

**Zeitschrift:** IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen  
**Band:** 4 (1969)  
**Artikel:** Prediction of behaviour of steel columns under load  
**Autor:** Tall, Lambert / Alpsten, Göran A.  
**DOI:** <https://doi.org/10.5169/seals-5939>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 24.01.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

**Prediction of Behavior of Steel Columns Under Load**

Le comportement des poteaux en acier soumis à la compression

Das Verhalten von belasteten Stahlstützen

LAMBERT TALL      GÖRAN A. ALPSTEN  
 U.S.A.                      Sweden

INTRODUCTION

The prediction of the behavior of compression members under load depends on a knowledge of material properties and geometry. There may be considerable scatter in both--in particular, residual stresses and out-of-straightness are predominant factors. Residual stresses are the initial stresses existing in a member before the application of external load. Out-of-straightness is used here to refer to all deviations which result in an eccentrically loaded column, that is, initial curvature, eccentric application of load, and unsymmetrical residual stress distribution.

This paper summarizes some aspects of a continuing general study of the stability of plates and columns underway at Lehigh University for the past two decades. The initial work, concerned mainly with small to medium-size rolled steel shapes, formed the basis for design recommendations subsequently incorporated into the U.S. specifications. Later investigations have included welded column shapes also. Current column research at Lehigh University deals with welded shapes built up from flame-cut plates and with very heavy shapes, rolled as well as welded members, of sizes up to 1122lb/ft.

Although studies at Lehigh University have considered simple columns, beam-columns, and framed columns, this paper includes only the simple columns, since a large number of variables have been considered in its study, and since it is, essentially, the basic column, to which the strength of other columns may be referred.

BASIC COLUMN STRENGTH

The strength of a simple column may be typified by its maximum (or ultimate) load. For any particular column cross section and material, the maximum load depends both on the magnitude and distribution of residual stresses within the cross section, and on the initial out-of-straightness. For the hypothetical case of zero initial out-of-straightness, the column remains straight under increasing load until the tangent modulus load is reached. The level of the tangent modulus load is greatly affected by the residual stresses. At the tangent modulus load, the column bifurcates and then continues deflecting under increasing load, reaching the maximum load, after which it starts unloading. See Fig. 1.

While laboratory testing techniques may simulate closely the behavior of a perfectly straight column (See Fig. 2), practical columns show an initial out-of-straightness which will cause the column to deflect immediately upon

loading. The deflection will increase gradually under increasing load up to the maximum load, as shown in Fig. 1. The maximum load of the column with initial out-of-straightness is reduced as compared to the perfectly straight column with other conditions the same. The maximum load and the shape of the load-deflection curve are affected by residual stresses and out-of-straightness. The unloading characteristics may be important when considering the framed column member in a structure--it is normally desirable that the column can sustain loads at or close to the maximum for relatively large deflections.

#### EFFECT OF VARIATIONS IN RESIDUAL STRESSES AND OUT-OF-STRAIGHTNESS

As noted above, the mechanical and geometrical properties of the column, including in particular residual stresses existing in the member and initial out-of-straightness, are of the utmost importance in their effect on column strength. These properties can vary considerably between different members, as well as between different elements of same fabrication conditions and cross-sectional geometry, and also within the member itself. This variation or scatter has been studied extensively, and some results of the variation in yield strength and residual stresses have been summarized in Ref. 1.

The formation of residual stresses is dependent on the manufacturing and fabrication processes used, as well as on the size and geometry of a particular member. [2] Thus, it may be expected that the fabrication and geometry are important factors in determining the strength of steel columns. The variations in manufacturing and fabrication processes, and in the member size and geometry, all lead to a scatter in the residual stresses, which when combined with the variation of material properties, will lead to a scatter in column strength both in the behavior under load, and in the maximum load. Similarly, the out-of-straightness characteristics are a result of the manufacture and fabrication which will introduce scatter in column strength. Indeed, a summary of all column test results obtained shows a tremendous variation, even when compared on the basis of equal yield strength as shown in Fig. 3. It should be noted that the testing method used for most of the column tests included in Fig. 3 involves a special alignment procedure, [3] designed so that the effect of out-of-straightness is minimized. Thus, it may be expected that the consideration of full variations in out-of-straightness would lead to additional scatter in the column test results of Fig. 3.

Most of the variation in column results, however, can be attributed to predictable variations in the residual stresses or other factors such as out-of-straightness, which could be controlled in the design or fabrication process. The strength of columns, and the consideration of the scatter in material properties, may be considered in either of two basic ways: (1) a statistical study of strength irrespective of causes, or (2) a theoretical study of mathematical models where all the variables may be considered either independently or together. The former is experimental, and the latter is theoretical with experimental correlation.

The Lehigh University studies of column strength have followed the second consideration--typical and possible variations in the influencing factors were considered and it was investigated theoretically whether these made significant variations in structural behavior. The verifying experiments were deterministic, rather than probabilistic, in nature. This approach was chosen

mainly for reasons of economy and time, and for the fact that the influence of each variable could be considered separately in order to understand fundamental behavior. The variables considered were residual stress, out-of-straightness, yield strength, manufacturing and fabrication processes (for instance, hot-rolled or welded) and details (for instance, weld method and heat input), and size and geometry of the cross section. The mathematical models used considered the simultaneous elastic and plastic regions at all stages during the loading process. Some effects, such as residual stresses, predominate in these column studies, and efforts were made to find ways of changing the residual stress distribution into a more favorable one. It is not believed that purely statistical studies would lead to methods of improving strength.

#### PREDICTION OF COLUMN STRENGTH

Two methods for the forecasting of the structural behavior of a simple column will be considered here. These methods are based upon the tangent modulus load concept ("T.M. prediction") and the maximum load of the column ("M.S. prediction"), respectively. The tangent modulus prediction, as generalized to include the effect of residual stresses, [4,5], considers a fictitious, perfectly-straight column with centric load application and symmetrical residual stresses. (See also Fig. 1.) It may be shown that the tangent modulus prediction under certain assumptions applicable to members of structural carbon steel is a function of the moment of inertia of the elastic part of the cross section, [5] or

$$\frac{P_{TM}}{A} = \frac{\pi^2 E I_e}{(L/r)^2 I} \quad (1)$$

where  $P_{TM}$  is the tangent modulus load,  $A$  is the cross-sectional area,  $E$  is the elastic modulus,  $I_e$  is the moment of inertia of the elastic part of the cross section,  $I$  is the total moment of inertia about the axis considered,  $L$  is the effective length of the column, and  $r$  is the radius of gyration of the cross section. The extension of the elastic areas of the cross section is dependent on the residual stresses and the applied strain. Typical column curves from tangent modulus predictions are shown in Fig. 4. ( $P_{cr}$  is the critical load, in this case the tangent modulus load.)

The maximum strength prediction is somewhat more complex to calculate. The basic concepts, however, are very simple--the theory is based upon equilibrium conditions for the deflected position of the column. The theory may be applied to the prediction of the post-buckling strength of the initially straight centrally loaded column as well as to the more practical case with initial out-of-straightness. The maximum load marks the position where, under increasing deflection, the rate of the resisting internal moment in the column is equal to the rate of the externally applied moment. Several studies have considered methods to calculate the maximum load, including the effect of residual stresses and initial-out-straightness. [6 through 11] An example of a maximum strength curve is given in Fig. 5, and compared with the corresponding tangent modulus curve. [12] In this particular case, the maximum strength

curve, based upon predicted residual stresses in a hot-rolled 14WF730 "jumbo" shape with an initial deflected curve of  $\delta_{\max}/L=0.001$ , falls slightly below the tangent modulus curve.

For a general investigation of the column strength as affected by accurate residual stress distributions and out-of-straightness, the numerical computations will become quite cumbersome and tedious, necessitating the use of an electronic computer. General programs have been developed for tangent modulus as well as maximum strength predictions. However, simplifying assumptions of various degree can be made, which may reduce the amount of necessary numerical operations to such a level that these methods may be used without the computer for practical estimates or for design. Thus, for small and medium-size rolled H-shapes it may be shown [13] that the following equation approximates the tangent modulus load

$$\frac{P_{TM}}{A} = \frac{\pi^2 E \left( \frac{E_t}{E} \right)}{(L/r)^2} \quad \text{for major-axis bending}$$

and

$$\frac{P_{TM}}{A} = \frac{\pi^2 E \left( \frac{E_t}{E} \right)^3}{(L/r)^2} \quad \text{for minor-axis bending.}$$

where  $E_t$  is the tangent modulus of the complete cross section. Figure 6 gives the computational procedure schematically.

For maximum strength predictions, the approximate method discussed in Refs. 8 and 10 may be sufficiently accurate and useful for many practical purposes. The method is based upon the assumption that the initial deflected curve and the curvature under load may be described by half-sine waves. The mid-height section of the column is considered only. By differentiating the deflected curve function twice, it is possible to obtain a simple relationship between the deflection at mid-height of the column ( $\delta_{mh}$ ) and the curvature at the same point ( $\phi_{mh}$ )

$$\phi_{mh} = \frac{\pi^2}{L^2} \delta_{mh}$$

After choosing arbitrarily a value of  $\delta_{mh}$ , the corresponding curvature  $\phi_{mh}$  is obtained directly from the equation above. The axial strain which produces equilibrium in the cross section can be found by an iterative procedure. The iteration is continued until an equilibrium equation for the mid-height section of the column, that is,

$$P (\delta_{init} + \delta_{mh}) = M$$

is satisfied.  $P$  is the axial load,  $\delta_{init}$  the initial mid-height deflection and  $M$  the internal moment corresponding to the stress distribution in the mid-height section.

Since methods are now available for a more rational column design procedure, there is no longer any need for complicated formulas using various correction factors for estimated fictitious eccentricities or initial deflections--in the past, such factors had been determined to take into account the transition in the column curve from the Euler curve to the yield strength load for short columns. It seems more logical to base an accurate column analysis upon the actual conditions, including measured or estimated residual stresses, out-of-straightness, and mechanical properties.

#### SOME TEST RESULTS: COMPARISONS WITH THEORY

Figures 7 through 12 illustrate the effect of various parameters on the column strength. The diagrams are included here to illustrate a few important points related to the effect of variations in residual stresses due to different manufacturing and fabrication conditions of steel columns.

A comparison between column test results for rolled wide-flange shapes and welded shapes of H and box section, built up from universal-mill plates, is shown in Fig. 7. [14] It is apparent from the diagram that there is a substantial variation between the results obtained for these four kinds of columns. The data of the rolled shapes, all of small to medium-sized cross section, fall reasonably close to the CRC Basic Column curve, suggested by the Column Research Council to describe the strength of columns, [15] and adopted as the design curve by the American Institute of Steel Construction. On the other hand, all the data points for welded shapes are below this curve, for some cases by as much as 30 per cent.

The effect of the column bending axis on column strength is shown in Fig. 8 for rolled wide-flange shapes. [16] Normally, such shapes will have compressive residual stresses at the flange tips, [5,12,13,16] which will reduce the column strength comparatively more for buckling about the minor axis.

Figure 9 shows the effect of the geometrical size of the cross section. Theoretical studies had indicated that the size of a hot-rolled member is an important variable in the formation of residual stresses--the stresses tend to increase with increasing size of a rolled member. [12] This would lead to reduced column strength for heavy rolled columns. The curves in Fig. 9 are tangent modulus predictions based upon the residual stresses predicted in a heavy rolled "jumbo" section 14WF730 and a smaller rolled H-shape. [11,17] It should be noted that the situation probably will be the opposite for welded shapes, because of the fact that welding residual stresses will decrease with increasing size of the structural member. [2]

An important factor which will affect the strength of welded H-columns is the manufacture of the component plates prior to welding. Several tests have shown that flame-cut plates show a more favorable residual stress distribution, which leads to improved strength of H-columns fabricated from such plates, as compared to similar columns built up from universal-mill plates. [10] See Fig. 10. The diagram in Fig. 10 also shows that the tangent modulus prediction estimates the column strength of the flame-cut welded shapes



fairly well. This means that the post-buckling reserve above the tangent modulus load of a fictitious perfectly straight column is of approximately the same magnitude as the reduction in strength due to unintentional out-of-straightness of a practical column. Thus, the tangent modulus concept may be used for the design of such members, including the effect of residual stresses. For the shapes of universal-mill plates in Fig. 10, the post-buckling reserve is considerable and an accurate maximum strength analysis is necessary to obtain close correlation with data.

Figure 11 illustrates the effect of the yield strength level on column strength. [18] Generally speaking, the higher the yield strength, the greater is the column strength, also when compared on a non-dimensional basis as in Fig. 11. The effect may be attributed to the fact that the magnitude of residual stresses often is relatively independent of the yield strength of the steel. [18] Thus, the residual stress to yield strength ratio will be lower for high-strength steels, leading to improved column strength. This trend is accentuated further for quenched and tempered steels, such as A514 steel, which have comparatively small magnitudes of residual stress due to the heat treatment.

Figure 12 shows the column strength of shapes which have been specially treated after manufacture--by an annealing that removes the major portion of residual stresses, and by a reinforcement accomplished merely by laying a weld bead along the flange tips. [19] The improved strength in the reinforced columns is achieved through the reversal of residual stresses at the flange edges.

### CONCLUSIONS

Methods for forecasting the structural behavior of steel columns based upon variations in different relevant parameters, in particular residual stresses, have been discussed in this paper. Examples were given for the influence of various parameters on column strength. The results indicate that the strength and behavior of columns under load can be predicted, and that the various influencing factors may be included in the prediction. While a summary of all column tests shows a tremendous scatter, most of this variation can be attributed to parameters which may be controlled in the design and fabrication. Thus, methods and extensive data are available for the rational design of centrally loaded steel columns.

The large scatter in results, and the consideration that this variation is caused largely by controllable factors, makes clear that the use of one design curve for all columns penalizes certain groups of columns, whereas other types of columns having a comparatively low strength will be designed to a lower real factor of safety. It appears logical that the specifications for the design of columns should be reconsidered in this light.

### ACKNOWLEDGEMENTS

This paper presents the results of a long-term study into the column strength of structural steel members, underway at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania. The National Science

Foundation sponsored the most recent part of the study.

Acknowledgements are due to the many colleagues who contributed to the various phases of the study and to whose work reference was made in the paper. In particular, Lynn S. Beedle, Director of Fritz Engineering Laboratory, provided guidance in the early stages of the investigation, and inspiration throughout the course of the study.

#### REFERENCES

1. L. Tall and G. A. Alpsten  
ON THE SCATTER IN YIELD STRENGTH AND RESIDUAL STRESSES IN STEEL MEMBERS, Fritz Engrg. Lab. Report No. 337.22, Lehigh University; to be presented at the IABSE Symposium, Theme IVa, London, September 1969.
2. G. A. Alpsten and L. Tall  
RESIDUAL STRESSES IN HEAVY WELDED SHAPES, Fritz Engrg. Lab. Report No. 337.12, Lehigh University, January 1969; to be published in the Welding Journal.
3. F. R. Estuar and L. Tall  
TESTING OF PINNED-END STEEL COLUMNS, Test Methods for Compression Members, ASTM STP 419, American Society for Testing and Materials, 1967.
4. W. R. Osgood  
THE EFFECT OF RESIDUAL STRESS ON COLUMN STRENGTH, Proc., First National Congress Applied Mechanics, June 1951.
5. C. H. Yang, L. S. Beedle and B. G. Johnston  
RESIDUAL STRESS AND THE YIELD STRENGTH OF STEEL BEAMS, The Welding Journal, Vol. 31 (4), 1952.
6. A. W. Huber and R. L. Ketter  
THE INFLUENCE OF RESIDUAL STRESS ON THE CARRYING CAPACITY OF ECCENTRICALLY LOADED COLUMNS, IABSE Publications, Zurich, December, 1958.
7. Y. Fujita  
ULTIMATE STRENGTH OF COLUMNS WITH RESIDUAL STRESSES, Journal, Society of Naval Architects of Japan, January 1960.
8. L. Tall  
THE STRENGTH OF WELDED BUILT-UP COLUMNS, Ph.D. Dissertation, Lehigh University, May 1961; University Microfilms, Inc., Ann Arbor.
9. R. H. Batterman and B. G. Johnston  
BEHAVIOR AND MAXIMUM STRENGTH OF METAL COLUMNS, Journal of the Structural Division, Proc. ASCE, Vol. 93, ST2, April 1967.
10. R. K. McFalls and L. Tall  
A STUDY OF WELDED COLUMNS MANUFACTURED FROM FLAME-CUT PLATES, The Welding Journal, Vol. 48 (4), April 1969.



11. G. A. Alpsten and L. Tall  
COLUMN STRENGTH OF HEAVY SHAPES - A PROGRESS REPORT, Fritz Engrg. Lab. Report No. 337.16, In preparation.
12. G. A. Alpsten  
THERMAL RESIDUAL STRESSES IN HOT-ROLLED STEEL MEMBERS, Fritz Engrg. Lab. Report No. 337.3, December 1968; to be published in the Welding Journal.
13. A. W. Huber and L. S. Beedle  
RESIDUAL STRESSES AND THE COMPRESSIVE STRENGTH OF STEEL, The Welding Journal, Vol. 33, December 1954.
14. F. R. Estuar and L. Tall  
EXPERIMENTAL INVESTIGATION OF WELDED BUILT-UP COLUMNS, The Welding Journal, Vol. 42, April 1963.
15. B. G. Johnston (Ed.)  
GUIDE TO DESIGN CRITERIA FOR METAL COMPRESSION MEMBERS, Column Research Council, 2nd Ed., Wiley, 1966.
16. L. S. Beedle and L. Tall  
BASIC COLUMN STRENGTH, Journal of the Structural Division, Proc. ASCE, Proc. Paper 2555, Vol. 86 (ST-7), July 1960.
17. G. A. Alpsten  
EGENSPÄNNINGAR I VARMVALSADE STÄLPROFILER ("RESIDUAL STRESSES IN HOT-ROLLED STEEL PROFILES"), Institution of Structural Engineering and Bridge Building, Royal Institute of Technology, Stockholm, June 1967.
18. Y. Kishima, G. A. Alpsten and L. Tall  
RESIDUAL STRESSES IN WELDED SHAPES USING FLAME-CUT PLATES, Fritz Engrg. Lab. Report No. 321.3, Lehigh University, In preparation.
19. L. Tall  
WELDED BUILT-UP COLUMNS, Fritz Engrg. Lab. Report No. 249.29, Lehigh University, April 1966.

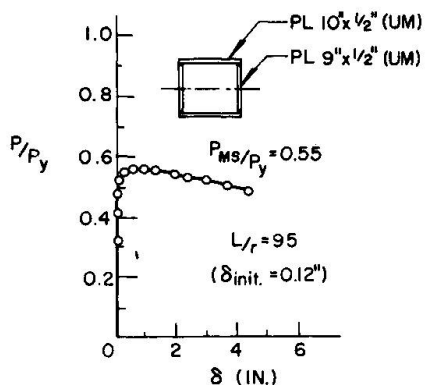


Fig. 1 Schematic load-deflection curves for simple columns

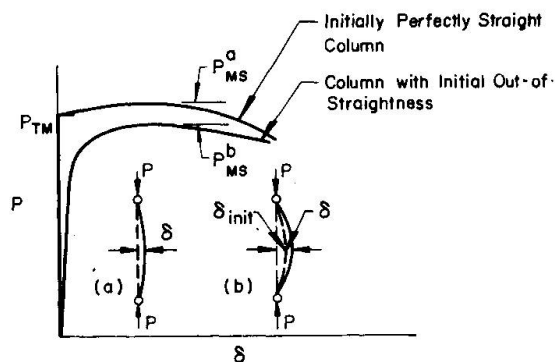


Fig. 2 Experimental load-deflection curve for a centrally loaded column

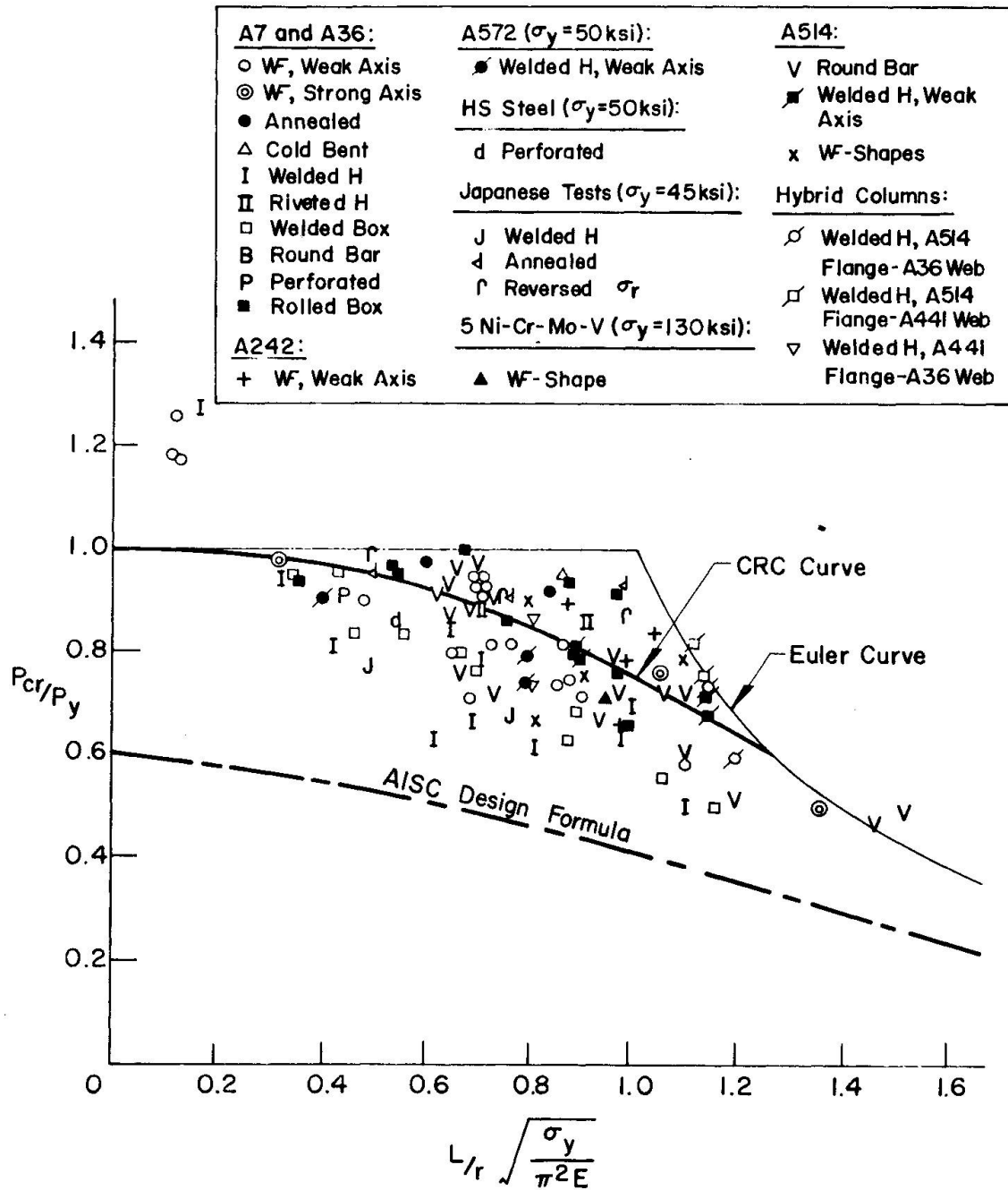


Fig. 3 Column test results

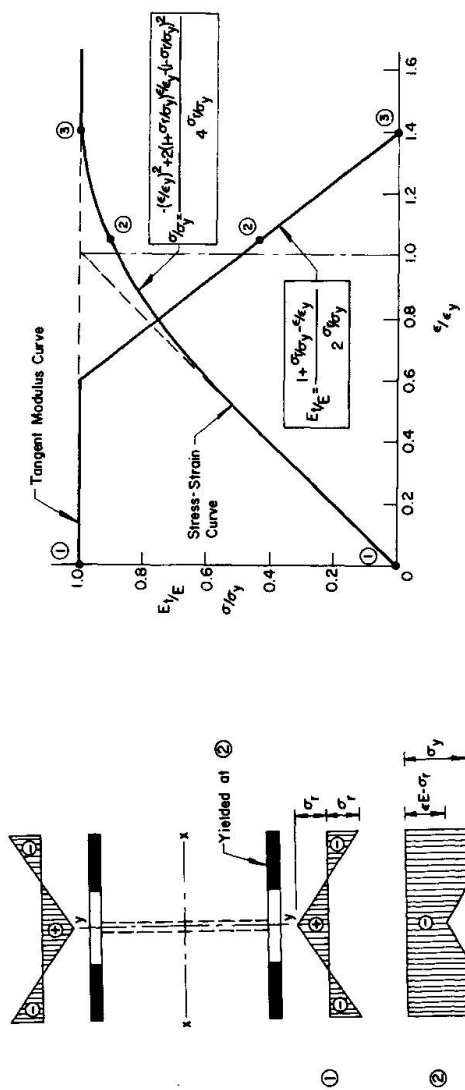


Fig. 4 Typical tangent modulus curves for a rolled WF-shape

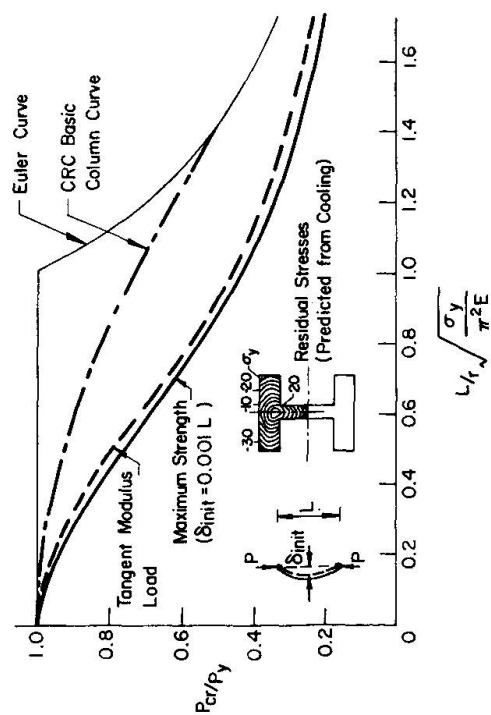


Fig. 5 Maximum strength curve for a heavy rolled "jumbo" shape 14WF730, weak axis

(a) Total Stress Distribution

(b) Stress-Strain and Tangent Modulus Curves for Cross Section

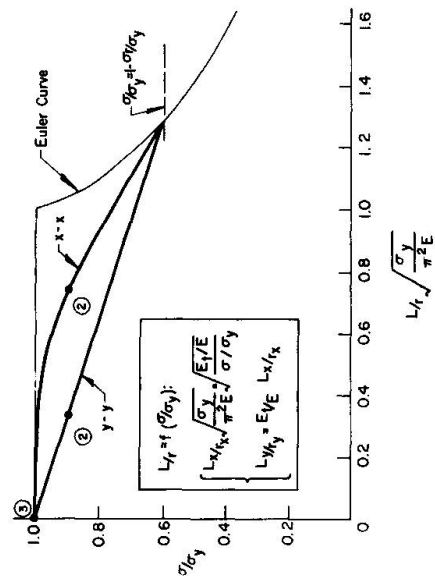


Fig. 6

Principle of approximate method for tangent modulus analysis

(c) Column Curves

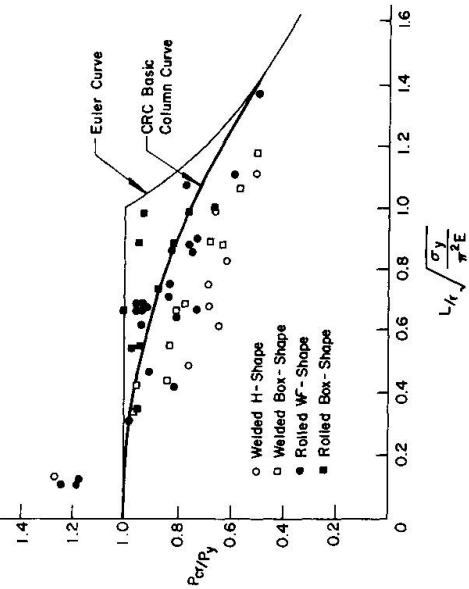


Fig. 7 Column test results for rolled and welded shapes

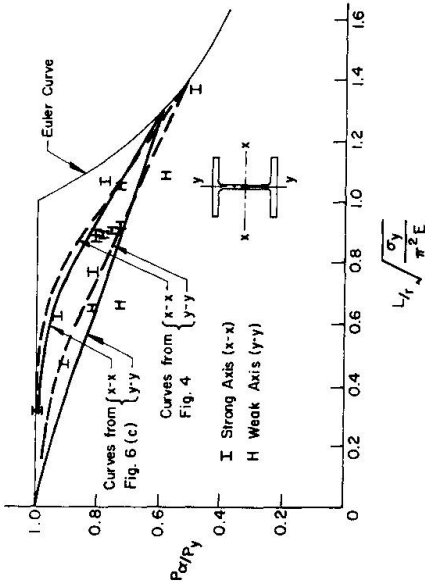


Fig. 8 Effect of bending axis on column strength, rolled WF-shapes

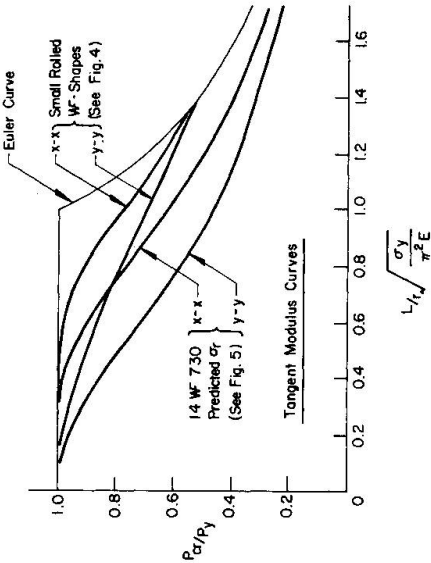


Fig. 9 Effect of geometrical scale of cross section on column strength

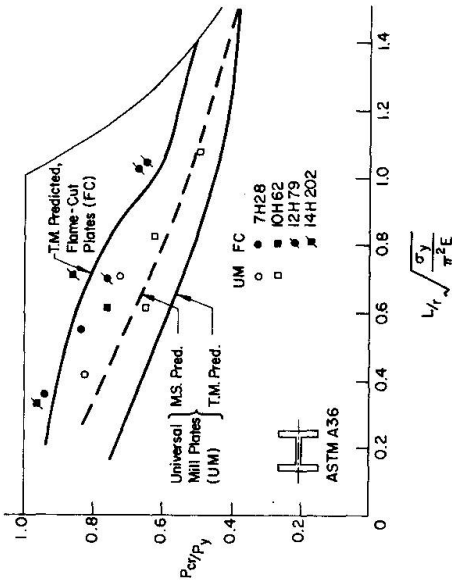


Fig. 10 Column strength of welded H-shapes of flame-cut and universal-mill plates.

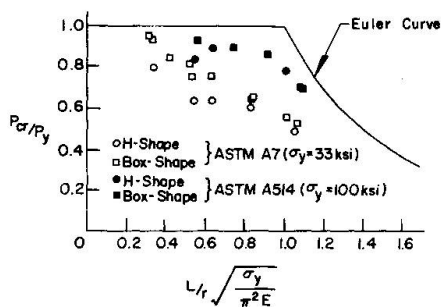


Fig. 11 Effect of yield strength on column strength

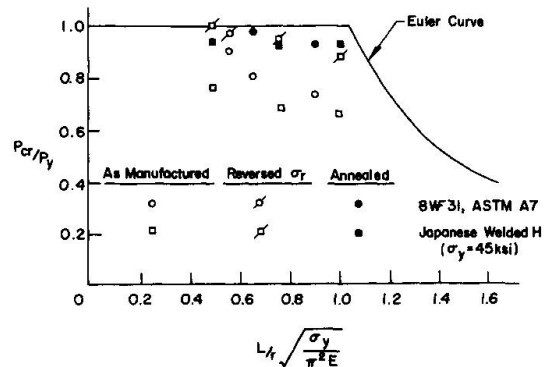


Fig. 12 Strength of H-columns; as manufactured, with reversed residual stress, and annealed

## SUMMARY

The discussion summarizes some results obtained in a study of residual stresses and column strength of rolled and welded steel shapes. Methods for forecasting the structural behavior and maximum strength of steel columns based upon variations in different relevant parameters, in particular residual stresses, are reviewed. Examples are given for the influence on column strength of various parameters, including manufacturing and fabrication procedures, bending axis, geometry of cross section, yield strength, and strengthening operations.

## RESUME

Les résultats obtenus lors d'une étude sur les contraintes résiduelles et la résistance des poteaux laminés ou reconstitués sont discutés. Quelques méthodes pour déterminer le comportement de la charge ultime des poteaux en acier, selon différents paramètres pertinents, en particulier celui des contraintes résiduelles sont revues. Plusieurs exemples montrent l'influence des paramètres sur la résistance des poteaux. Les paramètres étudiés sont les procédés de fabrication, l'axe d'inertie, la géométrie de la section droite, la limite d'élasticité et les opérations de redressage.

## ZUSAMMENFASSUNG

Dieser Beitrag fasst jene Ergebnisse zusammen, die durch Untersuchungen über die Eigenspannungen und über das Tragverhalten an gewalzten und geschweissten Stahlprofilen erhalten wurden. Es werden Berechnungsmethoden für die Voraussage des Tragverhaltens und der Traglast aufgrund der Veränderung wichtiger Parameter, insbesondere der Eigenspannungen, behandelt. Beispiele zeigen den Einfluss der verschiedenen Parameter-einschliesslich Bearbeitungs- und Herstellprozess, Knickachse, Geometrie des Querschnittes, Streckgrenze und Reckungen- auf die Stützspannung.