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## DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

**Statistical Concepts in Aseismic Design**

Concepts statistiques dans les analyses paraséismiques

Statistische Methoden für den Entwurf bei Erdbeben

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INTRODUCTION

In aseismic design, a dynamic analysis is usually required to evaluate the performance of important structures subjected to earthquakes that are likely to occur at a given site. The response of a lightly damped system depends strongly on the history of recorded motions; but since the time histories of future motions corresponding to a given intensity are unpredictable, present seismic design is based on earthquake environments prescribed in the form of a smooth (maximum) response spectrum. The available rules for estimating the responses of multi-degree-of-freedom elastic systems from a given response spectrum, however, are based on heuristic arguments and limited comparisons with results obtained through integration of recorded motions for certain responses [1]. Moreover, these rules do not give a direct procedure for taking the dispersion of the maximum response into account.

A stochastic approach to aseismic design, in principle, could provide a consistent and systematic means for designing against a set (or ensemble) of motions of prescribed intensity. With this approach, all responses of interest can be considered and explicit consideration can be given to the observed dispersions. Recognition of this fact has attracted much attention to the development of stochastic earthquake models [2], and to the formulation of approximate methods for calculating the distribution of maximum response under random earthquake-type motions [3, 4, 5, 6, 7, 8, 9].

For the stochastic approach to be of practical value, however, it is necessary to clarify the earthquake models that should be used and the technique of maximum response evaluation that is adequate for certain categories of structures. For linear structures considered herein, the results obtained thus far are quite encouraging and indicate the feasibility for practical implementation of certain random vibration results to aseismic design.

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\*Bracketed numerals refer to the corresponding references cited.

DISTRIBUTION OF MAXIMUM RESPONSE

Poisson Assumption for Up-Crossings

For structures subjected to earthquake-type ground motions, the exceedance of a high response level can be approximately described as a nonhomogeneous Poisson process with an exceedance rate equal to the rate of the up-crossings. This approximation yields the survival probability as

$$P_s(t) = P \left[ |X(\tau)| < b; 0 \leq \tau \leq t \right] = e^{-2 \int_0^t v_b(\xi) d\xi} \tag{1}$$

with

$$v_b(\xi) = \int_0^\infty \dot{x} f_{X, \dot{X}}(b, \dot{x}; \xi) d\dot{x} \tag{2}$$

in which  $f_{X, \dot{X}}$  is the joint density of the random response of interest  $X$  and its derivative  $\dot{X}$  at time  $t = \xi$ . The probability of exceeding the response level  $X = \pm b$  at least once within  $(0, t]$  is then obtained from

$$P_e(t) = 1 - P_s(t) = 1 - e^{-2 \int_0^t v_b(\xi) d\xi} \tag{3}$$

If the response is a stationary process throughout  $(0, t]$ ,  $v_b(\xi)$  becomes a constant  $\bar{v}_b$ ; ignoring a small probability of premature failure, Eq. (1) becomes, in this case,

$$\bar{P}_s(t) = e^{-2 \bar{v}_b t} \tag{4}$$

Eq. (4), which will be referred to as the homogeneous Poisson approximation, has been applied to the response of single-degree-of-freedom systems by a number of authors [8, 9].

For long durations and high response levels, Eq. (4) is asymptotically correct [10]; otherwise, up-crossings are correlated and the homogeneous Poisson assumption would not strictly be valid. Also, for small values of the integral, Eq. (3) gives results very close to the upper bounds in [5, 7]; however, the homogeneous Poisson assumption may not always provide an upper bound. This is particularly true for small values of  $t$ .

To examine the applicability of the Poisson assumption in practical situations of interest, the values of  $P_e(t)$  for the displacement response of a linear single-degree-of-freedom system subjected to a base motion described as a Gaussian white noise excitation are compared in Fig. 1 for a system with a 2-second period and for two values of damping,  $\beta = 0.02$  and  $0.08$ . For this type of excitation, the response and its derivative form a Markov vector and the numerical scheme described in [4] was used to obtain accurate results which are then compared to those obtained from the homogeneous and

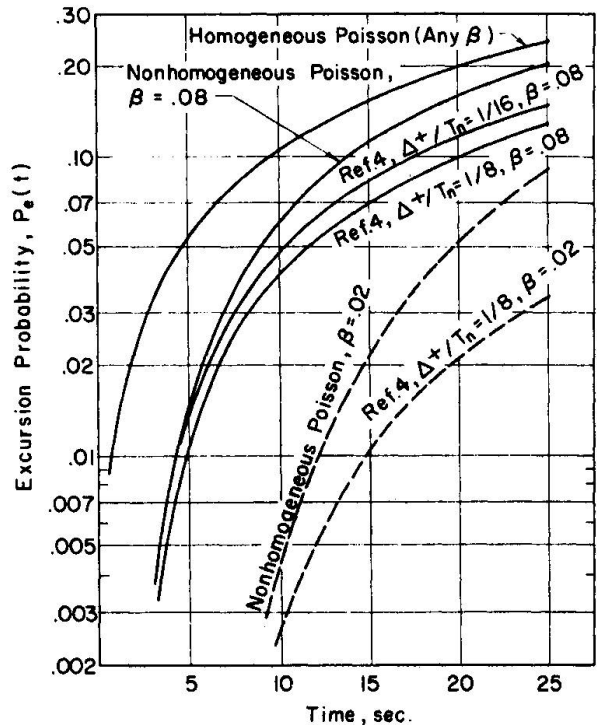


FIG. 1 COMPARISON OF CALCULATED PROBABILITIES OF EXCURSION -  $T_n = 2$  sec.,  $b = 3\sigma_0$ , WHITE NOISE EXCITATION

nonhomogeneous Poisson assumptions. The response level is  $b = 3 \sigma_o$ , in which  $\sigma_o$  is the rms value of stationary response, i.e.

$$\sigma_o^2 = \frac{\pi S_o}{2 \beta \omega_n^3}, \quad \omega_n T_n = 2 \pi \tag{5}$$

It is seen that for durations of 25 seconds or so the nonhomogeneous Poisson assumption improves the results over that of the homogeneous Poisson assumption. This is particularly true for the lower damping value. Zero initial conditions were assumed; hence, the response of a 2-second system with 2% damping is quite nonstationary for the durations considered, which is the reason for the need for a nonhomogeneous Poisson procedure. For higher damping values and systems with shorter periods, the difference between the homogeneous and nonhomogeneous processes becomes less significant for durations of the order of 25 seconds.

Another measure of the reasonableness of the above Poisson process approximation can be seen in terms of the response levels corresponding to a specified probability of exceedance  $P_e$  in a given duration. For a 2-second system with  $\beta = 2\%$  and  $P_e(25) = 10\%$ , the required response levels are respectively,  $3.3 \sigma_o$ ,  $3 \sigma_o$ , and  $2.6 \sigma_o$  for the homogeneous and nonhomogeneous Poisson approximations, and the accurate Markov scheme of calculation.

Probabilistic Response Spectra

On the basis of Eq. (1) the median pseudo velocities,  $V = \omega_n b$  corresponding to  $P_e(25) = 0.5$ , computed for a Gaussian excitation with spectral density

$$G_{Y_1}(\Omega) = S_{o1} \frac{1 + 4 \beta_{f1}^2 \left(\frac{\Omega}{\omega_{f1}}\right)^2}{\left[1 - \left(\frac{\Omega}{\omega_{f1}}\right)^2\right]^2 + 4 \beta_{f1}^2 \left(\frac{\Omega}{\omega_{f1}}\right)^2}, \quad \begin{matrix} -\infty < \Omega < \infty \\ \beta_{f1} = 0.642 \\ \omega_{f1} = 15.5 \text{ sec}^{-1} \end{matrix} \tag{6}$$

are compared in Fig. 2 with the average pseudo velocities obtained by Housner [11]. The value of  $S_{o1}$  used to compute these results is  $0.0052 \text{ ft}^2/\text{sec}^3$ . In [11, 12], which used simulation techniques to generate member functions from Eq. (6), it was found that  $S_{o1} = 0.00614 \text{ ft}^2/\text{sec}^3$  provides a good fit with Housner's spectra. It should be remembered that when using simulation studies, the average spectra calculated do not correspond to a specific value of probability of exceedance; rather, the spectra obtained from a simulation approach are simply the averages of the observed maximum responses at each damping and frequency. Furthermore, in [12] some sensitivity of  $S_{o1}$ , required for good fit, to the time interval used for generation of artificial earthquakes was reported. In view of these observations the value of  $S_{o1} = 0.0052 \text{ ft}^2/\text{sec}^3$  which gives a good fit between median pseudo velocity spectra and the Housner spectra is

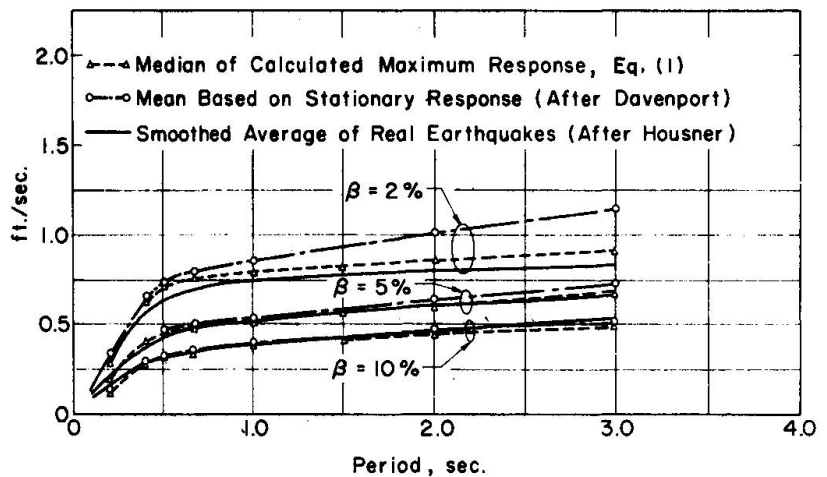


FIG. 2 SMOOTH VELOCITY SPECTRA

It should be remembered that when using simulation studies, the average spectra calculated do not correspond to a specific value of probability of exceedance; rather, the spectra obtained from a simulation approach are simply the averages of the observed maximum responses at each damping and frequency. Furthermore, in [12] some sensitivity of  $S_{o1}$ , required for good fit, to the time interval used for generation of artificial earthquakes was reported. In view of these observations the value of  $S_{o1} = 0.0052 \text{ ft}^2/\text{sec}^3$  which gives a good fit between median pseudo velocity spectra and the Housner spectra is

considered to be a good agreement between the Poisson assumption and simulation studies. Of course, by adopting the Poisson process approximation, the tedious process of simulation is avoided.

In Fig. 2 are also shown the mean response values computed from an expression due to Davenport [8], which assumes a homogeneous Poisson process. A duration of 25 seconds was used throughout.

Thus, for purposes of earthquake engineering design of linear systems, the significant maximum response statistics can be adequately determined on the basis of the nonhomogeneous Poisson process described above for a reasonably wide range of frequencies. If the excitation, system period, and damping are such that the nonstationarities in the excitation and response can both be ignored, the exceedance process may also be approximated by a homogeneous Poisson process as has been suggested previously [8, 9].

DESCRIPTION OF GROUND MOTIONS

Influence of Nonstationarity

In Fig. 4 are presented the first passage time probability density,  $f_T(t)$ , and reliability function  $P_s(t) = 1 - \int_0^t f_T(\tau) d\tau$ , corresponding to the shot noise and truncated white noise inputs shown in Fig. 3. These results were determined with the computational method of [4] for a system with a 2-second period and  $\beta = 0.02$ ;  $b = 3\sigma_0$ . The comparison shown in Fig. 4 is of interest because the shot noise excitation has been shown to represent quite well the nonstationarity observed in strong-motion records such as El Centro, Taft, and Olympia [14]. Moreover, it is known that long-period systems are more sensitive to input nonstationarity [12] and that for such systems the earthquake excitations can be treated as an uncorrelated process since the effective correlation time of earthquake motions is approximately 0.1 sec.

The comparison of the probability densities in Fig. 4 shows that the influence of input nonstationarity is controlled by the tail portion of the shot noise; therefore, for long duration responses the computed reliabilities may appreciably be in error. However, for durations of 25 sec. or so which apply to earthquakes of the type considered, the difference in the system reliabilities does not appear to be significant. This is further substantiated by the results of Table 1, which gives the response levels corresponding to an exceedance probability of  $P_e(25) = 5\%$  in a 25-second duration.

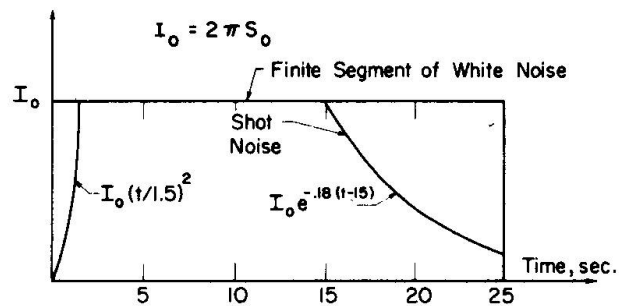


FIG. 3 INTENSITY FUNCTIONS FOR UNCORRELATED EXCITATIONS CONSIDERED

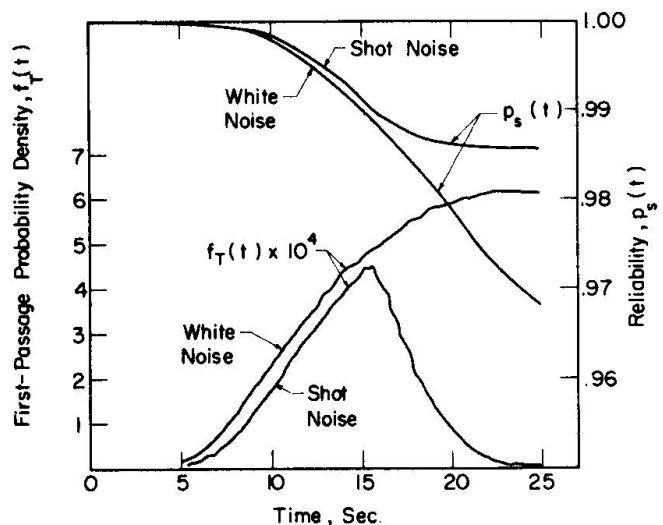


FIG. 4 FIRST-PASSAGE TIME PROBABILITY DENSITY AND RELIABILITY FUNCTIONS --  $T_n = 2$  sec.,  $\beta = 0.02$ ,  $b = 3\sigma_0$ , Excitations in Fig. 3

TABLE 1: RESPONSE LEVELS FOR 5% PROBABILITY OF EXCEEDENCE

Elastic systems with 2% damping; duration = 25 sec.

Period, Sec.	Response Values*	
	Shot Noise	White Noise
1	3.00	3.25
2	2.70	2.85
3	2.32	2.60

\* Measured in terms of  $\sigma_0$ , Eq. (5)

The insensitivity of the maximum response of linear damped systems to nonstationarity in the earthquakes of the type considered indicates that in specifying the earthquake environment for purposes of probabilistic design of such systems, primary attention should be given to the specification of the input spectral density.

It should be emphasized that the above conclusion is restricted to damped linear systems. Some exploratory results for elasto-plastic systems show that the effect of nonstationarity tends to increase with increasing values of the ductility factor [13].

**Definition of Intensity**

For dynamic seismic analysis, the earthquake environment is normally defined in terms of a smooth response spectrum; for firm ground conditions, an example would be the Housner's spectra [11]. Simulation studies [14, 15] have also shown that smooth average pseudo velocity spectra may be obtained from the spectral density of Eq. (6) or any of the following:

$$G_{Y_2}(\Omega) = S_{02} \frac{1}{\left[1 - \left(\frac{\Omega}{\omega_2}\right)^2\right]^2 + 4\beta_2^2 \left(\frac{\Omega}{\omega_2}\right)^2}, \quad \begin{matrix} -\infty < \Omega < \infty \\ \beta_2 = 0.5 \\ \omega_2 = 3.14 \text{ sec}^{-1} \end{matrix} \quad (7)$$

and

$$G_{Y_3}(\Omega) = S_{03}, \quad |\Omega| < 62.8 \text{ sec}^{-1} \quad (8)$$

Fig. 5 shows these spectral densities with  $S_{0i}$  determined so that the density amplitudes are the same for periods ranging from 2 to 4 sec. Clearly, the shapes of these densities are not the same. However, the median pseudo velocity spectra computed for the above densities using Eq. (1), and a low damping ( $\beta = .02$ ), are fairly close to each other as shown in Fig. 6a. This is in agreement with the findings of simulation studies and indicates that once a smooth response spectrum is selected for design, any of several proposed spectral densities may be used with Eq. (1) and the spectral density amplitude may be obtained from the specified response spectrum values.

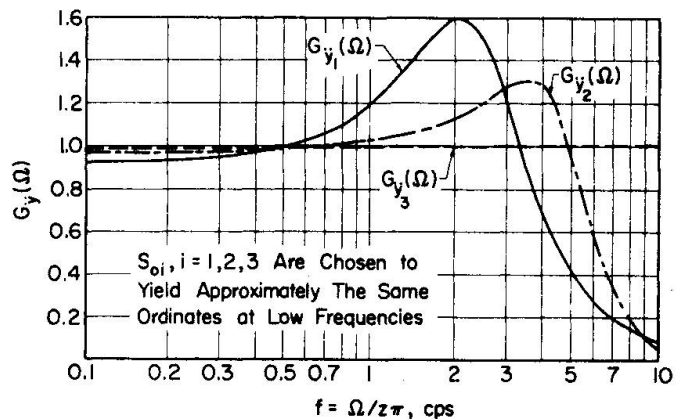


FIG. 5 NORMALIZED POWER SPECTRAL DENSITIES

Fig. 6b shows the pseudo velocity spectra obtained from the spectral densities of Eqs. (6), (7), and (8) in which the constants  $S_{0i}$  are selected so as to yield the same rms ground acceleration. These pseudo velocity spectra differ from each other.

Therefore, for purposes of consistency in practical aseismic design, the environment should be defined on the basis of a smooth design response spectrum; and from this, a corresponding spectral density can be obtained for use in the stochastic approach.

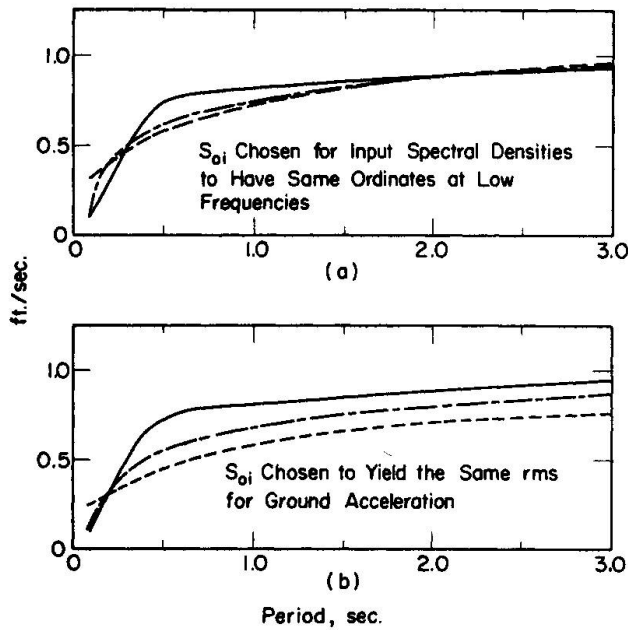


FIG. 6 MEDIAN PSEUDO VELOCITY SPECTRA CORRESPONDING TO TWO NORMALIZATIONS OF POWER SPECTRAL DENSITIES—  $\beta = 2\%$

PRACTICAL IMPLICATIONS

The stochastic approach described above is intended to improve practical design against earthquake forces; the implications are especially significant with reference to the design of multi-degree-of-freedom systems as illustrated below. An important aspect of earthquake motions that seems to be neglected in present designs, or at least is not explicitly considered, is the fact that the response to actual earthquakes of the same intensity has a wide dispersion as will be illustrated in Table 2.

An Illustrative Application

For the purpose of illustrating the practical implications alluded to above, the system shown in Fig. 7 is considered and the relative story distortions are studied. The ratio  $k/m$  is selected to give fundamental mode period of 1sec., and 5% damping is used in all the modes. All the 5 modes are considered in the results reported in Table 2, which provides a comparison for three sets of calculations.

TABLE 2: RESULTS FOR SYSTEM IN FIG. 7

Response x 100. ft. (1)	Normalized Records			Random Vib.		Response Spectrum	
	Average (2)	Range (3)	Second Highest (4)	$P_e = .5$ (5)	$P_e = 0.10$ (6)	SRSS (7)	ABS (8)
$U_1$	353	243 to 587	442	380	434	310	380
$(U_2 - U_1)$	321	216 - 548	402	341	389	282	319
$(U_3 - U_2)$	283	190 - 476	333	290	330	236	292
$(U_4 - U_3)$	221	158 - 351	248	224	251	179	246
$(U_5 - U_4)$	133	109 - 187	152	130	145	102	159

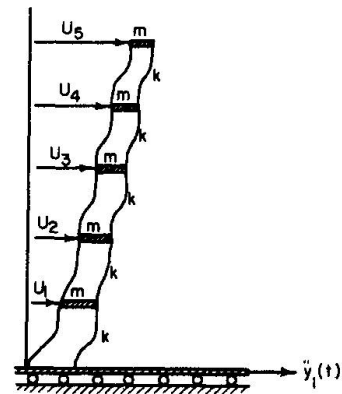


FIG. 7 SYSTEM CONSIDERED

The power spectral density of Eq. (6) is used with  $S_{01} = 0.0052 \text{ ft}^2/\text{sec}^3$ . On the basis of Fig. 2, stationary response assumption is acceptable for the period and damping values considered. Assuming Gaussian response,  $\bar{v}_b$  of Eq. (4) is

$$\bar{v}_b = \frac{1}{2\pi} \frac{\sigma_{\dot{X}}}{\sigma_X} e^{-\frac{1}{2} \left(\frac{b}{\sigma_X}\right)^2} \tag{9}$$

in which  $\sigma_X$  and  $\sigma_{\dot{X}}$  are the standard deviations of the response  $X$  and its derivative  $\dot{X}$ , respectively. For example, if  $X$  is taken to be the second story deformation, then

\*It has been numerically verified that the coupling effect of modes can be ignored in this structure because of separation in modal frequencies.

$$\sigma_X^2 \sim \sum_{k=1}^5 \gamma_k^2 \left[ \Phi_k(2) - \Phi_k(1) \right]^2 \int_{-\infty}^{\infty} G_{\ddot{Y}_1}(\Omega) H_k^2(\Omega) d\Omega \quad (10)$$

in which  $\gamma_k$  is the participation factor of mode  $k$ ,  $\Phi_k(i)$  is the amplitude of point  $i$  in mode  $k$ , and  $H_k(\Omega)$  is the modal frequency response function. The integral in Eq. (10) is adequately approximated by the following:

$$\int_{-\infty}^{\infty} G_{\ddot{Y}_1}(\Omega) H_k^2(\Omega) d\Omega = \begin{cases} \frac{\pi G_{\ddot{Y}_1}(\omega_k)}{2\beta\omega_k^3} ; & \omega_k \leq 2\omega_{f_1} \\ \frac{\pi G_{\ddot{Y}_1}(\omega_k)}{2\beta\omega_k^3} + \frac{\omega_{f_1}^2(1+4\beta_{f_1}^2)}{2\beta_{f_1}\omega_k^4} \pi S_{01} ; & \omega_k > 2\omega_{f_1} \end{cases} \quad (11)$$

Similar simplifications are possible for  $\sigma_Y$ .

Results of such calculations are tabulated in columns 5 and 6 of Table 2, representing the median and the 90 percentile response values.

The results for the normalized records, in Table 2, were obtained through a step-by-step integration of the equations of motion for the two horizontal components of the following records: El Centro (1934, 1940), Taft (1952), and Olympia (1949). Before processing, however, the records were normalized, following Housner [11], to have the same area under the undamped spectrum curve (from  $T = 0.1$  sec. to  $T = 2.5$  sec.); the responses thus normalized were averaged and fitted to the Housner's spectrum with zero damping. This gave the appropriate factors by which each record was multiplied and then used to obtain eight different responses for the multi-degree-of-freedom system. The average, the observed range of each response, and the second highest among the eight values are given in columns 2, 3, and 4 in Table 2. For this structure the highest values of all responses came from the same earthquake, NS El Centro 1934, but different records gave second highest values listed for the five response quantities summarized in Table 2.

The results listed under "response spectrum," columns 7 and 8 of Table 2, were obtained using the median response spectrum of Fig. 2 with  $\beta = 5\%$ . The absolute sum (ABS) and the square root of sum of the squares (SRSS) of the peak modal responses thus obtained are given in the columns indicated.

### Discussion

For the example structure, almost all of the response  $U_1$  (measure of base shear) is due to the first mode whereas the higher modes increase in importance for the response  $U_5 - U_4$ . With this in view, the agreement between the  $U_1$  responses in columns 2 and 5 merely indicates that a reasonable normalization of the records has been achieved since it is intended that the collection of records and earthquake model used produce roughly the same average effect on a single-degree-of-freedom system. On the other hand, the agreement between the  $(U_5 - U_4)$  values in columns 2 and 5 verifies the applicability of Eq. (1) to multi-degree-of-freedom systems.

Clearly there is a wide range in the responses of the structure to the normalized records; the proposed stochastic procedure with proper selection of  $P$  provides a consistent means for taking this range into account.

The values in columns 7 and 8 should be compared with those in column 5 because these are all responses to the same average base motion, described by the median response spectrum in Fig. 2. Note that the values of column 7 consistently underestimate those of column 5 whereas the values of column 8 are not consistent for all the responses in the structure.

### CONCLUSIONS

The main conclusions of the paper may be summarized as follows:

1. The Poisson approximation for up-crossings, Eq. (1), provides a flexible means for obtaining maximum response statistics under random earthquake-type motion; for single-degree-of-freedom systems, the differences with theory are not large enough to be significant, Fig. 1, and the results thus obtained are in agreement with those from simulation studies, Fig. 2. For the multi-degree-of-freedom system considered, Eq. (1) appears to produce results in fair agreement with those obtained from a normalized set of recorded accelerograms.

2. Responses to a set of normalized recorded motions having the same average response spectrum, which are commonly pooled together, vary in a wide range, Table 2. Unless quantitative means become available to separate differences among these records, it is necessary to take this variability into account. A random vibration approach provides a consistent manner for implementing this goal in practice.

3. For damped linear systems, the influence of nonstationarity of the input motions analogous to El Centro, Taft, and Olympia records can be ignored, Table 1. Starting from an average smooth response spectrum it is then possible to arrive at an appropriate spectral density to be used in a stochastic procedure for design.

### ACKNOWLEDGMENT

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## SUMMARY

Responses to a set of normalized recorded motions vary in a wide range. It is shown that for linear systems, approximate but adequate methods are available for systematically applying a stochastic approach to aseismic design. For certain commonly used accelerograms nonstationarity of the motions can be ignored when considering linear systems; in these cases the spectral density to be used should be selected after a design intensity is defined by a smooth response spectrum.

## RESUME

Le comportement d'un système soumis à un ensemble normalisé de mouvements enregistrés varie énormément. Il est démontré que, pour les systèmes linéaires, il existe des méthodes approximatives mais adéquates pour appliquer systématiquement l'approche stochastique dans les analyses paraséismiques. La non-stationarité des mouvements peut être négligée pour certains accélérogrammes largement utilisés et appliqués aux systèmes linéaires. Dans ces circonstances, la densité spectrale utilisée doit être déterminée à l'aide de l'intensité définie par une réponse spectrale lisse.

## ZUSAMMENFASSUNG

Das Verhalten eines Systems gegenüber einer Reihe von beobachteten und normalisierten Bewegungen variiert über einen weiten Bereich. Es wird gezeigt, dass für lineare Systeme ausreichende Näherungsmethoden vorhanden sind, um systematisch stochastische Methoden auf den Entwurf von Konstruktionen gegen Erdbebenbeanspruchung anzuwenden. Für bestimmte, häufig verwendete Beschleunigung-Zeit-Beziehungen kann für lineare Systeme die Veränderung des Ruhepunktes der Bewegungen vernachlässigt werden. In diesen Fällen sollten die zu verwendenden Spektraldichten (PSD) gewählt werden, nachdem die Intensität durch ein ausgeglichenes Verhaltensspektrum definiert ist.

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**Statistical Evaluation of Load Factors in Structural Design**

Calcul statistique des coefficients de sécurité dans la conception des ouvrages

Statistische Berechnung von Sicherheitsfaktoren im Entwurf

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**I Introduction**

The current design procedures in structural engineering involve the use of load factors to account for uncertainties that may exist in the applied loads and in the resisting strength of the structural members. The associated level of reliability depends on the probabilistic character of both loads and resistance. A procedure leading to the determination of load factors is formulated based on the following criteria: 1) The load factors should provide a desired level of structural safety with respect to all loading configuration. 2) This desired level of reliability may be determined by economic factors such as cost of construction and the cost of failure of the structural system. 3) Load factors for individual element design may be determined by the desired level of reliability of the whole system. 4) Additional information about the loading environment and the material strength characteristics can be systematically incorporated into the procedure to obtain a better set of load factors. 5) The procedure to evaluate load factors should be simple in form and easy for application. In the formulation that is presented here, dead, live, wind and earthquake loads are included. First, the load factors for a given reliability level for a structural element are calculated and second, the optimal reliability level of the structural system, its relationship with element reliability and the economic considerations in determining optimal reliability are considered. Step by step procedure is outlined and numerical example is worked out to illustrate the simplicity of the procedure.

**II Determination of Load Factors for Dead, Live, Wind and Earthquake Loads**

In addition to dead and live load, a structure may be subjected during its life span to loads like earthquake and wind. In previous works, Tang (7), Niyogi (3), Shah and others (4) have initiated such studies under dead and live loads. Incorporation of high wind loads and earthquake loads are especially important for structures with long service life and located around regions where occurrences of earthquakes or hurricanes or both are frequent. Given the location and the desired life span of a structure, the maximum magnitude of earthquake and wind loads that will act on the structure are random variables. In the formulation presented here, only the largest magnitudes of wind and earthquake loads are considered. The effects of repeated occurrences of minor

earthquakes and hurricanes are neglected. If we exclude the possibility that earthquake and wind would occur at the same time, then the possible loading configurations are:

$$\begin{aligned} W_1 &= DL + LL ; & W_2 &= DL + WL ; \\ W_3 &= DL + EL ; & W_4 &= DL + LL + WL ; \\ W_5 &= DL + LL + EL . \end{aligned} \quad 1$$

where DL, LL, EL and WL represent dead, live earthquake and wind loads respectively. If the mean values of these loads are taken as nominal loads to which load factors are multiplied, the corresponding design load values are:

$$\begin{aligned} W_1^* &= \alpha_{11}\mu_1 + \alpha_{12}\mu_2 ; & W_2^* &= \alpha_{21}\mu_1 + \alpha_{23}\mu_3 \\ W_3^* &= \alpha_{31}\mu_1 + \alpha_{34}\mu_4 ; & W_4^* &= \alpha_{41}\mu_1 + \alpha_{42}\mu_2 + \alpha_{43}\mu_3 \\ W_5^* &= \alpha_{51}\mu_1 + \alpha_{52}\mu_2 + \alpha_{54}\mu_4 \end{aligned} \quad 2$$

where  $\mu_1, \mu_2, \mu_3$  and  $\mu_4$  are the mean dead, live, wind and earthquake loads respectively. Note that load factors

$$\alpha_{13} = \alpha_{14} = \alpha_{22} = \alpha_{24} = \alpha_{32} = \alpha_{33} = \alpha_{44} = \alpha_{53} = 0 \quad 3$$

In general, for the  $i$ th combination of loading, we can write:

$$W_i^* = \sum_{j=1}^4 \alpha_{ij}\mu_j \quad 4$$

Stochastically, if all the load components are assumed normally distributed, the design load for the  $i$ th loading configuration is

$$W_i = \mu_{W_i} + k_i \sigma_{W_i} = \sum_j \mu_j + k_i \left( \sum_j \sigma_j^2 + \sum_{\substack{k,l \\ k \neq l}} \rho_{kl} \sigma_k \sigma_l \right)^{1/2} \quad 5$$

where  $\rho_{kl}$  denotes the coefficient of correlation between the  $k$ -th and  $l$ -th component. The index  $j$  depends on the  $i$ th loading configuration. Thus,

$$\begin{aligned} j &= 1, 2 \quad \text{when } i = 1 ; & j &= 1, 4 \quad \text{when } i = 3 \\ j &= 1, 3 \quad \text{when } i = 2 ; & j &= 1, 2, 3 \quad \text{when } i = 4 \\ j &= 1, 2, 4 \quad \text{when } i = 5 . \end{aligned} \quad 6$$

In terms of the component loads, this design load may also be written as

$$W_i = \sum_j l_{ij} = \sum_j \mu_j (1 + k_i V_j) \quad 7$$

where  $V_j$  = coefficient of variation of the  $j$ th load component. Equating 4, 5 and 7 and assuming that the resistance is Gaussian with coefficient of variation  $V_R$  and reliability coefficient  $k_i$ , ( $k_i$  measures the number of standard deviations in the standardized Gaussian distribution corresponding to the  $i$ th load), a general expression for the load factor can be obtained

$$\alpha_{ij} = \frac{1}{1 - k_i V_R} \cdot \frac{l_{ij}}{\mu_j} = \frac{1 + \frac{(\sum_j \sigma_j^2 + \sum_{\substack{k,l \\ k \neq l}} \rho_{kl} \sigma_k \sigma_l)^2}{\sum_j \sigma_j^2}}{1 - k_i V_R} k_i V_j \quad 8$$

If  $k_i^*$  denotes the level of the overall reliability for the  $i$ th loading combination, the relation between  $k_i$  and  $k_i^*$  can be shown to be (ref. 7)

$$k_i = \frac{\sqrt{(V_R \mu_R)^2 + \sigma_{L_i}^2}}{V_R \mu_R + \sigma_{L_i}} k_i^* \tag{9}$$

where

$$\sigma_{L_i} = \left( \sum \sigma_j^2 + \sum_{\substack{k, \ell \\ k \neq \ell}} \rho_{k\ell} \sigma_k \sigma_\ell \right)^{1/2} \tag{10}$$

For any desired level of overall reliability corresponding to  $k_i^*$ ,  $k_i$  is determined from equation 9. Load factors are then computed by equation 8. In order to test for the sensitivity with respect to the type of distribution that is assumed, Extreme Type II (largest value) distribution has been assumed for the wind and earthquake loads, keeping the dead load and live load distributions Gaussian. It was observed that the values of design loads and load factors obtained for any desired reliability level does not change appreciably from the all-Gaussian model.

III Summary of Procedure

A step by step procedure is outlined below for the evaluation of load factors under dead, live, wind and earthquake loads. (1) Compute the magnitudes of dead, live, wind and earthquake loads acting on the structure, based on the empirical formula in the existing code. (2) Through structural analysis, obtain the design moments (stresses) due to each load component. Select the critical section for the member to be designed. (Numerical example is given in section 6.) Let  $\mu_1, \mu_2, \mu_3$  and  $\mu_4$  be values of such design moments (stresses) at the critical section due to dead, live, wind and earthquake load respectively. (3) Based on the available data, determine the coefficients of variation  $V_1, V_2, V_3$  and  $V_4$  for each load component. Compute  $\sigma_1, \sigma_2, \sigma_3$  and  $\sigma_4$  by using equation  $\sigma_i = \mu_i V_i, i = 1, 2, 3, 4$ . (4) From available data, determine coefficient of variation of resistance ( $V_R$ ). (5) Choose the value of the overall reliability for the member to be designed, say  $\bar{u}$ . Determine the corresponding reliability coefficient  $k^*$  from tables of normal integrals. (6) Compute the values of  $\sigma_s, \mu_s, \sigma_L, V_L$  and  $k$  for each loading configuration. All  $\rho_{ij}$  except  $\rho_{14}$  may be assumed to be zero. The value of each  $k_i$  ( $i = 1$  to  $5$ ) is determined from the values  $k^*, V_R$  and  $V_L$ , by using equation 9 or from charts similar to that in Figure 4. (7) For each loading configuration  $i$ , compute load factors  $\alpha_{ij}$  by

$$\alpha_{ij} = \frac{1 + \left( \frac{\sigma_L}{\sigma_s} k V_j \right)_i}{1 - k_i V_R}$$

Note that  $\alpha_{13}, \alpha_{14}, \alpha_{22}, \alpha_{24}, \alpha_{32}, \alpha_{33}, \alpha_{44}, \alpha_{53}$  are zero. (8) Compute the design load

$$W_i = \sum_{j=1}^4 \alpha_{ij} \mu_j \quad i = 1 \text{ to } 5 .$$

The example given in Section 6 would help to illustrate the above procedures.

IV System Analysis

In general, a structure is made up of structural components called members or elements. For a floor system consisting of four T-beams, each beam is called an element. The reliability of the floor system depends on the reliability of

each beam as well as the type of framing of the system. We will consider two types of systems, namely series system and parallel system.

(A) Series System - The series system is defined as one which fails if any one of its elements fails. When any element fails, that is, when its strength capacity is exceeded by the applied load, it does not continue to deform sufficiently so that its adjacent elements can take on the extra load. If we further assume for the four-beam floor system that (i) The probabilistic properties of each T-beam are identical. (ii) The event that any one beam fails is independent of that of the others. (iii) The loads are evenly distributed to each beam. Then the probability of failure of the floor system is

$$p_{FS} = 1 - (1 - p_F)^4 \approx 4p_F \quad (\text{for small } p_F) \quad 11$$

where  $p_F$  = probability of failure of each element. This may be generalized such that for a series structural system of  $N$  elements, its probability of failure is  $N$  times that of the individual element, that is

$$p_{FS} = N \cdot p_F \quad 12$$

(B) Parallel System - On the other hand, if all the elements are designed with sufficient ductility such that they do not lose their load capacity until all elements reach ultimate conditions, the structural system is called a parallel system. Ideally, this system will fail when all the elements fail. However in practice, even for the most ductile system, the structural system will fail when only a fraction of the total number of elements fails. In addition to the assumption listed in Case (A), if we assume for the same floor system that the system will fail if two or more beams fail, then using the concept of Bernoulli Trials,

$$p_{FS} = \binom{4}{2} p_F^2 (1 - p_F)^2 + \binom{4}{3} p_F^3 (1 - p_F) + \binom{4}{4} p_F^4 \quad 13$$

For small value of  $p_F$ , then

$$p_{FS} = \binom{4}{2} p_F^2 = 6p_F^2 \quad 14$$

This may be generalized for a parallel structural system with  $N$  elements, if the failure of  $M$  or more elements lead to system failure, then the probability of system failure is approximately

$$p_{FS} \approx \binom{N}{M} p_F^M \quad 15$$

Since reliability is defined as

$$\bar{u}_S = 1 - p_{FS}, \quad \bar{u} = 1 - p_F \quad 16$$

we can see from equations 12 and 15 that simple relations do exist between system reliability and element reliability.

#### V Cost Analysis

It is known that the total design load increases as the desired level of reliability for one design increases. If we assume that cost of construction is linearly proportional to this value of design load, and let  $c_2$  be the loss when the structure fails, and assume all other costs negligible, an expected total cost function can be defined as

$$\begin{aligned} TC &= \text{cost of construction} + \text{expected cost of failure} \\ &= c_1 W^*(\bar{u}) + c_2 (1 - \bar{u}_S) \end{aligned} \quad 17$$

where  $W^*$  = total design load which is a function of reliability;  $c_1$  = construction cost per unit design load for the system; and  $\bar{u}$  = reliability of an element.

For a series structural system, applying Equations 12 and 16, this becomes

$$TC(\bar{u}) = c_1 [W^*(\bar{u}) + C(1 - \bar{u})] \quad 18$$

where  $C = \frac{\text{cost of failure}}{\text{cost of construction per element per unit design load}}$

In other words, the cost coefficient  $C$  is a measure of how important the consequences of structural failure are relative to the unit cost of construction. Thus, a large value of  $C$  corresponds to a case where failure involves great losses. In the numerical work which follows, the mean dead load is assumed not as a constant but to be 1 psf per 5 psf of the total design load. The expected total cost function is evaluated for the sample data used for the Gaussian model. Figure 1 shows, for a given value of  $C$ , that a distinct minimum cost does exist at a certain level of reliability. As  $C$  increases, the optimal design should have a higher level of reliability so that the expected loss due to failure is decreased. The optimal reliability level as a function of the cost coefficient for various coefficients of variation of loads and resistance is shown on Figure 2. The variation in the load does not seem to affect this optimal relation. However as the coefficient of variation of resistance increases, the optimal reliability level does decrease for any given cost coefficient  $C$ . This implies that the optimal decision, when the resistance is highly uncertain, is to cut back the cost of construction by using smaller load factors and to risk a higher probability of failure. The analysis is similar in the case of a parallel system. The total cost function for the four-beam floor system example is computed to be

$$TC(\bar{u}) = c_1 [W^*(\bar{u}) + 1.5C(1 - \bar{u})^2] \quad 19$$

The relation between the cost coefficient and the optimal level of element reliability is given in Figure 3 for various values of the coefficient of variation of the resistance. This relation is again very insensitive to the statistical variation in the applied loads. For a given value of cost coefficient  $C$ , the optimal level of element reliability is much less for the parallel system than for the series system. Therefore, for the two types of systems, namely, series and parallel, once we know the values of  $C$  and  $V_R$ , the optimal level of reliability for element design may be easily determined.

#### VI Numerical Example - Dead, Live, Wind and Earthquake Loads

A one-story plane frame structure is chosen to illustrate how load factors can be determined in a step by step procedure for a desired level of reliability. The frame's dimension and member stiffness are shown in Figure 5. Assume that only bending moment failure is of interest. (1) Compute the magnitudes of dead, live, earthquake and wind loads acting on the structure. Assume a tributary span of 20 feet perpendicular to the frame. The dead load is represented by a uniform load acting on the girder BC. Its magnitude is given by

$w_1 = DL \times 20' = 60 \times 20 = 1.2 \text{ K/ft}$ . Similarly, the magnitude for the live load is  $w_2 = LL \times 20' = 80 \times 20 = 1.6 \text{ K/ft}$ . For the earthquake load, the Seismic

Engineering Association of California (SEAOC) recommends the following equation for the equivalent static lateral load. (Ref. 8)

$$Q = K C_o W Z_o \quad 20$$

We may assume  $Z_o = 1$  (in California),  $D = 1$  for the type of framing and  $C_o = 0.1$  for a one-story structure. Then  $Q_1 = DL \times \text{floor area} \times Z_o \times K \times C_o = 60 \times 20 \times 20 \times 1 \times 1 \times 0.1 = 2.4 \text{ K}$ . For the wind load, an equivalent static force is given by (ref. 9)

$$F = .00256 C_d A V^2 \quad 21$$

We may assume  $C_d = 1$  for flat wall,  $V = 80$  mph for a 50-year occurrence period in San Francisco for a building height of 50 feet, and  $A = 10 \times 20 = 200$  square feet for a tributary span of 20 feet and an exposed height of 10 feet. Then  $Q_2 = .00256 \times 1 \times 200 \times 80^2 = 3.3^k$ . (2) Compute bending moments due to each load. The moments (in kip-ft.) at critical locations are

	DL	LL	EL	WL	DL+LL	DL+EL	DL+WL	DL+LL+EL	DL+LL+WL
A	13.4	18.0	34.3	47.1	31.4	47.7	60.5	65.7	78.5
B	-26.7	-35.7	-25.7	-35.4	-62.4	-52.4	-62.1	-88.1	-97.8 *
C	-26.7	-35.7	27.7	35.4	-62.4	-1.0	8.7	-36.7	-27.0
D	13.4	18.0	-34.3	-47.1	31.2	-17.9	-33.7	-3.1	-15.9
E	33.3	44.5	0	0	77.8	33.3	33.3	77.8	77.8

Take location B for our further analysis. (3) Compute mean and standard deviation for each component.

$$V_3^2 \approx V_{z_o}^2 + V_k^2 + V_{c_o}^2 + V_{w_o}^2$$

$$= (0.1)^2 + (0.08)^2 + (0.2)^2 + (0.08)^2 = 0.0628 \text{ (say)}$$

$$\mu_1 = 26.7 \quad V_1 = 0.083 \quad \sigma_1 = 2.22$$

$$\mu_2 = 35.7 \quad V_2 = 0.25 \quad \sigma_2 = 8.93$$

$$\mu_3 = 35.4 \quad V_3 = 0.2 \quad \sigma_3 = 7.08$$

$$\mu_4 = 25.7 \quad V_4 = 0.25 \quad \sigma_4 = 6.43$$

The index 1, 2, 3, 4 correspond to dead, live, wind and earthquake load contributions respectively. The values of the coefficient of variation  $V_1, V_2, V_3, V_4$  are assumed for numerical illustration. The correlation coefficients  $\rho_{ij}$  are assumed zero, except for  $\rho_{14} = 0.5$ . (4) Assume coefficient of variation of resistance, say, 0.1 in this case. (5) Choose the desired overall reliability  $u = 0.9999$ . (6) Compute the values of  $\sigma_s, \mu_s, \sigma_L, V_L, k^*, k$ .

Case	Loading Combination	$\sigma_s$	$\mu_s$	$\sigma_L$	$V_L$	$k^*$	$k$
1	DL + LL	11.15	62.4	9.2	0.147	3.72	2.68
2	DL + WL	9.3	62.1	7.41	0.12	3.72	2.7
3	DL + EL	8.65	52.4	7.8	0.15	3.72	2.68
4	DL + LL + WL	18.23	97.8	11.6	0.12	3.72	2.7
5	DL + LL + EL	17.58	88.1	11.85	0.134	3.72	2.7

(7) Compute load factors  $\alpha_{ij}$  and design load  $w_i$ .

Case	Loading Combination	$\alpha_{i1}$	$\alpha_{i2}$	$\alpha_{i3}$	$\alpha_{i4}$	$w_i$ (k-ft)
1	DL + LL	1.61	2.13	0	0	119
2	DL + WL	1.61	0	1.96	0	113
3	DL + EL	1.61	0	0	2.08	96.8
4	DL + LL + WL	1.57	1.97	1.85	0	177
5	DL + LL + EL	1.58	1.97	0	1.97	163

## VII Conclusion

An approach is presented in this paper to formulate a procedure for quantitative evaluation of load factors for dead, live, wind and earthquake loads. Various models which describe the statistical characteristics of the loads and resistance are studied under the formulation. Load factors obtained in each case for any desired level of reliability do not differ appreciably. The Gaussian

model for both the loads and resistance appear to be the most convenient one to work with. Step-by-step procedure are outlined to illustrate the simplicity in the evaluation of load factors. From system and cost analyses, the optimal level of reliability for the structural member design are mainly determined by two parameters. They are the cost coefficient  $C$  which represents the ratio of cost of failure and the cost of construction, and the coefficient of variation of resistance  $V_R$ . Once this desired reliability level is given, together with the coefficients of variation of each load component, the corresponding load factors may be computed by the procedures formulated.

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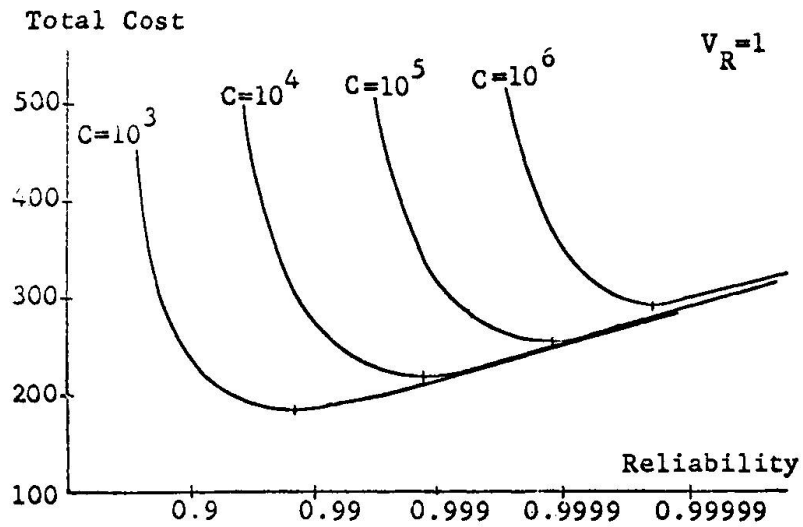


FIG. 1 TOTAL COST VS. ELEMENT RELIABILITY--  
SERIES SYSTEM

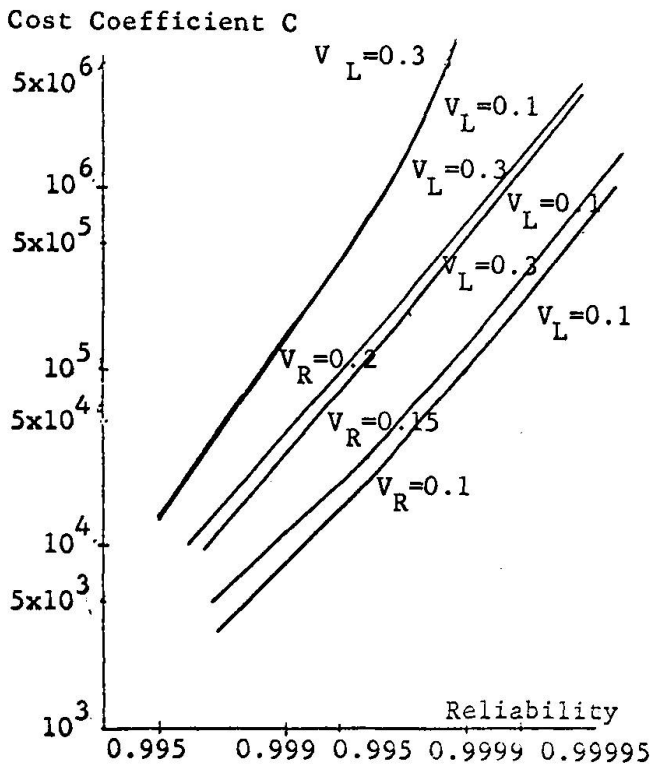


FIG. 2 COST COEFFICIENT VS. OPTIMAL  
ELEMENT RELIABILITY--SERIES  
SYSTEM

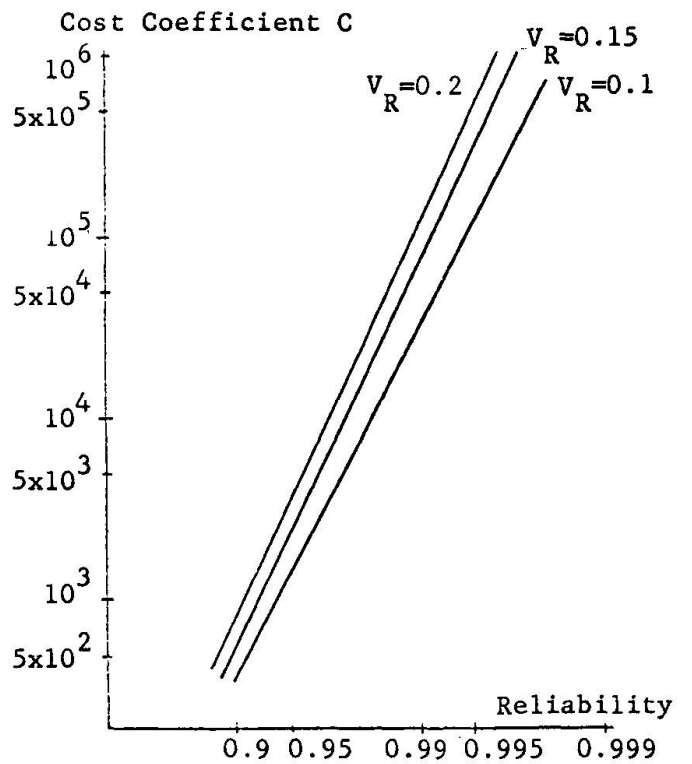


FIG. 3 COST COEFFICIENT VS. OPTIMAL  
ELEMENT RELIABILITY--  
PARALLEL SYSTEM

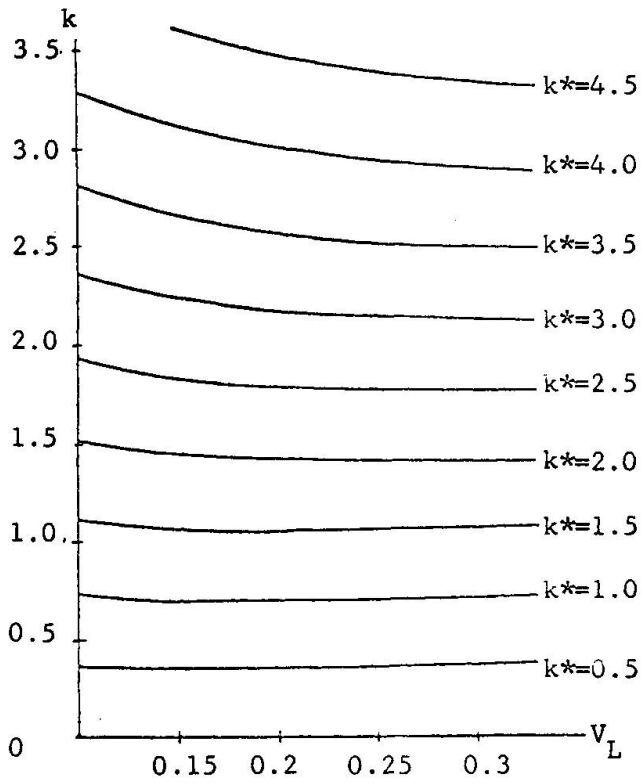


FIG. 4 CHART FOR DETERMINING RELIABILITY COEFFICIENT  $k - V_R = 0.15$

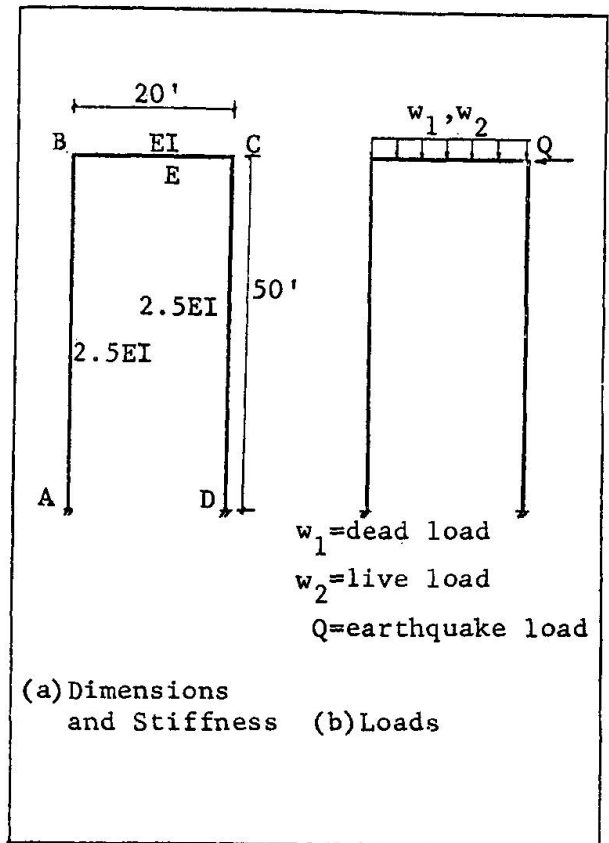


FIG. 5 PLANE FRAME EXAMPLE

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### III

#### **On Fatigue Damage Estimation of Railway Bridge Members Through Actual Train Loading**

Sur l'estimation des dommages causés aux éléments de ponts-rails par la fatigue due aux essieux

Über die Schätzung von Ermüdungsschäden an Eisenbahnbrücken-Teilen durch Zuglasten

FUMIHITO ITOH  
JAPAN

#### INTRODUCTION

The major load acting on railway bridges is the train load which makes a short-term variation. Its amplitude of variation is considerably wide, therefore in discussing the safety of railway bridge members it is important to consider not simply the maximum value of load but also the decline of strength due to repeated loading, i.e., the fatigue damage. It is for this reason that in many countries the specifications or codes for designing the railway bridges set an allowable unit stress for fatigue in bridge members, and information on how to decide the safe limit of fatigue strength is being eagerly sought by engineers.

Fatigue life depends on the number of repeated stress cycles as well as on the maximum value of stress. Therefore the thing to be known is not only what is the maximum value of stress developed under passage of a train but also to what number of stress cycles with an amplitude of the maximum stress in that condition corresponds the fatigue damage occurring in bridge members under passage of that train.

According to Prof. Pelikan, who investigated German trains, the number of repeated stress cycles mentioned above depends on the span length of a railway bridge. This trend must be the same with the Japanese railways, too. JNR has constructed Tokaido SHIN KANSEN as a line devoted to the operation of multiple-unit electric railcar trains. On this line, the train load is uniformly distributed but with a larger wheel base than in locomotives, the number of repeated loadings tends to be large. In case of bridges on SHIN KANSEN, with the above fact taken into account, the designing was so made as to let the bridge bear a larger load than really encountered but the difference due to bridge span length was ignored.

The purpose of this paper is to investigate the situation in more

detail. The points to be elucidated are: How best to analyze the stress-time relation; how best to utilize the results of this analysis for estimation of fatigue life; how to relate these results to train category or span length of a bridge; or what are the best statistical quantities for checking the safety. The present paper sums up what has so far been achieved on this problem by the author in his own way which is even short of the statistical approach.

#### STRESS-TIME RELATION UNDER TRAIN LOAD

The stress-time relation developing in bridge members under train passage has different features from those of airplane wings in turbulent air or of axles in running cars. The stress in a wheel axle varies randomly on both sides of the mean level which is virtually constant, with no definite correlation existing between maximum and minimum.

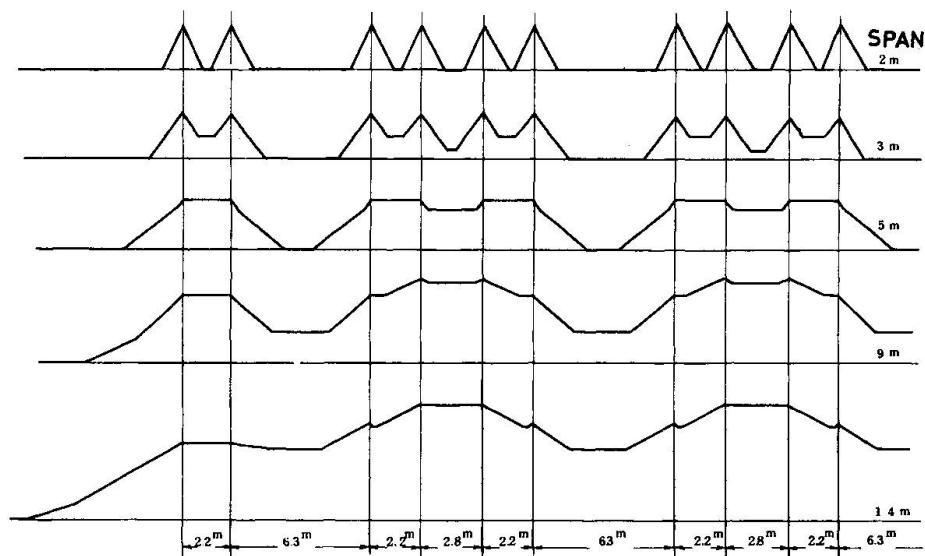


Fig. 1 Theoretical stress-time relations under Tokaido SHIN KANSEN N-load train.

By contrast, the stresses in bridge members are, though short in duration, characterized by mean stress variation which has a stress-time relation depending on train category, span length of the bridge and functions of members, and a relatively small oscillation added around this stress. Figure 1 illustrates the theoretical wave form appearing Tokaido SHIN KANSEN passes over simple supported girders of several different span lengths, the wave form is broadly similar to the above-mentioned mean stress variation. From this it is realized how influential is the span length.

#### STATISTICAL COUNTING METHODS FOR ANALYZING LOAD-TIME HISTORIES TO BE APPLIED FOR ESTIMATION OF FATIGUE LIFE

In the discussion of fatigue life, it is not enough to find simply the magnitude of maximum stress intensity and its probability of occurrence. Because the fatigue strength is not associated with the magnitude and frequency of maximum stress intensity only, but, more important, with those of stress amplitude. Thus, the counting method for this purpose must be one that can permit conversion to the magnitude and

frequency of stress amplitude so that the counted stress waves appearing in railway bridge members may be related to the fatigue strength. If several such methods are available, the most reasonable and most practicable one must be selected from among them.

The author checked the following nine as such counting methods:

- a) Peak Count Method.
- b) Mean-Crossing Peak Count Method.
- c) Level-Crossing Count Method.
- d) Fatigue Meter Count Method.
- e) Range Count Method.
- f) Range-Mean Count Method.
- g) Modified Range Count Method.
- h) Range-Pair Count Method.
- i) Modified Range-Pair Count Method.

Out of these nine, the four (a)-(d) are rejected for the present purpose because they cannot give stress amplitude from counted result on stress-time relations such as developed in railway bridges. The methods (e) and followings, which deal with stress amplitude from the first, are satisfactory in this respect, but the other three except (h) and (i) have the drawback that not only the results vary depending on the position and number of artificially selected counting levels, but also a major amplitude is apt to be overlooked as the result of minor vibration components, if any, being counted. Since it is undesirable to adopt for the solution of a scientific problem a method whose data depend on something artificial, the author thinks it advisable to refrain from use of such methods.

The remaining two methods (h) and (i) produce absolutely the same results. The instruments available for Range-Pair Count Method have a slow response, and to count the stress so rapidly changing as those in railway bridge the instrument becomes too large to be fit for field use. Thus, Modified Range-Pair Count Method has been found best for this purpose.

This is a method devised by Shiraishi; in this method the maximum and minimum values in stress-time relation are arranged in the order of their occurrence and they as reduced to a pulsating load are counted. Shiraishi performed the reduction graphically, but the author changed the procedure to do it numerically with no resort to the counting level. Measured stress-time relations are recorded on magnetic tape and, after reduced by the data processor to a series of extreme values, are fed to the electronic computer for necessary conversion.

#### METHOD TO BE USED FOR ESTIMATION OF FATIGUE LIFE

Numerous studies have been made to search for a damage law that can predict the number of repeated cycles to failure of members subjected to variable load such as to be able to agree with experimental results, but there is yet no theory established about which of these studies can give the most reliable results.

Here the author is going to discuss not in terms of determining which of these studies is generally the most accurate but in those of finding which of them will be the most convenient for the practical purpose. In this line of thinking certain errors would be tolerated.

From this standpoint the so-called Miner's method or its improvement, for instance, method of Corten & Dolan seems to be the most preferable one. As steel, when corroded, has its endurance limit reduced, here for the purpose of simplifying the calculation, the endurance limit of steel is to be disregarded.

Then putting the stress in a material as  $\sigma_i$ , its number of cycles to failure at this stress level as  $N_i$  and  $k$  as a constant, it is assumed that the following holds:

$$\sigma_i N_i^k = \sigma_j N_j^k.$$

Under this assumption a fatigue failure occurs when the following holds:

$$\sum_i \left( \frac{n_i}{N_i} \right) = 1.0.$$

If the number of stress amplitudes  $\sigma_i$  appearing under passage of one train as live load is  $n_i'$ , the number of trains,  $N_t$ , to a fatigue failure will be:

$$\frac{N_t}{N_1} \sum_{i=1}^m n_i' \left( \frac{\sigma_i}{\sigma_1} \right)^{1/k} = 1.0.$$

Therefore, if

$$N_{e1} = \sum_{i=1}^m n_i' \left( \frac{\sigma_i}{\sigma_1} \right)^{1/k},$$

$N_t$  will be given as the ratio of repeated numbers  $N_1$  and  $N_{e1}$  to the representative stress  $\sigma_1$ . If the maximum stress due to a train load is taken as representative stress  $\sigma_1$  and if the values of a different reference stress  $\sigma^*$  and the repeated number  $N^*$  corresponding to this stress are known,  $N_t$  will be given by

$$N_t = \frac{N^*}{N_{e1}} \left( \frac{\sigma^*}{\sigma_1} \right)^{1/k}.$$

If the number of trains in category  $j$  passing over the bridge in one year is  $n_{tj}$ ,  $\sigma_1$  for  $j$ th train category is  $\sigma_j$ , etc., and the number of train categories are  $s$ , the serviceable number of years  $T$  for the bridge will be given by

$$T = \frac{N^*}{\sum_{j=1}^s n_{tj} N_{ej} \left( \frac{\sigma_j}{\sigma^*} \right)^{1/k}},$$

or finding the equivalent maximum stress intensity for each train, i.e.,  $\sigma_t = \sigma_1 N_{e1}^k$ ,  $T$  will be given by

$$T = \frac{N^*}{\sum_{j=1}^s n_{tj} \left( \frac{\sigma_{tj}}{\sigma^*} \right)^{1/k}}.$$

If  $\sigma_i$  follows a logarithmic normal distribution with  $\sigma_m$  as median, putting the total number of trains in a year as  $N_0$  and using the

standard deviation  $S_0$  of  $\log \sigma_t$ , the following is calculated:

$$N_{eq} = N_0 \exp\left[\frac{S_0}{2k^2}\right],$$

and accordingly will be found as

$$T = \frac{N^*}{N_{eq}} \left(\frac{\sigma^*}{\sigma_m}\right)^{1/k}.$$

### ESTIMATION BY STATICAL CALCULATION

As mentioned above, the fatigue damage under train passage depends on the equivalent repeated cycles  $N_{e1}$ , as well as on the magnitude of maximum stress. Much attention has been paid to the maximum stress, but  $N_{e1}$  is an entirely novel conception and the following discussion will center around  $N_{e1}$ .

#### (1) Bridge Span vs. Train Categories.

Calculations were made with four categories of train and the relations between the number of trains in each category that can be passed until failure of bridge members and the span length of bridge was established as in Fig. 2. The categories of trains adopted for calculation and their symbols were as follows:

- 30 x N; Standard freight electric railcar train for designing Tokaido SHIN KANSEN, composed of 30 cars.
- 12 x P; Standard passenger electric railcar train for designing Tokaido SHIN KANSEN, composed of 12 cars.
- 12 x P'; Revenue passenger electric railcar train operated on Tokaido SHIN KANSEN, composed of 12 cars.
- K18 + 30F; A locomotive of K18 standard construction load in Japan hauling 30 four-axle freight cars 10m long with 15 ton axle weight.

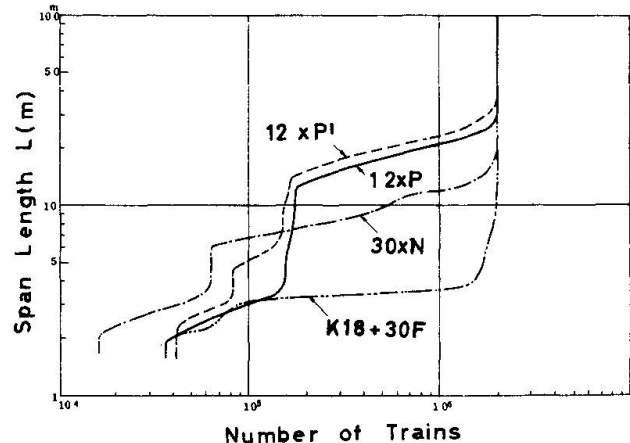


Fig. 2. Span-life relations by statical calculation.

Bridge members considered in this calculation were cord members at span center or flange with  $k = 0.20$  taken on assumption that they were designed to be able to stand just 2 million loadings at maximum stress.

From these calculations it may be concluded:

- $N_{e1}$  of a girder with shorter span length than the shortest wheel base is practically equal to the number of axles and constant regardless of span length.
- For a train hauled by a heavy locomotive, practically  $N_{e1} = 1.0$  will hold, if the span length of the bridge is longer than 4m.
- For an electric railcar train, too,  $N_{e1} = 1.0$  will hold, if the bridge span length is more than 1.3 ~ 1.5 times the car length.
- Under an electric railcar train,  $N_{e1}$  is approximately equal to the number of car couplings on a girder with a span longer than the wheel-base at the coupling and shorter than the wheel-base between the second and the third axles of a car.

(2) Difference Depending on Position of Section Under Consideration.

So far as the bending moment is concerned, the value depends little on the position of the member considered in the axial direction of bridge. The results of calculations about 7 sections in a span under passage of JNR series 181 10-units electric car train fitted all into the shaded region in Fig. 3.

(3) Load-Spreading Effect of Rails.

In JNR, there has never been a single case of even a short-span bridge developing a fatigue failure in its members. This may raise a doubt about the reliability of the above theory.

If the bridge is supposed to have been designed to have a strength that can stand for 2 million repetitions of loading in accordance with the customary practice and the axle weight is assumed to be spread actually as Fig. 4 on account of rail- and sleeper rigidity, the calculation about the same train as in Fig. 2 produces the results as indicated in Fig. 5.

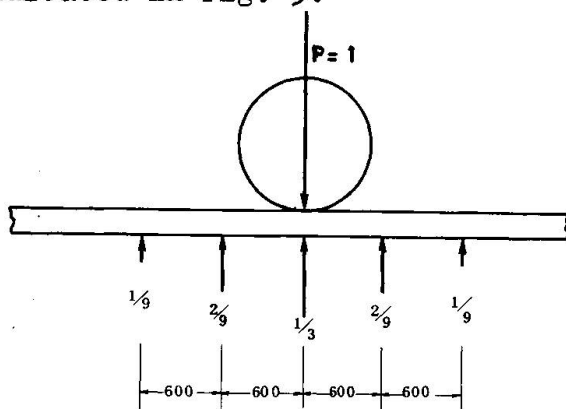


Fig. 4. Assumed distribution of load under sleepers.

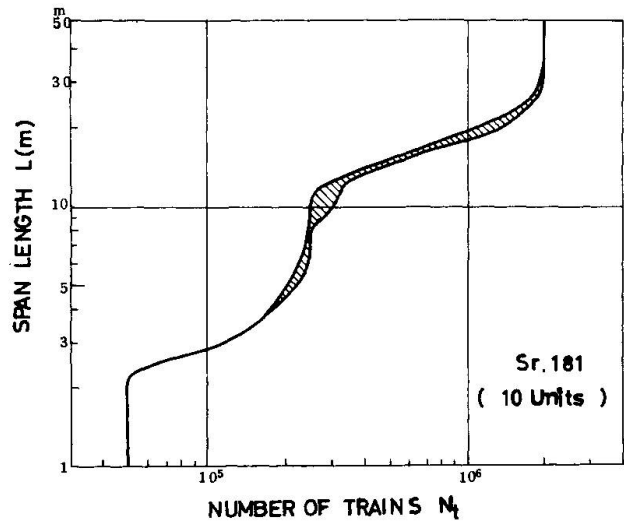


Fig. 3.  $L - N_t$  curve of simple girder under 10 units car train (Series 181).

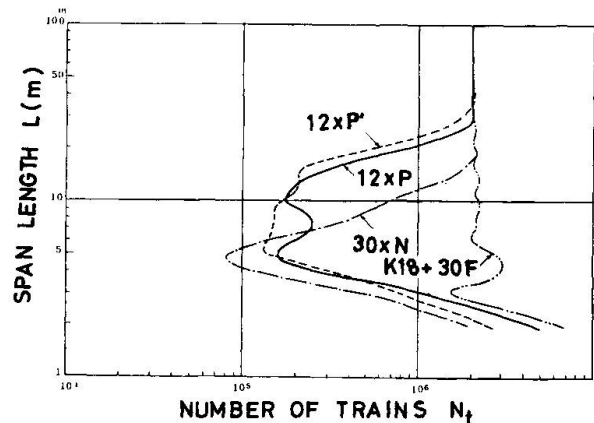


Fig. 5.  $L - N_t$  curve under effect of rail and sleeper rigidity.

Namely, so long as the bridge has been customarily designed and the rails supported by the sleepers, the hazard of fatigue failure in a span shorter than 4 m may be rather discounted. Thus, it may be practically presumed that a train hauled by a heavy locomotive necessarily develops a maximum stress intensity only once in its passage. Concerning the electric railcar trains, bridges calling for the most elaborate checking exist in the range of span lengths between 4 m and 20 m.

ANALYSIS USING MEASURED WAVE FORM

Real stress waveforms emerging in bridges are not always in agreement with the results of customary calculations. As well known, all stress-time relations make a smooth change with no inflections such as

observed on calculated curves. Meanwhile, an increased train speed is accompanied with a vibration which is not susceptible of routine calculation. Thus it becomes necessary to ascertain to what extent the above-mentioned calculation is applicable to the real waveforms. Here are to be cited a few examples in recent times.

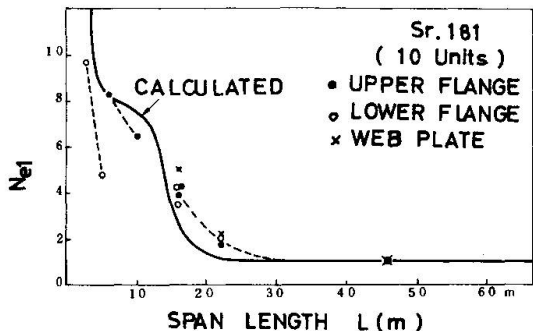


Fig. 6. L - N<sub>e1</sub> relation by electric railcar train (Series 181, 10 units).

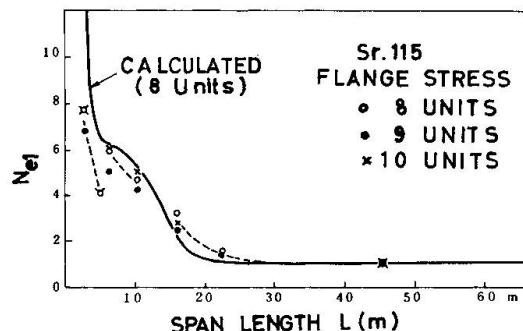


Fig. 7. L - N<sub>e1</sub> relation by electric railcar train (Series 115) measured on flanges of girder.

Fig. 6 and Fig. 7 illustrate the L - N<sub>e1</sub> relations as obtained from a similar analysis to the above of measured stresses in three plate girders, respectively with span lengths of 16.0m, 22.3m and 45.5m, in these figures, in addition to the data on the main girders, data using the measured stresses in 3.2m and 5.1m stringers as well as cross beams 6.4m and 10.2m long in the influence line are entered. Cross beam may be equated to main girder, but the curve for stringer characterized by continuity cannot agree with the curves plotting the results about main girders or cross beams.

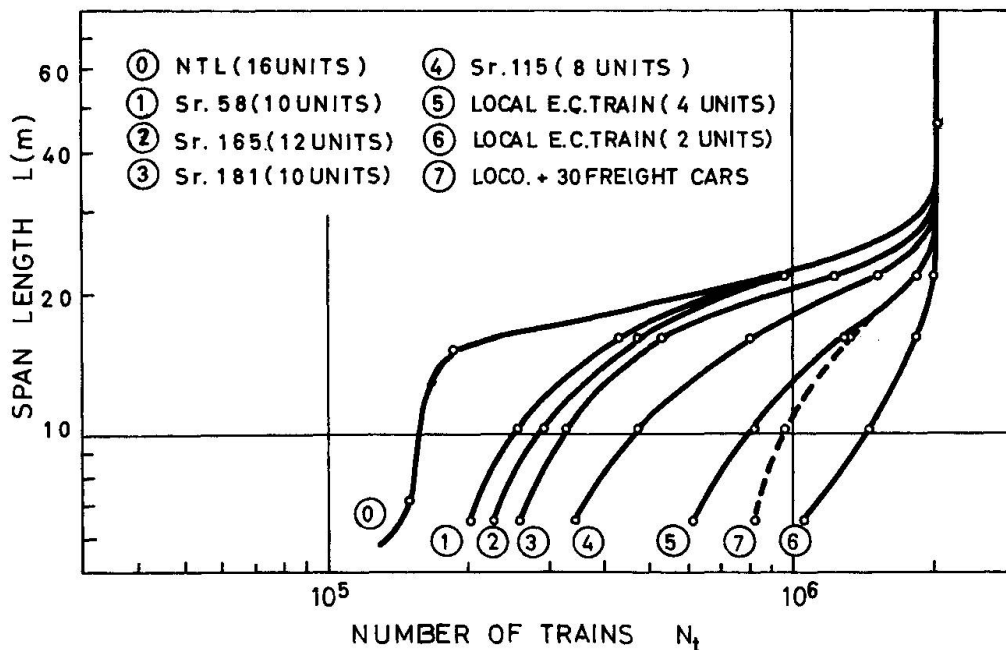


Fig. 8. Relation between span length and number of train passage to failure.

Figure 8, summerizing the results of such measurements, illustrates the relation between span length and number of train passages to failure just like in figure 2. Thus it can be said that the results of analysis based on measured waveforms are in the same tendency as the results of static calculations. So far as the range measured here is concerned, a slight difference in the stress-time relation and the small amount of vibration seem to have no large bearing on fatigue.

This, however, does not mean that  $N_{e1}$  is not sensitive to the speed. The author has some data which show extremely great influence beyond certain critical speed. In spite of these results, nothing conclusive can be said about the general tendency in the effect of speed on  $N_{e1}$ , because even among girders looking similar, some are influenced heavily by the speed, and others are little influenced by it, while still others are almost free from the influence of speed.

After all, above-mentioned equivalent numbers of loading, instead of crude statistical distributions of counted data on stress-time relations in railway bridges, seems to have a merit or possibility to be used as one of statistical quantity to measure the remaining life to fatigue failure under actual train loadings.

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#### SUMMARY

A kind of fatigue life estimating method combining that of Corten and Dolan with modified range-pair count method is suggested, which is practical to calculate the fatigue damage of existing railway bridge members.

The result of the illustrative applications of this method on simply supported girders shows that equivalent numbers of loading cycle depend on the span length of the girder and the arrangement of the wheel axles.

#### RESUME

L'auteur présente une méthode pour prédire la résistance à la fatigue des éléments de ponts ferroviaires. Il s'agit de la combinaison de deux méthodes; l'une permettant d'évaluer la limite de fatigue, proposée par Corten et Dolan, et l'autre étant une modification de la "range-pair count method".

Les résultats d'application de cette méthode à la poutre simplement appuyée nous montre que la fréquence équivalente des charges dynamiques dépend considérablement de la portée de poutre et de la répartition des essieux du train.

#### ZUSAMMENFASSUNG

Für die Ermüdungsdauer wird eine Schätzmethode, jene von Corten und Dolan mit dem abgeänderten Verfahren des "Schwingungsweite-Zählens" kombinierend, vorgeschlagen, die erlaubt, das Ermüdungsversagen für Eisenbahnbrückenteile einfach zu berechnen.

Das Ergebnis der Anwendungen auf einen einfach aufgelegten Träger zeigt, dass die Anzahl Lastwechsel von der Trägerspannweite und der Radachsenanordnung abhängen.

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**Aspects statistiques concernant les surcharges climatiques et la sécurité des constructions**

Die statistische Seite der Klimabelastung und der Bauwerkssicherheit

Statistic Aspects on the Climatic Loadings and the Safety of Structures

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L'étude de la sécurité des constructions [2], [5], [15], [22], [19], et des surcharges climatiques [1], [3], [4], [6], [9], [12], [13], [14], [17], [20], [21] ont été l'objet de nombreuses recherches. Les hommes de science ont cherché à déterminer avec exactitude les surcharges qui agissent sur les bâtiments et les efforts dans les éléments des bâtiments, pour pouvoir envisager un coefficient de sécurité minimal, qui ne compromette pas la résistance, la stabilité et les conditions d'exploitation des bâtiments.

Les nouvelles méthodes et les possibilités d'exécution ont permis de réaliser des toitures légères ayant un petit rapport poids propre sur grandeur des surcharges climatiques, ce qui augmente l'importance relative des surcharges climatiques.

Quand on néglige la possibilité des accumulations maximales de neige [9], [10], de son épaisseur maximale probable [13], [14], ou de la vitesse maximale du vent [12], [15], on risque l'écroulement partiel ou total ou la perte de la stabilité lors de l'exploitation.

Dans notre pays, le Prof.Dr.Ing. Cristea Mateescu, membre correspondant de l'Académie, a entrepris des recherches très intéressantes au sujet des accumulations de neige sur les toitures par l'action du vent, en effectuant des essais sur modèles dans un tunnel aérodynamique avec des matériaux pulvérulents et en établissant les critères des similitudes mécaniques [9].

En même temps, le Dr.Ing. Horia Sandi a effectué des études concernant la sécurité des constructions soumises aux surcharges temporaires à l'aide du calcul des probabilités, spécialement pour les toitures légères avec différentes durées d'exploitation [19].

Dans le calcul des constructions à l'état limite, on tient compte de la variabilité dans la qualité des matériaux, les charges et les surcharges, et dans les conditions de travail, en séparant le coefficient unique de sécurité en trois coefficients divers, à savoir les coefficients de surcharge  $n$ , d'homogénéité  $k$ , et des conditions de travail  $m$  (à l'aide duquel on réduit soit la résistance des matériaux  $m_m$ , soit la capacité portante de l'élément  $m_e$ ).

La liaison entre la valeur du coefficient unique ( $c$ ) et les coefficients diversifiés ( $n, k, m_m, m_e$ ) peut être exprimée par la relation [11] :

$$c = \frac{n}{m_m m_e k} \quad (1)$$

ou quand  $m_m = 1$

$$c = \frac{n}{m_e k} \quad (2)$$

Pour vérifier un élément constitué d'un matériau homogène, on peut employer la relation:

$$\sum n S^n \leq m_e m_m k R_F^n \quad (3)$$

dans laquelle  $F$  est la caractéristique géométrique de la section de l'élément considéré. Dans le cas de l'élément exécuté à l'aide de plusieurs matériaux, cette relation devient:

$$\sum n S^n \leq m_e m_m k_i R_i^n F_i \quad (4)$$

Les résistances de calcul sont données dans les prescriptions techniques et quand on tient compte des conditions de travail des matériaux, cette relation a la forme:

$$\sum n S^n \leq m_e \sum R_i F_i \quad (5)$$

qui, dans le cas plus général où  $m = 1$ , devient

$$\sum n S^n \leq \sum R_i F_i \quad (6)$$

La somme des sollicitations normées multipliées par le coefficient de surcharge, nommée sollicitation de calcul  $\sum n S^n = S$ , représente la sollicitation maximale possible.

Les normes françaises "élaborées par la Commission des spécialistes français sous la présidence de Monsieur N. Esquillan" considèrent comme surcharge normale la surcharge qui peut être atteinte une ou plusieurs fois dans l'année, et comme surcharge extrême la surcharge atteinte une seule fois pendant la durée de service de la construction.

On a fait des études statistiques pour les surcharges climatiques (vent et neige) [12], [13], [14], [15], en utilisant les observations météorologiques enregistrées [23]. Les fonctions de distribution théoriques qui correspondent le mieux à la distribution empirique des surcharges climatiques sont celles de Fisher-Tippot, de Gumbel, la fonction logistique, exponentielle, la loi de Poisson et de Pearson. On a obtenu des résultats convenables avec la méthode de Gumbel, appliquée aux surcharges climatiques en utilisant les études théoriques de Monsieur G. Demarre [3].

La vitesse maximale du vent à Bucarest-Filaret a été étudiée en utilisant une série d'observations de longue durée (65 années); la valeur maximale enregistrée est de 41,9 m/s et la moyenne de

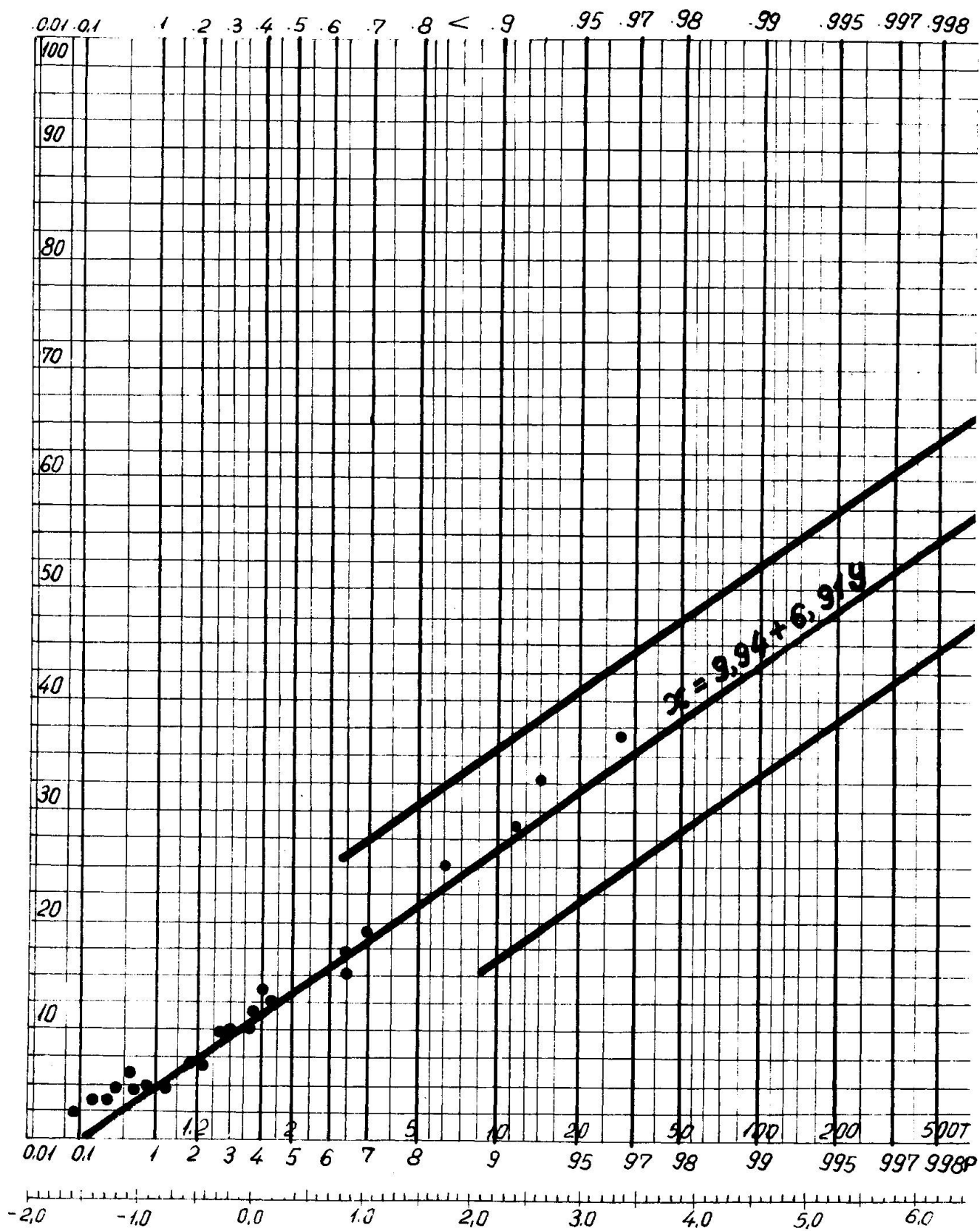


Fig. 1 La droite de Gumbel pour la vitesse du vent à Bucarest - Filaret

13,74 m/s (fig. 1).

A l'aide de la droite de Gumbel, nous avons fait des extrapolations pour différentes périodes de temps, en calculant à l'aide de la formule de Newton les valeurs correspondentes de la pression dynamique de base.

Dans la littérature spécialisée, on prend en général la valeur 1,2 pour le coefficient de surcharge  $n$  des bâtiments habituel [16]. Pendant les 65 ans d'observation, cette valeur a été dépassée en atteignant un coefficient de surcharges 1,57. On mentionne que les normes françaises indiquent un rapport entre la pression dynamique normale et extrême de 1,75 [18].

Pour la surcharge de neige, on a analysé deux aspects: l'épaisseur maximale probable de la couche de neige et la densité volumétrique de la couche de neige.

On a évalué l'épaisseur maximale probable de la couche de neige à Bucarest-Filaret en utilisant les observations effectuées pendant 48 ans. La valeur maximale enregistrée sur le terrain a été de 150 cm et la moyenne seulement de 47,5 cm. On prend pour la surcharge due à la neige un coefficient de surcharge  $n = 1,4$ . Les normes françaises [18] prennent 1,65 pour le rapport entre la surcharge extrême et la surcharge normale. Le poids de la neige pour 1 m<sup>2</sup> de surface horizontale dépend naturellement de la densité volumétrique de la neige et de son épaisseur.

L'Institut Météorologique Central de Bucarest a effectué ces dernières années des enregistrements systématiques de la densité volumétrique de la neige. La moyenne de la densité maximale mensuelle était 0,255 t/m<sup>3</sup>, le maximum maximum de 0,74 t/m<sup>3</sup> étant enregistré exceptionnellement au mois de février 1968. Au mois de janvier 1969, la densité volumétrique de la neige sur les toitures à Bucarest a atteint 0,30 - 0,55 t/m<sup>3</sup>. On peut accepter pour la neige tombée successivement pendant l'hiver une densité volumétrique de 0,22 t/m<sup>3</sup> [11], en apportant des corrections à l'aide du coefficient de surcharge  $n$  et en fonction de la durée de service de la construction et du mode de l'action (direct, indirect etc.) de la neige sur l'élément de construction.

Les dernières recherches astronomiques font une liaison entre les éléments climatiques de la terre et le soleil; les éruptions du soleil, etc. On remarque dans les phénomènes solaires l'existence de cycles ayant des durées différentes: le plus connu est de 11 ans, et son existence a été marquée dans notre pays par des hivers exceptionnels (fig. 2), 80 et 400 ans sont aussi des cycles dont l'existence a récemment été étudiée. Nos observations météorologiques ne contiennent pas encore un si grand intervalle de temps et on cherche maintenant à utiliser les méthodes statistiques mentionnées plus haut pour l'étude de la variation de longue durée de phénomènes solaires.

On emploie en général dans divers pays la classification des bâtiments d'après leur durabilité; on distingue trois catégories de

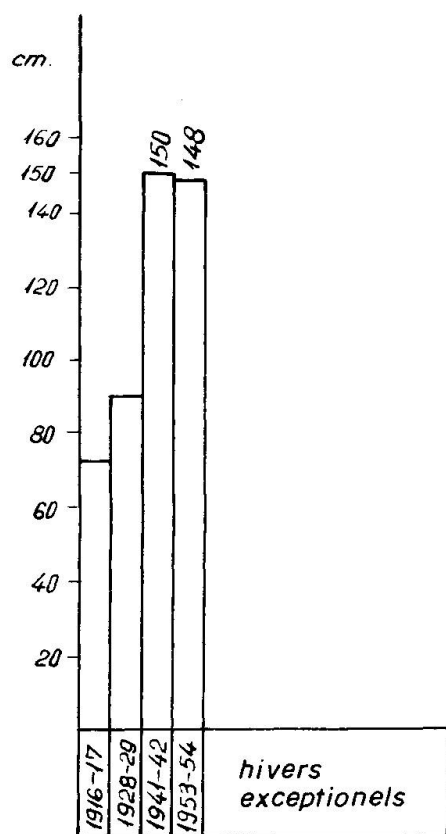


Fig. 2 L'épaisseur maximale de la neige lors des hivers exceptionnels à Bucarest - Filarest

de bâtiments caractérisées par des durées de 20, 50 et 100 ans. On peut calculer différentes valeurs du coefficient de surcharge  $n$ , selon la durée d'exploitation de 20, 50 ou 100 ans, en fonction des enregistrements statistiques.

Il en résulte, par exemple, comme surcharge maximale extrême dans la zone de la ville de Bucarest, un coefficient de surcharge de 1,50 pour une durée de 80 ans, alors que la valeur utilisée dans quelques prescriptions est de 1,2. Les mêmes considérations peuvent être valables pour la surcharge extrême due à la neige. Il en résulte ainsi la nécessité d'établir les maxima maximorum (extrêmes) des surcharges climatiques pour un bâtiment, en fonction de 20, 50, 100 ans ou plus, et de la valeur des surcharges.

Evidemment il faut tenir compte du fait que les valeurs établies à l'aide des données statistiques doivent être appliquées différemment selon l'importance de l'élément de construction (principal ou secondaire), c'est-à-dire en fonction du mode d'application ou de transmission de la surcharge climatique, directement ou par l'intermédiaire d'autres éléments. Dans le cas des éléments de construction indirectement sollicités, il faut faire des réductions correspondantes, différentes dans le cas de la neige ou du vent.

Pour le calcul des constructions soumises à l'action des surcharges climatiques, le rapport entre la charge accidentelle (surcharge) et le poids propre de l'élément (poids mort) présente aussi une certaine importance. Les études effectuées par le Dr. Ing. Horia Sandi [19] ont abouti à différentes valeurs du coefficient de sécurité des toitures (fig. 3), en fonction de la durée d'exploitation de la construction  $T$ , et du rapport  $Z^n/P^n$  (poids de la neige/poids propre de la toiture).

Dans le cas de la surcharge due à la neige, il faut aussi tenir compte de la possibilité de l'accumulation de la neige qui dépend de la forme de la toiture, de l'action du vent et des autres facteurs. On notera que la forme architectonique des toitures légères, qui peut très largement varier, est importante pour la surcharge due à la neige; l'étude sur modèle établie par le Dr. Ing.

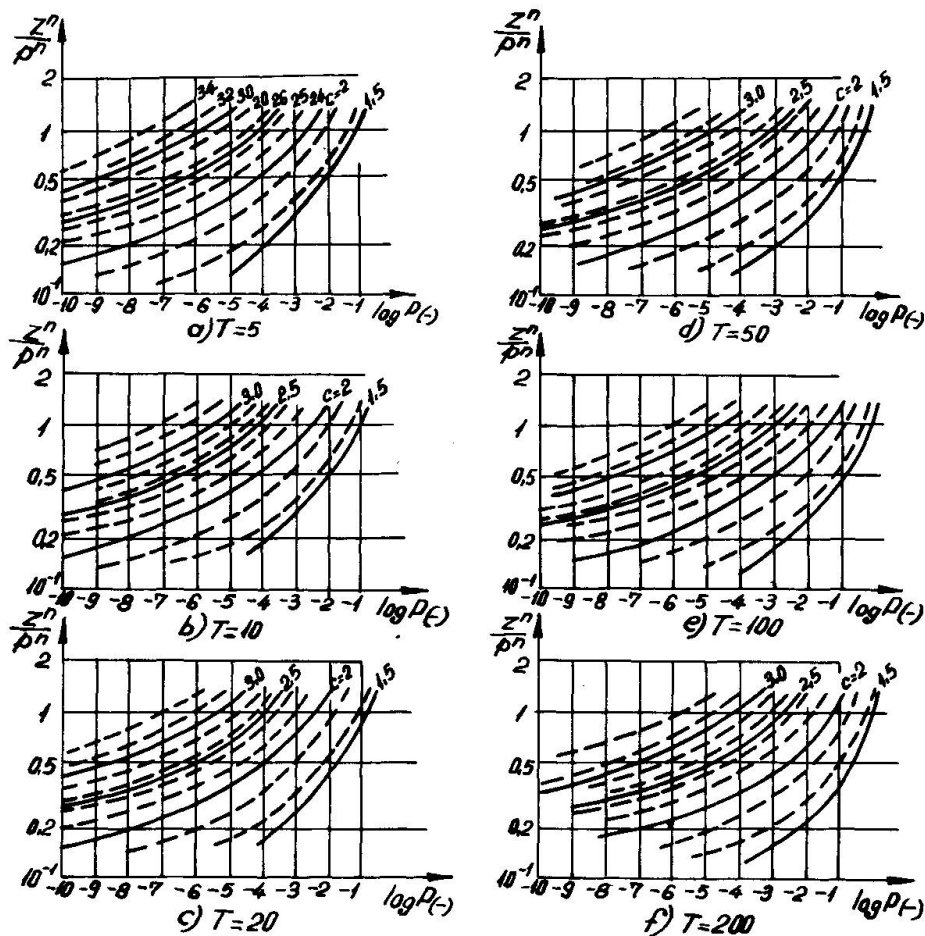


Fig. 3 Courbes des coefficients de sécurités

Cristea Mateescu peut être appliquée avec une certaine exactitude dans ce cas [9] .

L'étude statistique des phénomènes météorologiques permet de déterminer les surcharges climatiques maximales probables pour la durée convenable de 20, 50, 100 ans ou plus. On peut ainsi utiliser pour les bâtiments des coefficients de surcharges différents pour 20, 50, 100 ans ou plus.

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## Statistical Distribution of Axle-Loads and Stresses in Steel Railway Bridges

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The paper presented by the author at the 8th Congress of IABSE in New York dealt with the non-stationary random vibration of a beam loaded by a moving random load (1). This theoretical paper has found applications in connection with bridge problems and it has, therefore, been supplemented by some experimental research work carried out on three steel railway bridges.

Here, I would like to explain some of the fundamental results and to discuss some of the experimental observations.

First of all, the instantaneous, i.e., the static and dynamic, axle loads were measured and the results were evaluated by means of histograms and distribution functions. It was determined that the mean value of the instantaneous axle load is 13,1 Mp. This is much lower than the Czechoslovak Building Code value, which is 24 Mp. The root-mean-square deviation, 5,4 Mp, is, on the contrary, very high. The speeds of the trains and the number of axles were also evaluated from the statistical point of view.

The stresses in the main structural members of the bridges, i. e., in the bridge girders and in the cross- and longitudinal beams, were also measured under the usual service load. The stresses were classified with respect to the transient time and to the crossings of 50 kp/cm<sup>2</sup> thresholds. The number of cycles of stresses was also evaluated by means of histograms and distribution functions. We determined that the mean stresses in all the main bridge members under traffic load were approximately 200 to 300 kp/cm<sup>2</sup>. These average values are much lower than the standard code values, which are about 1000 kp/cm<sup>2</sup> in this particular case. However, the root-mean-square deviations of the stresses reach very high values, up to 200 kp/cm<sup>2</sup>, and they are caused more by the statistical deviations of the static axle loads than by their dynamic effects. Moreover, we attempted to compare the series of local maximum stresses with the corresponding series of axle loads, but the results so far are not satisfactory.

We also attempted to measure and to evaluate the higher statistical and probabilistic functions necessary for the random vibration concept, i.e., the correlation functions and the spectral densities of variation of the stresses in some bridge members. The

results, however, appeared to be extremely heterogeneous, so that hitherto no conclusions could be drawn from them and further research work seems to be necessary.

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### III

#### Effets du vent sur les constructions

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Le remarquable rapport de C.W. Newberry, traitant d'un sujet des plus importants et actuels pour la construction aurait pu faire l'objet d'amples et intéressantes discussions. Il est donc regrettable qu'il n'ait donné lieu à aucune intervention, préparée ou non. Une raison de cette carence réside peut-être dans le fait que cet exposé technique n'aborde pas l'aspect probabiliste de la question, aspect essentiellement lié au thème général du Symposium.

Il me paraît indispensable de souligner l'importance de cet aspect, puisque, tant que les sollicitations extérieures - et notamment celles dues à des phénomènes aussi aléatoires que le vent - n'auront pas été définies dans le sens probabiliste, l'évaluation du degré de sécurité des ouvrages reste illusoire. En effet, la plupart des règlements actuels imposent, pour les vents dits "normaux" ou exceptionnels", des valeurs plus ou moins arbitraires, parfois modifiées au hasard des conclusions tirées d'un événement spectaculaire local. Or, il faudrait que ces valeurs puissent être assorties d'indications sur leur probabilité - ce qui implique la nécessité de disposer de données statistiques suffisantes - et que cette probabilité soit normalisée sur le plan international, afin de rendre comparables les règles pour le calcul des constructions dans les divers pays.

Le rapport présenté constitue une excellente synthèse des connaissances actuelles dans le cadre traité. Je suis donc d'accord sur son contenu, mais je ne le suis pas pour autant sur son titre.

En effet, le problème de la sécurité des constructeurs vis-à-vis du

vent présente deux parties distinctes : l'action du vent et les effets de cette action, c'est-à-dire le comportement de la structure. Or, le rapport traite de la première partie et non de la seconde. J'estime donc qu'il devrait être intitulé en conséquence.

Il ne s'agit pas là d'une simple querelle de mots, car à travers une telle imprécision de terminologie on risque de faire passer au second plan, voire de faire oublier, le deuxième volet du problème, qui est celui qui importe en fin de compte, en tant qu'élément essentiel de la sécurité de l'ouvrage, les sollicitations extérieures n'étant que les données de base pour l'étude du problème.

On peut d'ailleurs remarquer que les moyens d'investigation mentionnés pour l'action du vent ne sont pas tous applicables pour déterminer la réponse de la structure. Ainsi, les études en soufflerie ne sont d'aucun secours dans ce domaine, car on ne peut pas réaliser, à l'échelle d'un modèle réduit, la réplique fidèle du comportement très complexe d'un bâtiment complet. Les études sur bâtiments réels - coûteuses et difficiles à interpréter, mais qui finalement devraient être plus révélatrices - n'en sont qu'à leur début. Actuellement, on est donc limité aux études théoriques sur modèle mathématique, d'une valeur scientifique certaine, mais fondées sur des hypothèses simplifiées et plus ou moins arbitraires. On n'a donc pas la garantie de serrer la réalité d'assez près.

On est ainsi obligé de reconnaître l'insuffisance de nos connaissances, notamment sur les effets du vent dans la structure des bâtiments à étages, ce qui conduit à prendre des marges de sécurité probablement excessives dans les calculs. L'équilibre que l'on doit rechercher, entre les impératifs de la sécurité et de l'économie, s'en trouve compromis.

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A propos de l'aperçu historique donné dans le rapport, je voudrais remarquer incidemment que, à côté de Sir B. Baker, Irminger et Stanton, il convient de citer Gustave Eiffel, pionnier des études aérodynamiques et analyste clairvoyant du comportement des structures, dont les publications revêtent aujourd'hui encore un caractère d'actualité.

**La protection antisismique des structures****PANAIT MAZILU**

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Le problème essentiel dans ce genre de sollicitation réside dans la possibilité d'une structure d'absorber par sa déformation l'énergie cinétique imprimée par le séisme. La réserve d'énergie de déformation dont peut disposer une construction par sa déformation au delà de la limite élastique, dans le domaine plastique, ne peut et ne doit pas, en principe, être négligée. Il y a, d'ailleurs, une littérature technique sur ce sujet.

Evidemment, il y a encore beaucoup de difficultés pour élaborer une théorie complète énergétique, à cause, entre autres, de la différence qui existe, d'un côté, entre le procédé global d'évaluer l'énergie de déformation d'une structure et, d'autre côté, le caractère local de la rupture qui peut entraîner la ruine totale de la construction.

Mais il ne faut pas ignorer l'existence de cette énergie de déformation plastique, dans les conditions d'une construction rationnellement conçue.

La manière de traiter l'action d'un tremblement de terre à l'aide des forces sismiques est certainement conventionnelle. On ne doit pas oublier qu'en réalité ces forces n'existent pas comme des forces extérieures; il s'agit en réalité d'une énergie cinétique qui peut être absorbée par l'énergie de déformation plastique, du moins dans un cas extrême d'une sollicitation sismique, toujours possible, supérieure à celle prévue par les normes officielles et les données statistiques de la probabilité des sollicitations défavorables.

C'est pour cela que le problème de la sécurité des structures dans les régions sismiques doit être traité d'une manière un peu différente.

Il s'agit non seulement des considérations théoriques, mais aussi des conséquences pratiques pour l'élaboration des projets.

A titre d'exemple, à Bucarest, pour la construction d'un hôtel en béton armé de 80 m de hauteur, ayant un plan compliqué, on a prévu des ouvertures étroites de grande hauteur, sans béton, pour réduire la rigidité excessive des diaphragmes verticaux, mais avec une armature d'acier doux, capable de supporter des déformations plastiques et, par conséquent, d'accumuler une importante énergie de déformation.

Some remarks concerning the introduction of Mr. G.R. Mitchell  
on "loading on buildings"

F.K. LIGTENBERG  
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- 1) Exceptional loads occur in many cases only during a very short time (a few hours). Examples are moving of furniture, people gathering for a reception, fire, repair activities. It seems very improbable that these loads will be found in an inspection in a certain number of office blocks or something like that. This means, that the imagination of the man who does the research (or of the designer) must be directed towards visualising exceptional circumstances. Much can be learned from case studies where something has gone wrong.
- 2) It seems a good idea to control circumstances in some way. This would mean however, that we as structural engineers have the task to give good information (understandable for people without technical knowledge) to the users of a building. For a washing machine this is quite conventional, for a building not!
- 3) Partitions form a considerable part of dead weight loads. We ought not to introduce these as "uniform" loads without taking account of their structural action.
- 4) Point loads are very important for the right dimensioning of details of a structure (e.g. holes in floors). An "equivalent" uniform load on a small floor area is a point load.

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I should like to make several random comments on loadings based on recent research and experience.

1. Load Studies; Load studies are expensive and no little care should be taken to avoid collecting more information than is needed. It is important to remember that the interest is not in the data for its own sake, but for eventual use in guiding structural design. This simple observation has led to several data collection implications. For example,

a. If one is satisfied with estimating the member forces in supporting beams and columns it appears to be satisfactory to obtain rather gross information about the spatial disposition of the loads. Analysis suggests that the U.S. National Bureau of Standards scheme of recording the load location as simply being in one of nine sections within a room introduces negligible uncertainty in the member force prediction.

b. The load data uses seem to dictate a need for either extreme load data or simple means and variances, but not for complete descriptions.

i) For design of slabs and members sensitive to "local" loads, data from the upper tail of the load probability distribution is needed. This can probably be obtained most cheaply by training crews to sample "conditionally," e.g., with orders to measure only rooms which they estimate by quick visual check to have loads in excess of  $x$  pounds per square foot.

ii) For design of members with respect to non-failure limit states (e.g., deformation, cracking, etc.) and for members, such as major columns and footings, which support the sum of many room or bay loads, it appears to be satisfactory to estimate only the mean and variance. Sufficiently accurate estimates can be obtained with only ten to twenty rooms per building (or perhaps per firm.) Obtaining estimates of the building-to-building variation is very important, if, as some suspect, this variation is significant compared to within-building variation. The reason will be demonstrated below.

c. The degree of spatial correlation among loads in a building is important in major members, such as columns, which support many individual loads. If a column supports two floor loads (with common variance  $\sigma^2$ ) the variance of the column load is  $2\sigma^2(1+p)$ , in which  $p$  is the correlation coefficient between the loads. Since  $p$  is probably positive in this case, the estimate of the column variance can be underestimated by a factor as large as 2 if the common simplification of independence ( $p = 0$ ) is adopted.

d. A primary source of this spatial correlation can be among or building-to-building variation. A discussion by R.B. Corotis and me (in the July 1969 Journal of the Structural Division of ASCE) shows that, even if there is no within-building spatial correlation, the correlation coefficient between the two floor loads is

$$\rho = \sigma_A^2 / (\sigma_A^2 + \sigma_w^2) = (\text{among}) / (\text{among} + \text{within}).$$
 Clearly this number will be significantly larger if among-building variation is large compared to within-building variation. This conclusion supports the need for adequate sampling of many different buildings, not simply careful sampling within buildings.

e. As others have mentioned these loads, being measured as they are, at effectively random points in time, do not represent observations of the maximum peak loads during a building's lifetime. Mr. Mitchell's suggestion of treating occupancy changes as being randomly selected from the (spatially measured) population seems quite reasonable. For smaller members, at least, some consideration must be given also to loads due to concentrations of people. The N.B.S. is recording open, unloaded area as a simple measure of the potential for loads due to people. Rooms heavily loaded with static loads can be expected to have less potential for loads due to people, i.e., a negative correlation can be expected between static load and unloaded area (or "people-load potential").

2. Load Combinations; The problem of properly combining loads in probabilistically based codes has been referred to here several times. It is important to keep in mind in this regard that loads (or load effects) are in fact random functions of time. A variety of random variables associated with such random functions are important. When designing for peak gravity loads, the designer should be interested in the mean, variance, characteristic value, etc., of the random variable: peak live load during the structural lifetime. Design for wind combined with live load is another problem, however. As many have observed, it is unlikely that the peak lifetime wind load will occur simultaneously with the peak live load. Comparing the rapid versus slow fluctuation of the two random time functions and assuming that they are effectively uncorrelated functions, it would appear to be reasonable and practical to treat this combined loading by adding to the peak wind load random variable, the instantaneous (i.e., arbitrary point in time) live load random variable. This is, of course, precisely the variable which is being observed in the present load surveys.

3. Earthquake Risks; To support a previous discussion that illustrated that probabilistic methods are to be used in design, I can cite recent experience in using probabilistic methods (Cornell, B. Seis. Soc. Amer., V. 54, No. 5) to estimate the probability of exceeding design earthquake intensity values for nuclear power plants. Interestingly enough, when several different sites were analyzed in this manner and the probabilities calculated for

each of the two rather arbitrarily defined design levels ("maximum probable" and "maximum credible"), both of which had been previously, independently selected by rather arbitrary means, a surprising degree of consistency was found.<sup>3</sup> The former level usually corresponded to a return period of about 10<sup>3</sup> years and the second to 10<sup>5</sup> or 10<sup>6</sup> years.

### III

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I would like to add the following comments to Mr. Newberry's fine paper:

1. I wholeheartedly agree with Mr. Newberry's plea that wind-load requirements should not be lowered until more research in this area has been completed. In the past, buildings have been far stronger in resisting wind pressures than those for which they were designed; primarily, on account of the existence of non-load bearing partitions. However, the tendency today, at least in the United States, is for office buildings to be built with moveable partitions. Many partitions that are not moveable do not extend all the way from floor to ceiling. Therefore, we no longer are guaranteed the built-in added safety factor so frequently present in the past.

2. In ACI Committee 348 (Structural Safety), we consider serviceability to be one aspect of structural safety. Therefore, it is not enough to design a building to withstand wind pressure so that the building will not collapse. The building must also be comfortable for those inside it. This gets to be important as more of our tall buildings are apartment buildings, not only office buildings as in the past. Wind deflections which might be acceptable to workers in an office building, may be totally unacceptable to tenants living in an apartment building.

Concerning Mr. Mitchell's paper, I would like to add the following comments:

1. There is usually very little control of construction loads by the designing engineer and sometimes not even by the contractor. This is a problem which engineers should consider during their design and contractors in planning their construction sequence. Many more buildings collapse during construction than after they are completed. This is especially true of concrete buildings where frequently construction loads far in excess of the design live load are imposed on parts of the structure which have not yet attained their design strength and are not intended to for twenty-eight days.

2. For snow loads, the duration of the load must be considered together with the intensity of the load.

3. In addition to those mentioned there are two other load surveys being conducted in the United States; one by the Post Office Department of its facilities and the other by the National Bureau of Standards, the latter being confined to office buildings.