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Computer Experiments Concerning Random Nonlinear Structural Behaviour

Expériences sur ordinateur concernant le comportement aléatoire non-linéaire des structures

Rechnerexperimente für zufälliges, nichtlineares Bauwerkverhalten

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1 – INTRODUCTION

While rational probabilistic approaches to questions of structural safety and reliability have received considerable attention in recent years, the actual application of such concepts in practical structural design has been quite limited. Certainly part of the reluctance to employ probabilistic concepts in practice is explained by the inadequate state of our knowledge concerning the statistical character of structural loading and structural behaviour. While the stochastic character of the loads acting on structures must presumably be deduced from field observations, it is not likely that statistically meaningful information on structural behaviour, for structures of any complexity, can be obtained from field or laboratory tests because of practical limitations on the size of the statistical samples available.

These considerations suggest that it would be of value to develop analytical methods which would enable one to predict stochastic structural response characteristics from the knowledge of variability in the properties of structural materials. At present such a statistical theory of structures exists only for the study of brittle behaviour. It was established by Weibull (1). For other types of material behaviour, some particular results have been obtained (2).

The present paper employs a previously suggested numerical approach (3) to study randomness in the nonlinear behaviour of plane skeletal structures. A computer program developed to analyse framed structures possessing rather arbitrary nonlinear moment-curvature relations (4) is utilised to study the flexural behaviour of samples of randomly formed, nominally identical structures. The randomness considered derives from the uncertain nature of the mechanical properties of the materials, as reflected in the moment-curvature diagrams of the unit elements used to form the structures; structure geometry as well as conditions of loading and support are considered to be deterministic. The intent is thus to examine the manner in which the stochastic character of structural behaviour is influenced by the probabilistic nature of material properties.

In the examples presented, the element moment-curvature diagrams were assumed to belong to a prescribed statistical populations and prismatic beams were formed by combining elements chosen at random from this population. The tri-linear diagrams employed closely represent the nonlinear behaviour to be expected in conventionally reinforced concrete beams (5). The distribution of M for a given curvature was assumed to be Gaussian with a coefficient of variation of 10%. Load-deflection curves were determined for a sample of twenty beams, each containing forty elements, under several conditions of loading and support.

While the examples considered are relatively simple, it is felt that they offer some insight into the little understood subject of random nonlinear behaviour. The proposed method of computer simulated numerical experiments may be readily extended to include more general structures and statistical distributions.

2 – RANDOMNESS OF MOMENT-CURVATURE RELATIONS AND OF STRUCTURAL BEHAVIOUR

If several nominally identical flexural elements are tested in pure bending in the same manner, different moment-curvature relations M(θ) will be obtained, Fig. 1a, and a probabilistic distribution may then be defined for the set of relations. In what follows, it is assumed as a working hypothesis that the distribution of M for a given value of θ is normal, with a mean value $\overline{M}$ and a coefficient of variation c_v which is independent of θ. Thus a particular diagram $M_i(\theta_i)$ is related to the mean diagram $\overline{M}(\theta)$ by $M_i = \overline{M}(\theta_i)\,(1+\alpha\, c_v)$, where α measures (in standard deviations) the distance of the considered diagram from the mean diagram. Since the diagrams, in the regions of practical interest, are monotonic, it follows that

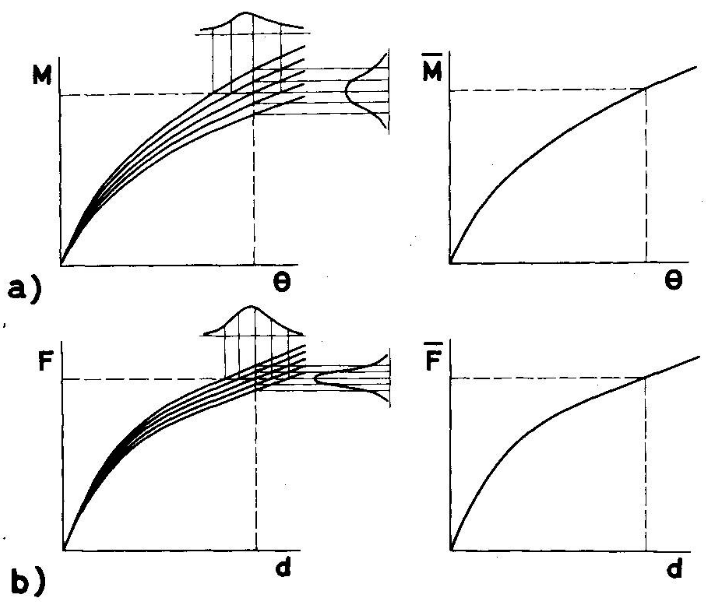


Fig. 1 – Randomness of mechanical properties and of structural behaviour

$$P_r(M < M_i | \theta_i) = P_r(\theta > \theta_i | M_i)$$

It should be noted that the distribution of θ for a given value of M is not normal, since this occurs only if the diagrams are straight and parallel. It is also clear from Fig. 1 that the distribution of θ is not symmetrical but is skewed toward the larger values of θ. The dispersion and skewness increase with increasing M .

Mechanical and statistical considerations suggest that the hypothesis of normality for the distribution $P_r(M < M_i | \theta_i)$ is a reasonable one. Since the bending moment M is merely the integral, taken over the cross section, of the stresses developed at elementary areas multiplied by corresponding distances to the neutral line, for a given state of deformation (curvature) the central limit theorem implies an approximately normal probabilistic distribution of M for the population of unit elements.

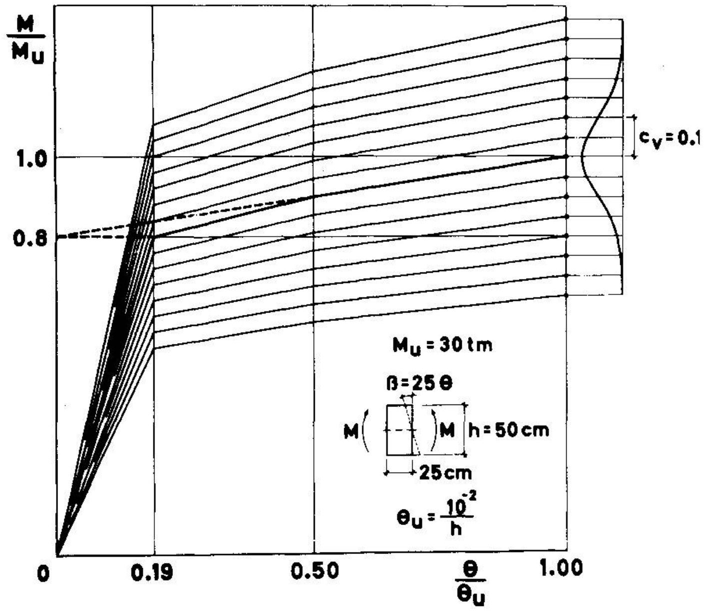


Fig. 2 – Trilinear moment-curvature diagrams.

If random combinations of these unit elements are used to form a sample of nominally identical structures, the behaviour of these structures under deterministic proportional loading may be investigated numerically and the results obtained represented by a family of force-displacement diagrams, as shown in Fig. 1b. Statistical methods may then be used to study the relation between randomness of mechanical properties and randomness of structural behaviour and to investigate questions of structural safety.

The particular M (θ) diagrams employed in the examples were of the tri-linear type shown in Fig. 2. The graph shown in heavy outline represents the mean behaviour $\overline{M}$ (θ), while the other diagrams represent a normal population. The coefficient of variation was assumed to be 10%.

3 – FORMATION OF BEAMS BY RANDOM SAMPLING

A sample of twenty beams was constructed for the study. To accommodate the available computer program (4), each of the twenty beams was formed by combining 39 whole-length unit elements with two half-length elements, one at each end. The length of the elements was thus 1/40 of the total beam length.

Fifteen different element types, corresponding to fifteen different M(θ) diagrams (or fifteen values of M_u), were used to form the beams. The diagrams were spaced at intervals of .05 $\overline{M}$; i.e., at intervals of one half of the standard deviation. Each sample beam consisted of a set of 41 elements chosen at random from this population. The selection of the elements was performed with the aid of tables of Random Normal Deviates (6), and the frequency distribution of element types is given in Table 1.

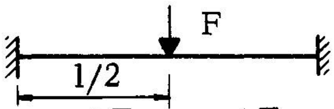
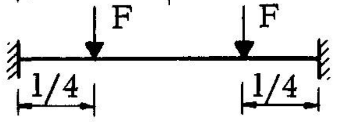
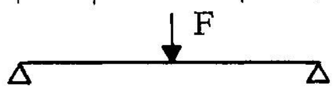
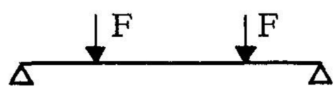
TABLE 1 – ELEMENT TYPES

Element Type	$\dfrac{M_u}{\overline{M}_u}$	α	Corresponding Range of Random Normal Deviate	Normal Frequency of Occurrence %
1	0.65	- 3.5	- ∞ to - 3.25	0.06
2	0.70	- 3.0	- 3.25 to - 2.75	0.24
3	0.75	- 2.5	- 2.75 to - 2.25	0.92
4	0.80	- 2.0	- 2.25 to - 1.75	2.79
5	0.85	- 1.5	- 1.75 to - 1.25	6.55
6	0.90	- 1.0	- 1.25 to - 0.75	12.10
7	0.95	- 0.5	- 0.75 to - 0.25	17.47
8	1.00	0	- 0.25 to 0.25	19.74
9	1.05	+ 0.5	0.25 to 0.75	17.47
10	1.10	+ 1.0	0.75 to 1.25	12.10
11	1.15	+ 1.5	1.25 to 1.75	6.55
12	1.20	+ 2.0	1.75 to 2.25	2.79
13	1.25	+ 2.5	2.25 to 2.75	0.92
14	1.30	+ 3.0	2.75 to 3.25	0.24
15	1.35	+ 3.5	3.25 to + ∞	0.06

4 – TEST STRUCTURES AND NUMERICAL EXPERIMENTS

Numerical experiments on the twenty beams were performed for four different conditions of loading and support as indicated in Table 2 which al-

TABLE 2 – TEST STRUCTURES

Ref. Number	Structure	Span l	Depth h	Ult. Moment M_u
1		10 m	50 cm	± 30 tm
2		10 m	50 cm	+ 10 tm - 30 tm
3		10 m	50 cm	+ 30 tm
4		10 m	50 cm	+ 30 tm

so gives the dimensions, mean ultimate moments, and ultimate curvature values used in the computations. Note that the ratio of positive to negative ultimate moments for the hyperstatic structures, 1 and 2, is based upon elastic design requirements.

Deflections at regular increments of increasing load were determined using two computer programs developed at LNEC for the nonlinear analysis of either isostatic or hyperstatic plane framed structures. Each permits the introduction of numerous arbitrary M (θ) data defining the nonlinear flexural behaviour of the various element types that make up the structure. For isostatic cases, deflections are obtained by a simple numerical integration of the beam differential equation, $y'' = \theta$, satisfying the appropriate boundary conditions. For hyperstatic cases, an iterative procedure based on the stiffness method is used to determine displacements, moments and shears for each level of loading. This program is described in considerable detail in a previous paper (4).

5 – RESULTS OF SIMULATED TESTS

The load-deflection data obtained for the four structures are presented graphically in Figs. 3, 4, 5 and 6. In addition to the twenty graphs for the sample beams, the figures show, in heavier outline, the diagrams obtained for a "standard" beam which possesses uniform mechanical properties throughout its length corresponding to the mean diagram $\overline{M}(\theta)$.

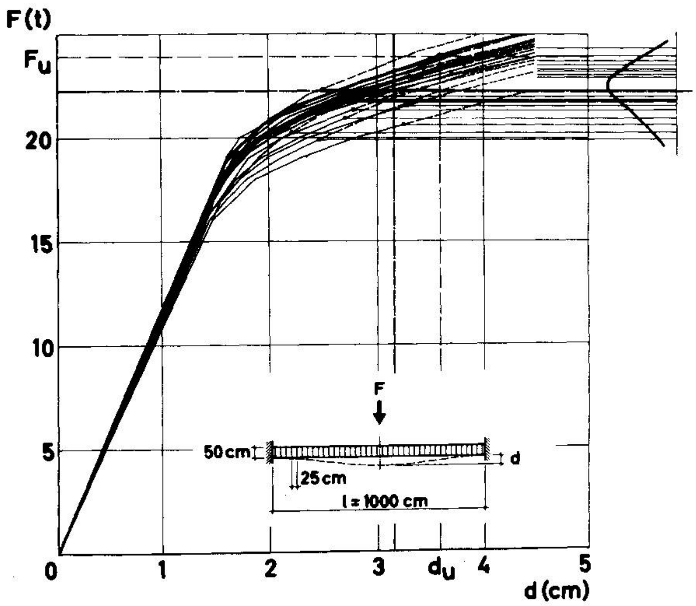


Fig. 3 – Load-deflection diagrams for structures 1.

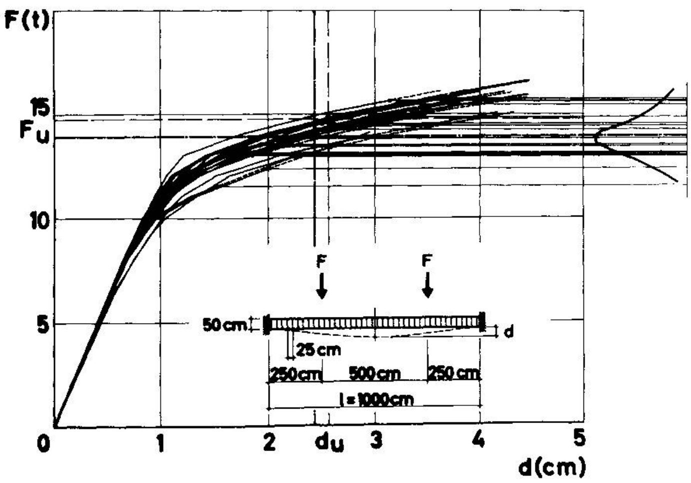


Fig. 4 – Load-deflection diagrams for structures 2.

The results, with respect to ultimate strength of the test structures, are summarized in Table 3. Two different limit states were considered in defining structural "failure" – a moment (rotation capacity) criterion and a deflection criterion. The former considers failure to occur when the bending moment M reaches the random ultimate value M_u anywhere in the beam; i.e., when θ reaches the limiting value θ_u anywhere. Failure loads according to this criterion are indicated by heavy dots on the load-deflection graphs. Failure, according to the latter criterion, occurs when a prescribed limiting deflection, d_u, is attained. This critical deflection was taken do be the midspan deflection of the standard beam when the moment capacity limit state is reached. For the standard beam therefore, the ultimate load by either criterion is the same.

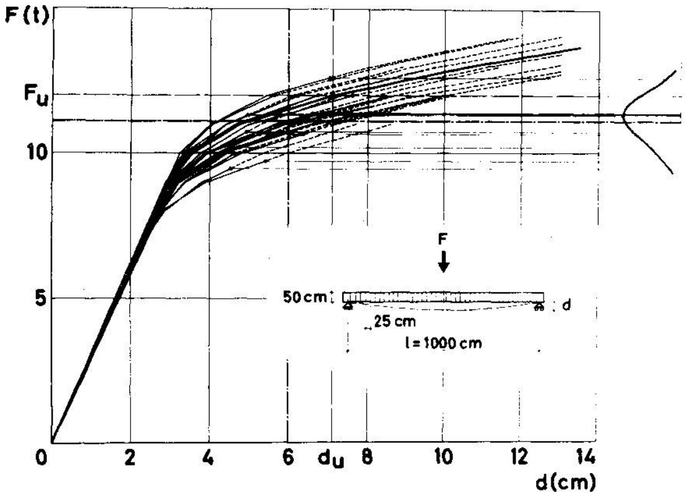


Fig. 5 – Load-deflection diagrams for structures 3.

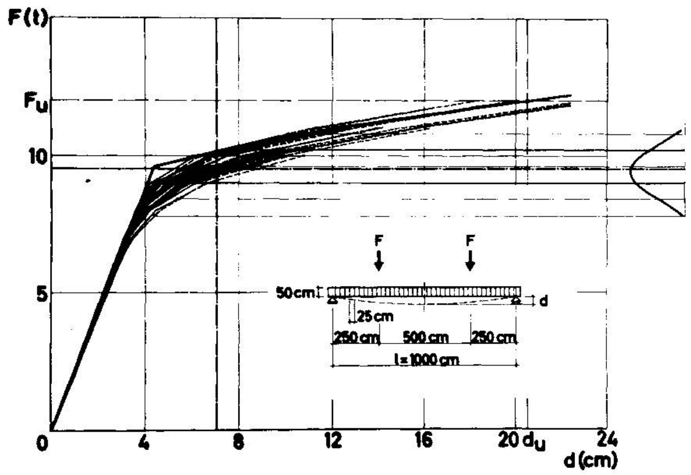


Fig. 6 – Load-deflection diagrams for structures 4.

6 – DISCUSSION OF TEST RESULTS

For all four structures it is obvious from the load-deflection diagrams that the mean deflection of the random beams exceeds that of the standard beam. This result is not difficult to understand in light of the skewness of the distribution $P_r(\theta > \theta_i | M_i)$ previously discussed. The bias toward large values of θ is simply carried over, through the numerical integration, as a bias toward large values of deflection. This aspect of the structural response is reflected in Table 3 by the fact that the mean ultimate strength of all structure samples, when judged by a deflection limit state, is lower than the strength of the nominal standard beam. It is also seen that the coefficients of variation of ultimate load values (3.2% - 5.3%) are significantly less than that (10%) of the original M_u values; i.e., the load-deflection diagrams exhibit considerably less dispersion than the original M(θ) diagrams. From Table 3 it is also apparent that the reduction in mean strength, as well as the variability of strength values, is greater for the moment capacity limit state than for the deflection criterion.

The significant difference in ultimate load probability distributions, depending on the choice of limit state, is explained by the fact that while structural deflection is influenced by material variability throughout the entire structure, moment-induced failure depends, essentially, on local material properties at a limited number of critical sections. More specifically, for the moment capacity criterion, structural strength is influenced by the following factors: the number of critical sections, the moment gradient in the vicinity of these sections, and, in the case of hyperstatic structures, the nature of the moment redistribution in the inelastic range.

The influence of these factors on mean strength may be readily discerned in the examples considered. Fig. 7 shows the bending moment distribution involved in the four test structures (based on elastic analysis for the hyperstatic cases). For isostatic structures, where moment redistribution is not a factor, the ultimate moment failure criterion corresponds essentially to a Weibull type "weakest link" theory under a prescribed stress

TABLE 3 – SUMMARY OF ULTIMATE LOAD DATA

Structure	Ult. Load Std. Beam, F_u t	Moment Criterion: Mean Ult. Load, t	% Reduction	Std. Dev., t	c_v %	Deflection Criterion: Mean Ult. Load, t	% Reduction	Std. Dev., t	c_v %
1	24	22.2	7.5	1.27	5.7	23.1	3.8	0.75	3.2
2	14.7	13.8	6.1	1.16	8.4	14.2	3.4	0.50	3.5
3	12	11.1	7.5	0.90	8.1	11.6	3.3	0.61	5.3
4	12	9.5	20.8	0.74	7.8	–	–	–	–

distribution. Regions of high uniform stress enhance failure probability, and it follows that Structure 4, with an extended region subjected to maximum bending moment (zero moment gradient), shows the most significant reduction in mean strength, 20.8%.

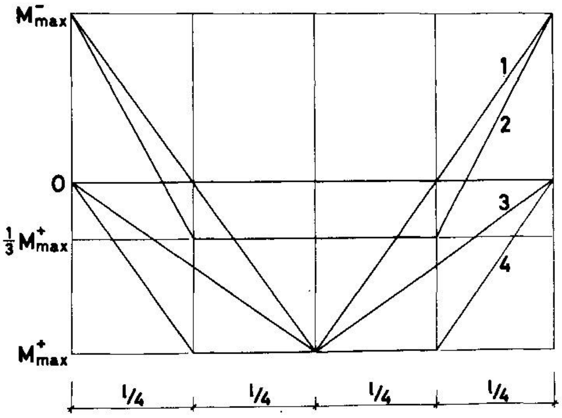


Fig. 7 – Moment gradients in test structures.

In comparing results for Structures 1 and 3, two opposing factors must be considered. While Structure 1 has three critical sections, against only one for Structure 3, this unfavorable factor is apparently balanced by its more favorable moment gradient, which is twice that of Structure 3. As a result, both structures show the same mean strength reduction, 7.5%. It should be noted that moment redistribution is not a significant factor in Structure 1. In fact, for the standard beam, the elastic moment ratio, $M^+_{max} / M^-_{max} = 1$, is maintained through the inelastic range and the randomly formed beams displayed only small variations in this ratio.

Although Structure 2 has an apparent extended critical region between the load points, moment redistribution is a significant factor in this case. For an elastically designed beam ($M^-_u = 3\ M^+_u$), redistribution in the inelastic range results in $M^-_{max} > 3\ M^+_{max}$, so that only the two support sections are critical. This structure also has the steepest moment gradient in the critical regions and, consequently, it shows the smallest reduction in mean strength, 6.1%.

While these simple examples give some insight into random nonlinear behaviour, particularly with respect to mean response and general variability, the sample size is not large enough to deduce the precise nature of the response probability distribution. To obtain more significant information concerning the small failure probabilities associated with the practical range of structural safety, the method can, of course, be employed with samples of larger size. Continually increasing computer capacity and speed make such an extension perfectly feasible.

7 – CONCLUSIONS

A computer simulation method has been presented which offers considerable promise for obtaining knowledge of the stochastic response characteristics of plane skeletal structures from a knowledge of variability in the properties of structural materials. From the results of the examples considered, the following general remarks may be made:

1. The mean strength of a population of structures is significantly less than that of a structure possessing the mean mechanical properties throughout.

2. The amount of this strength reduction depends on the moment gradient and, for statically indeterminate cases, on the nature of the moment redistribution.

3. The reduction in strength is less pronounced when failure is based on a deflection rather than a moment capacity limit state.

4. The dispersion of structural response is significantly less than that of the material properties.

The proposed method is perfectly general and may be extended to include larger samples, alternate input distributions, and more complex structures.

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SUMMARY

The randomness of structural behaviour as influenced by the probabilistic nature of material properties is studied on basis of numerical experiments.

The numerical experiments consisted in the statistical analysis of force-displacement diagrams obtained for several simply supported and perfectly built-in beams under one or two forces. Each beam was formed by combining elements chosen at random. These elements are defined by moment-curvature diagrams belonging to a prescribed statistical population.

The examples presented give some insight on the way the statistical distributions of deflection and rupture vary in function of the types and dimensions of the structures and of the statistical distribution of the mechanical properties.

RESUME

On étudie le caractère aléatoire du comportement des structures en tant qu'influencé par la nature probabilistique des propriétés des matériaux, prenant pour base des expériences numériques.

Les expériences numériques ont consisté dans l'analyse statistique de diagrammes forces-déplacements pour différents types de poutres, simplement appuyées et parfaitement encastrées, soumises à une ou à deux forces concentrées. Chaque poutre a été formée en combinant des éléments choisis au hasard et définis par des diagrammes moments-courbures appartenant à une population statistique.

Les exemples présentés permettent de comprendre la variation des distributions statistiques des flèches et de la rupture en fonction des types et des dimensions des structures et de la distribution statistique des propriétés mécaniques.

ZUSAMMENFASSUNG

Die Zufälligkeit des durch die Wahrscheinlichkeitsnatur der Materialeigenschaften beeinflussten Bauwerkverhaltens ist aufgrund numerischer Experimente untersucht worden.

Die numerischen Experimente bestanden in der statistischen Auswertung der Kraft-Verschiebungs-Bilder verschiedener einfach aufgelegter und starr eingespannter Träger unter ein oder zwei Kräften. Jeder Balken war aus zu kombinierenden Elementen zufällig zusammengesetzt worden. Diesen Elementen entsprechen Moment-Krümmungs-Bilder, die zu einer bestimmten Grundgesamtheit gehören.

Die Beispiele geben einigen Einblick wie die statistische Verteilung der Durchbiegung und des Bruches in Abhängigkeit der Form, der Abmessungen und der Verteilungsfunktion der mechanischen Eigenschaften ändert.

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