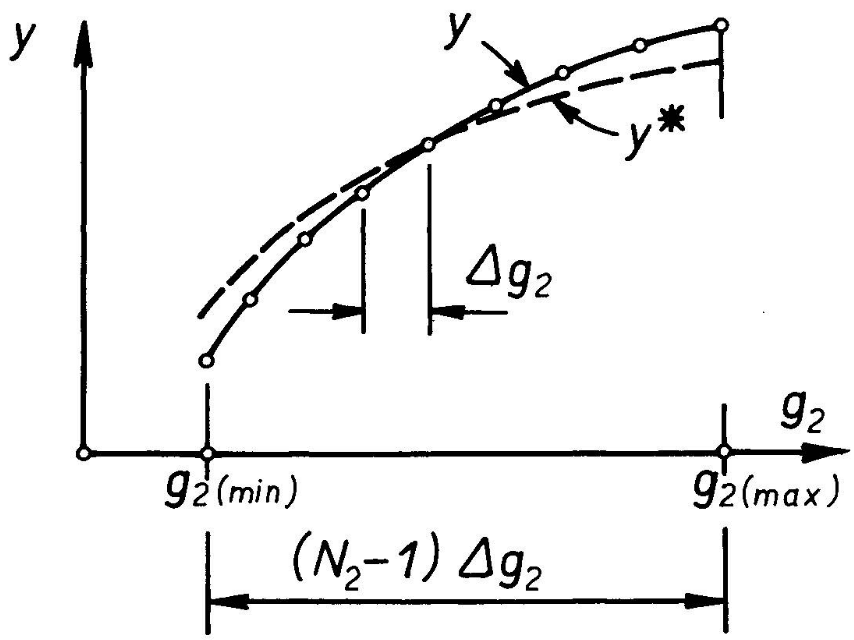

Fig. 2. - Substitue function derived from another defined function.

To generate the population of points, intervals for individual primary quantities g_i must be defined by lower and upper bound values, g_i^{I} and g_i^{II}, respectively. These intervals are divided into $(N_i - 1)$ divisions, the width of which is Δg_i. Thus, the number of points generated in this way will be given by the product

$$M = \prod_{i=1}^{n} N_i$$

Accepting Eq. (5), no interval is defined for g_1, so that $N_1 = 1$. For all primary quantities the number of divisions may either be equal or it might differ, taking into account the influence of the quantity on the result.

Quasi-randomness of the equation error

It appears from some tentative tests that the distribution of the quantity

$$\lambda = \frac{y^*}{y} \tag{6}$$

has a surprisingly random character, even if the number of points, M, is relatively small (of the order of 100). This seems to be rather illogical since both functions, y and y^*, are deterministically given. No explanation of the quasi-random behaviour of λ can be given today. And it cannot be generalised, neither.

The quasi-randomness of the equation error λ suggests the use of the statistical parameters of its distribution as quantities suitable for checking the effectiveness of the substitue function. These parameters should evidently be: the mean $\bar{\lambda}$, the standard deviation s_λ, and also the coefficient of skewness, a_λ, defined by

$$a_\lambda = \frac{\mu_3^c}{s_\lambda^3}$$

where μ_3^c is the third central moment of the statistical distribution of λ. Further statistical parameters are not considered, the above three being quite sufficient for the purpose.

The mean, $\bar{\lambda}$, should be near to unity. Actually, this has proven to be true in all cases investigated to date (for examples of practical application of the method see the author's papers /4/ and /5/). The mean is not greatly influenced by the number of generated points, M.

The standard deviation, s_λ, supplies the information on the spread of λ about the mean. It can be stated, that the less the s_λ, the better the fit. In general, the value of s_λ decreases with the number of points, M; the contribution of individual primary quantities not being uniform, a further decrease of s_λ may be achieved by concentrating the increase in M to some of the quantities only. Since a perfect fit is never possible, the standard deviation converges to some definite value.

A similar importance is attached to the coefficient of skewness, a_λ. In general its value differs from zero, towards negative or positive values. In practical cases values of λ either at the left-hand or at the right-hand tail of the statistical distribution are important, depending upon the nature of the problem. It cannot be said that a value of a_λ near to zero would be the most convenient one - a definite skewness may be often more favourable than a zero skewness.

If computing the statistical parameters of the population of values λ obtained for those M generated points which were used in deriving the unknown exponents α_i, incorrect conclusions might be drawn. Evidently, in this way the quality of the fit is checked just by those points from which α_i's has been established. Therefore, it is necessary to check the fit also for points which are removed as far as possible from the M points of the original population. Such points lie just between the original ones (for one-dimensional function this is schematically shown in Fig. 3). A new, control population of values y and y^* is generated for the control system of points and

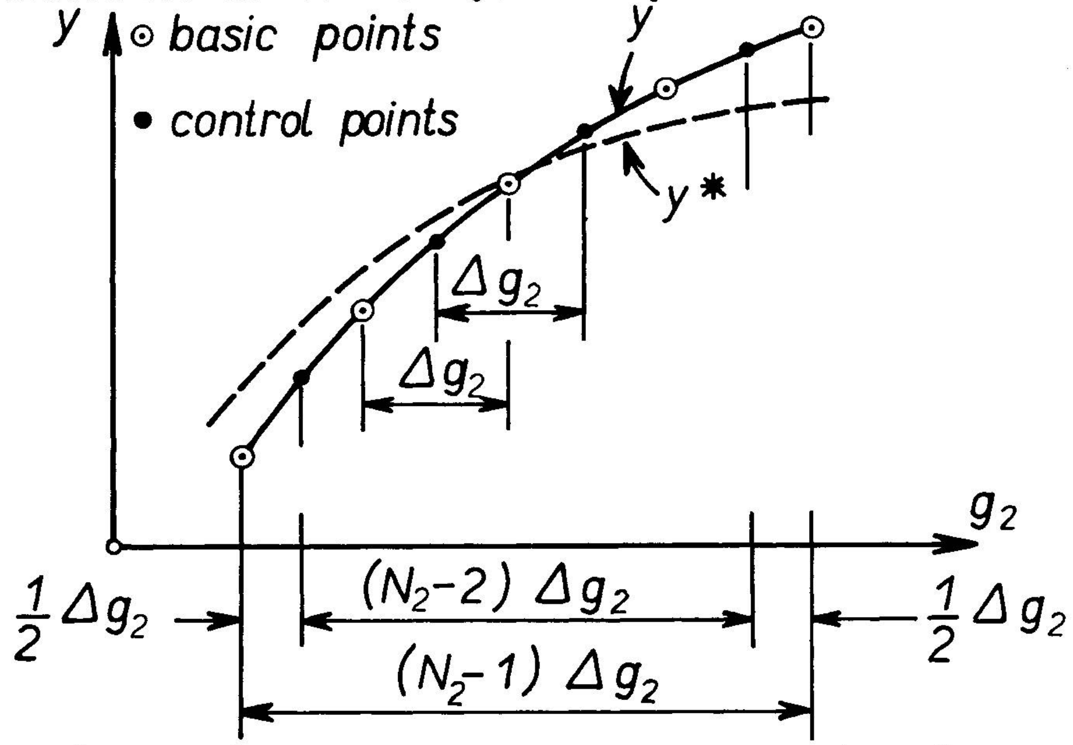

Fig. 3. - Basic and control system of values(y, g_2)for function y depending on one primary quantity.

again the statistical parameters of the equation error, λ , are found. It is obvious that the number of points of the control population will be smaller that M, being equal to

$$m = \prod_{i=1}^{m} (N_i - 1)$$

For the assessment of the new set of statistical parameters the same is valid as for the original population.

Partial safety factor

Modern structural code formats as introduced by the European Concrete Committee, CEB, International Building Council, CIB, and others use to cover the approximations of design assumptions the partial safety factor γ_a belonging to a wider family of factors γ. This factor is frequently introduced only in conceptual terms, or sometimes, directly by means of an empirical value; yet, no method of establishing γ_a has been so far presented.

Now, using the above quasi-randomness of the equation error, λ, the partial safety factor γ_a can be defined. The procedure is outlined as follows: Computing the value of y^* for a given set of q_i's it is known that in comparison with the exact value of y the result is charged by equation error λ. However, the magnitude of λ is not known. On the other hand it is known that the quasi-randomness of λ is described by the statistical parameters established for the whole population of points investigated. Assuming now a convenient statistical distribution for the description of the random behaviour of λ (Fig. 4) the adequate quantile, either λ_{min} or λ_{max}, can be found for a chosen probability $P(\lambda < \lambda_{min})$ or $P(\lambda > \lambda_{max})$.

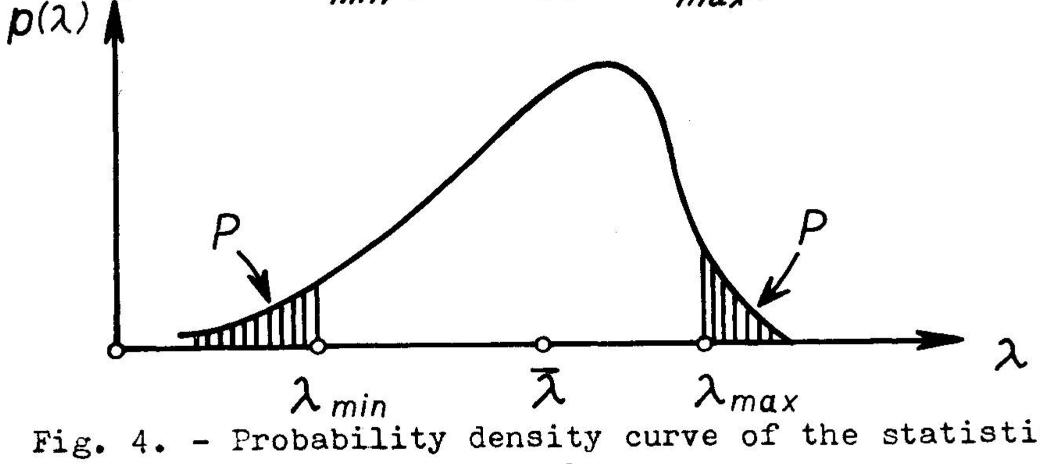

Fig. 4. - Probability density curve of the statistical distribution of λ.

The values of the quantiles are given by

$$\lambda_{min} = \bar{\lambda} + t_{min}\, s_\lambda \tag{7}$$

$$\lambda_{max} = \bar{\lambda} + t_{max}\, s_\lambda \tag{8}$$

where t_{min} (with negative sign) or t_{max} are values of standardized random variable (with zero mean and unit variance) depending on the type of the selected distribution and its statistical parameters, and on the probability $P(\lambda < \lambda_{min})$ or $P(\lambda > \lambda_{max})$, respectively.

At the present state of knowledge it is not possible to say in general which type of statistical distribution should be used. In the cases investigated until now (see /4/ and /5/) log-normal distribution proved to be adequate for modelling the quasi-randomness of λ (it must be mentioned here that the log-normal distribution can have both positive or negative skewness; this fact is not commonly known). In other cases, of course, other types od statistical distribution might be applied. It is evident that the optimum type would depend upon the type of the substitute function y^*.

Similarly, no definite answer, based on some theoretical analysis can be given with regard to the probability $P(\lambda < \lambda_{min})$, $P(\lambda > \lambda_{max})$. The problem must be solved within the whole context of statistical design. However, it may be tentatively said that a reasonable value of P would be 10^{-2} for cases of minor importance (e.g. checking the stiffness of a current beam) and about 10^{-3} in more important cases (ultimate strength, etc.). It must be stressed here that the probability P has nothing to do with the probability of failure P_U, since it represents entirely different statistical phenomenon. The use of the method is not restricted to a probabilistic code format only, it can be used also with the classical codes.

Assume, now, for instance, a quantity y the over-estimate of which in comparison with its real value is on the unsafe side in the design (e.g. the ultimate moment of a section). After finding the substitute value y^* it is obviously not known if its deviation from y is at the left or right-hand tail of the distribution of λ. To be on the conservative side the worst must be anticipated and, therefore, λ_{max} must be considered in the design, i.e. in order to obtain the design value y^d, y^* must be divided by λ_{max}, i.e.:

$$y^d = \frac{1}{\lambda_{max}} \cdot y^*$$

The same procedure is used for cases where negative deviations are unsafe:

$$y^d = \frac{1}{\lambda_{min}} \cdot y^*$$

Consequently, the partial safety factor γ_a covering the equation error is defined as

$$\gamma_a = \frac{1}{\lambda_{max}} \tag{9}$$

or

$$\gamma_a = \frac{1}{\lambda_{min}} \tag{10}$$

It is clear that for cases where λ_{min} is to be used negative skewness of the distribution will be more favourable than a positive one; the opposite is true for λ_{max}.

The use of the factor γ_a can be seen from the following example:

The condition of safety of a reinforced concrete section for the limit state of failure is e.g.

$$1.5\, M_D + 1.8\, M_L \leqq \phi\, M_U \tag{11}$$

where M_D, M_L are moments produced in the section by dead load and live load respectively, M_U is the ultimate moment of the section calculated by means of an exact formula and ϕ is the capacity reduction factor covering the random behaviour of the materials, dimensions etc. (actually, ϕ is again a partial safety factor).

Using now a substitute formula for the ultimate moment, value M_U^* is calculated and the condition of safety (11) must be changed to

$$1.5\, M_D + 1.8\, M_L \leqq \gamma_a\, \phi\, M_U^*$$

where evidently in thus case γ_a is defined by $1/\lambda_{max}$, i.e. $\gamma_a < 1$, since higher design ultimate moment, M^d, would give an unsafe result. For further examples see again /4/ and /5/.

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SUMMARY

When known deterministic formulas are approximated by more simple formulas artificial equation errors are encountered. The deviation between the results obtained by both formulas can be mathematically treated. It has been found in some cases that the behaviour of the artificial equation error is quasi-random, so that it can be described by a suitable statistical distribution. This fact has a signifiance for the practical design, since partial safety factors can be mathematically derived.

RESUME

Si des formules déterministiques connues sont approximées par des formules simplificatives, les erreurs d'équation artificielles entrent dans le calcul. Les dérivations entre les résultats donnés par les deux formules (originale et simplifiée) peuvent être traitées mathématiquement.

Il a été trouvé en quelques cas étudiés que l'erreur d'équation artificielle est quasi-aléatoire et qu'on le peut décrire par une distribution statistique convenable. Ce fait a une importance pour le calcul pratique: le coefficient partiel de sécurité de calcul approximatif peut être dérivé par des méthodes mathématiques.

ZUSAMMENFASSUNG

Wenn man genaue durch vereinfachte Funktionen darstellt, entsteht ein künstlich geschaffener Gleichungsfehler (wie er in der klassischen linearen Regressionsrechnung als im Quadrat zu minimierende Abweichung auftritt). Die Abweichung der durch die beiden Funktionen entstehenden Ergebnisse kann berechnet werden. Der künstliche Gleichungsfehler verhält sich sozusagen zufällig, so dass er durch eine Dichtefunktion dargestellt werden kann. Dieser Umstand hat einen für die Praxis unübersehbaren Vorteil, sintemal Teilsicherheitsbeiwerte mathematisch hergeleitet werden können.

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A Logical System for Partial Safety Factors

Un système logique de coefficients partiels de sécurité

Ein logisches System für Teilsicherheitsfaktoren

MILÍK TICHÝ
Building Research Institute
Technical University
Prague, Czechoslovakia

It is now generally accepted that a system of partial safety factors proves to be practical in structural design. These factors cover a wide spectrum of various influences which are due to the properties of loads or to the properties of structures. The character of the influences is very heterogeneous and thus the methods of establishing the values of partial safety factors are still discussed and not yet settled in general. In spite of this fact systems of partial safety factors were recently proposed by several international organizations, particularly by the European Concrete Committee, CEB /1/, International Building Council, CIB /2/, and International Standard Organization, ISO /3/.

However, all these systems have some of the following drawbacks:

a) they are not universal, i.e. they are often developed from the point of view of a particular type of structures;

b) they do not strictly separate factors according to the individual influences (e.g. factors attributed to loads and load-effects depend upon the material properties, methods of construction, etc.);

c) they are not flexible enough to enable continuous developments of design codes;

d) factors are distributed unevenly, i.e. some influences are stressed too much, others are disregarded at all.

Table 1. The Proposed System of Partial Safety Factors.

Origin of the influence	Character of the influence					
	Random		Non - random			
	Occurrence of unfavourable random events	Simultaneous occurrence of random events	Exactness of analysis, basic assumptions, etc.	Mode of occurrence of unfavourable events	Apparently non-random properties	Consequence of occurrence of unfavourable events
Structure	γ_{S1}	γ_{S2}	γ_{S3}	γ_{S4}	γ_{S5}	γ_{S6}
Load	γ_{L1}	γ_{L2}	γ_{L3}	γ_{L4}	γ_{L5}	γ_{L6}

The above drawbacks cause difficulties in communication among different national and international organizations preparing design codes. To avoid this, a system can be developed, with a two-way classification of partial safety factors: according to the origin of influences they should cover (loads, structures), and, secondly, according ot the character of the influences (random, non-random, etc.). The proposed system is shown in Table 1, where each factor must be considered as a symbol for a group of factors covering influences of the same origin and character. A more detailed explanation of factors in Table 1 will clear the idea:

<u>Factors γ_{S1}</u> cover random behaviour of separate:

- material properties (strength, moduli of elasticity, etc.),
- dimensions,

artificial stress states (prestressing force),

or, integrally, random behaviour of the structural resistances (ultimate load, cracking load), or other important quantities (width of cracks, deflection, etc.).

The main aim of factors γ_{S1} is to ensure a low probability of occurence of unfavourable events.

Similarly, <u>factors γ_{L1}</u> express random behaviour of separate loads, or load-effects.

<u>Factors γ_{S2}</u> take into account low probability of simultaneous occurrence of two or more unfavourable random events, e.g. occurrence of minimum strength of concrete and steel, minimum ultimate bending moments in a statically indeterminate structure, etc.

<u>Factors γ_{L2}</u> have the analogous meaning for loads.

<u>Factors γ_{S3} and γ_{L3}</u> cover

- intentional or unintentional approximations accepted in the analysis, simplifications of hypotheses, etc.,
- uncertainties in basic assumptions.

<u>Factors γ_{S4}</u> take into account the mode of occurence of unfavourable events in the structure (e.g.brittle fracture).

Factors γ_{L4} cover unfavourable modes of load action: impact, repeated loads, etc.

Factors γ_{S5} and γ_{L5} should cover all deviations from some average behaviour which cannot be treated statistically at the present time, e.g. corrosion, emergency loads.

Factors γ_{S6} and γ_{L6} take into account consequences of structural failure (in a wider sense of word). If the damages concern the structure (its serviceability and durability) factors γ_{S6} would apply, whereas γ_{L6} would be used if objects carried or protected by the structure are endangered (goods, people). Since the border between the two domains of application may be arbitrary in many cases, both groups of factors, γ_{S6} and γ_{L6} , might be unified into one.

The proposed system of partial safety factors can be used for any type of structures, structural materials and loads. The quantitative meaning of particular factors may be different in separate but the qualitative meaning will not change.

R e f e r e n c e s

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SUMMARY

In order to enable the communication between different national and international bodies working in the domain of design standardization, as well as between designers on the whole, a simple universal system of partial safety factors is proposed and discussed. The system is based on a two-way classification of origins and characters of influences occurring in the structural design.

RESUME

Pour simplifier la communication entre les différentes organisations nationales et internationales dans le domaine du calcul des constructions et aussi entre les ingénieurs de projet eux-mêmes, un système universel de coefficients partiels de sécurité est proposé. Le système est basé sur une classification bi-dimensionelle.

ZUSAMMENFASSUNG

Um die Verständigung zwischen verschiedenen nationalen und internationalen Organisationen auf dem Gebiete der Bemessung der Baukonstruktionen zu verbessern, ist ein einfaches allgemeines System der Teilsicherheitsfaktoren entworfen worden. Das System nützt eine zweidimensionale Klassifizierung der Einflüsse auf die Sicherheit aus.