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Autor: Anderson, David / Bijlaard, Frans / Nethercot, David A.
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Analysis and Design of Steel Frames with Semi-Rigid Connections

Analyse et conception des charpentes d'acier à assemblages flexibles

Berechnung und Entwurf von Stahlrahmen mit flexiblen Verbindungen

David ANDERSON
Senior Lecturer
University of Warwick
Coventry, UK

David Anderson, born 1944, received his degrees from the University of Manchester. His teaching and research interests at the University of Warwick are concerned with the analysis and design of steel structures. He is also engaged in the assessment and drafting of codes for steel and composite structures.

Frans BIJLAARD
Civil Engineer
IBBC-TNO
Delft, The Netherlands

Frans Bijlaard, born 1947, graduated as Civil Engineer at the Technical University of Delft. For the last 10 years he has been concerned with research on stability and connections in steel structures. At present, he is Head of the Department of Steel Structures of the TNO Institute for Building Materials and Structures.

David A. NETHERCOT
Reader
University of Sheffield
Sheffield, UK

David Nethercot, born 1946, obtained his degrees at University College Cardiff where he was also an ICI Fellow. He has been active in research and code drafting in steel and aluminium structures at Sheffield since 1971.

Riccardo ZANDONINI
Associate Professor
University of Trento
Trento, Italy

Riccardo Zandonini, born 1948, received his engineering degree from the Politecnico di Milano. After fourteen years as research associate, assistant and associate professor at the Politecnico di Milano, he was appointed at the University of Trento. His research work has been related to the strength and stability behaviour of steel and composite structures.

SUMMARY

A survey is given of methods of analysis for steel frames with semi-rigid connections. Various models are employed to represent the behaviour of the joint, the most common being relationships between moment and rotation based on experimental behaviour. The results of such analyses may be used as a basis for design. This approach has been found to be well-defined for braced frames using plastic theory, but for other cases reliance is placed at present on empirical methods. Research needed to provide more rational methods is identified and current knowledge is summarised.

RÉSUMÉ

Les méthodes d'analyse des charpentes en acier à assemblages flexibles sont présentées. Pour traduire le comportement des assemblages, divers modèles peuvent être utilisés. Le plus courant de ceux-ci repose sur des relations moment-rotation relevées lors d'essais. Les résultats de telles analyses peuvent servir de base de dimensionnement. Cette approche se révèle adéquate pour les structures contreventées calculées selon la théorie plastique; par contre, pour les autres cas, on continue à se fonder sur des méthodes empiriques. On met enfin en évidence les recherches encore nécessaires pour pouvoir dégager des approches plus rationnelles et on résume l'état des connaissances actuelles.

ZUSAMMENFASSUNG

Dieser Bericht soll eine Übersicht über die Methoden zur Berechnung von Stahlrahmen mit flexiblen Verbindungen geben. Verschiedene Modelle zur Beschreibung des Anschlussverhaltens werden vorgestellt, wobei das Anschlussverhalten am häufigsten durch die Momenten-Rotations-Beziehung charakterisiert wird. Die Ergebnisse solcher Rahmenberechnungen können für die Bemessung benutzt werden. Dies wurde für unverschiebliche Rahmen schon ausreichend nachgewiesen, während für andere Fälle die Methoden teilweise noch auf Erfahrung beruhen. Es wird eine Übersicht über den Stand der Technik gegeben, und es werden die Bereiche identifiziert, auf denen weitere Forschung nötig ist.

1. INTRODUCTION

Traditional approaches to the design of steel frames idealise the behaviour of the connections as either rigid or pinned. In the last decade though, interest in semi-rigid construction has increased. In comparison with pinned connections, significant savings can be made in the cost of material, without the expense of fully-rigid connections. Despite this, most design codes still ignore semi-rigid construction. Furthermore, no document is so far available giving a comprehensive account of present knowledge together with criteria and methods for design.

Aware of this problem, Technical Working Group 8.2 (Stability : Systems) of the European Convention for Constructional Steelwork decided in 1984 to establish a Task Group with the aim of preparing a reference document for designers. As a preliminary step the Task Group prepared a state-of-the-art report which forms the basis of this Survey.

The aim herein is to offer a critical assessment of current knowledge, concerning particularly:
- models developed to incorporate joint stiffness in frame analysis, and
- criteria and methods for semi-rigid design of frames under static loading.

Possible criteria for the further development of design methods are suggested and areas for further research are identified.

An understanding of the behaviour of connections and the ability to predict their load-deformation characteristics is essential to the semi-rigid approach. A forthcoming paper [1] reviews available knowledge on these topics, but a brief introduction is included herein for the benefit of readers.

2. JOINT BEHAVIOUR AND ITS MODELLING

An appreciation of the behaviour of joints requires an understanding of the relationship between the loads applied to the connection (moments in both planes, shears, axial force, torque and bimoment) and the corresponding deformations. However, since virtually no data are available for the full 3-dimensional case, this effectively reduces to a knowledge of, and in the case of analytical studies the ability to represent mathematically, the connection's moment-rotation or M-φ curve.

Details of connection behaviour may be found elsewhere [1]; in addition three major reviews [2-4] have been published, which cover virtually all the currently available experimental M-φ data. The semi-rigid nature of the connection is due to deformation of its components at the interface between the beam end and the column face. A selection of M-φ curves is presented in Figure 1. It is apparent that for semi-rigid connections, the curves are non-linear and their shape depends on the exact form of connection.

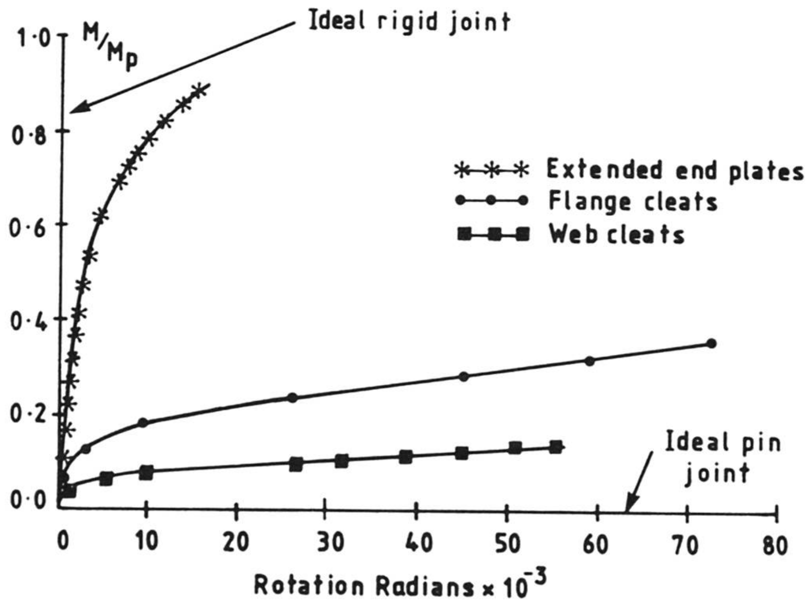


Fig. 1. Examples of moment-rotation curves

Although a considerable body of experimentally based M–φ data is available, that relating to any particular connection type is rather mixed. Specifically it varies in terms of:
- number of tests available;
- number of variables present in the joint;
- method of defining M and φ;
- quality of reporting;
- characteristics other than in-plane rotational response.

3. FRAME ANALYSIS

3.1 General

Interest in methods of analysis suitable for frames with semi-rigid connections was first shown more than 50 years ago [5–8]. This followed early investigations of joint behaviour [8] which had shown that savings could be achieved if designers took account of the stiffness of the connections [8,9]. Existing elastic methods for plane frames were modified to take account of flexural deformation of the connections, assuming linear M–φ characteristics. Other forms of deformation were neglected. More comprehensive and refined methods were only made possible by the development of electronic computers in the early 1960s [10,11]. Since then, progress in structural analysis has led to increasingly sophisticated approaches. These can now include non-linearity resulting from material behaviour and the geometry of the structure. For the design office, it is possible to adapt commonly available computer programs to account for joint flexibility, as illustrated for one such program by Edinger [12].

The connection is usually represented by fictitious structural elements at the ends of members. These elements are assigned pre-determined relationships between forces and displacements, so as to simulate the behaviour of the joint as a whole. The elements generally comprise assemblages of rigid and deformable components connected end-to-end [13], such as shown in Figure 2, although a trussed system was recently presented [14].

Fig. 2. Member with semi-rigid joints

The degree of refinement is related to:
- the sophistication of the assumed model, particularly the number of degrees of freedom considered and the accuracy of the force-displacement relationships;
- the possibility of allowing for interaction between different forms of end force, for example axial force and bending moment;
- the capability of allowing for the finite dimensions of the joint.

Concerning the choice of model, it is important that account is taken of the deformation of the column in the region of the joint, in addition to the flexibility of the connection itself. This is particularly true of the column web panel in welded beam-to-beam connections [15]. It has also been shown [7,8] that the finite dimensions of the joints have significant influence on the distribution of moments in the elastic range.

A review of existing methods of analysis is now given, concentrating on those aspects which are specifically related to the treatment of joint flexibility.

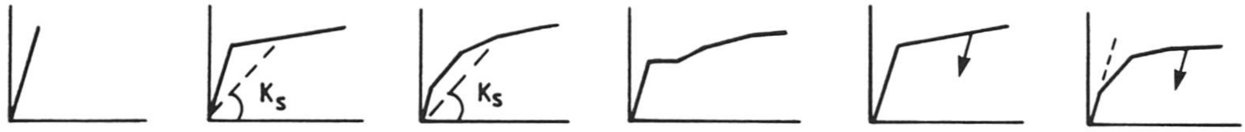
3.2 Elastic analysis

3.2.1 Linear analysis

The connections and the members are assumed to have linear force-displacement relationships (Figure 3a) and the effect of deformations on the equilibrium of the frame is disregarded. In its usual form, the analysis required the solution of a set of linear equations:

$$|F| = |KE|.|D| \quad (1)$$

in which the elastic stiffness matrix $|KE|$ takes into account the rigidity of the connections and possibly the size of the joints. The advantage of this approach is that the overall procedure is the same as that commonly adopted for rigid–jointed frames. Existing methods, including computer programs can therefore be easily modified to allow for joint flexibility.



(a) (b) (c) (d) (e) (f)

Fig. 3. Force-displacement relationships

Baker and others [6,7] presented long–hand methods, based on slope deflection and moment distribution, which allowed also for the finite dimensions of the joint. Monforton and Wu [10] were the first to incorporate the flexure of joints in a matrix displacement procedure. This requires an appropriate correction matrix to be applied to the member stiffness matrix and the fixed–end force vector. More recently, Lightfoot and Le Messurier [11] considered the general case of members restrained elastically against all displacement components. However, neither Monforton and Wu, nor the later authors, made allowance for the size of joints.

The assumption of linear behaviour is very approximate for most types of connection. It is only acceptable at very low values of displacement, or if the value of joint stiffness is chosen to reflect average behaviour over the full range of expected displacements.

3.2.2 Incremental analysis

The behaviour of the connection is modelled in a more refined way by a non–linear relationship. Assuming this is the only source of non–linearity, the analysis may be carried out in an incremental manner, as a series of linear analyses. At each step the stiffness matrix is corrected to allow for the changes in joint rigidity. This operation requires an iterative procedure if mathematical functions [5] are used to represent the behaviour of the joints.

Incremental analysis has been described by Romstad and Subramaniam [16] for the case of a bi–linear moment–rotation relationship (Figure 3b); the procedure can be easily extended to more complex piece–wise linear curves (Figure 3c), including allowance for slip (Figure 3d).

3.2.3 Iterative linear analysis

As an alternative to incremental methods, the secant stiffness (denoted K_S in Figures 3b and 3c) can be used to allow for non–linear joint behaviour. This approach requires iteration based on linear elastic analysis.

Different procedures have been proposed by Goverdhan [3] and by Cosenza, De Luca and Faella [13] for in–plane analysis, and by Ang and Morris [17] and by Lopetegui [18] for three–dimensional structures.

3.2.4 Second–order elastic analysis

The methods already presented are easily extended to allow for the influence of deformation on the equilibrium of the frame, by using techniques well–established in structural analysis. For example, a number of procedures for iterative analysis are discussed in references [13, 17].

3.3 Inelastic analysis

If account is to be taken of reversal of loading on the connections, the joint representation should include separate branches for loading and unloading (Figures 3e, 3f). Such reversal may influence the behaviour of a frame under non-proportional loading. Methods which include separate branches whilst disregarding the yielding of members are defined as inelastic. Incremental procedures are then applied to reduce the analysis to a series of linear steps [19].

3.4 Elastic-plastic analysis

The influence of material non-linearity can be accounted for approximately by incorporating the effect of yielding in the vicinity of the joints into the representation of the joint itself. Some of the methods described above can then continue to be used.

However, recent methods [14, 19-23] enable yielding in the members to be considered as a separate effect, in addition to non-linearity due to joint behaviour and geometric effects. These methods deal with the behaviour of the members and the joints at various levels of refinement. They originate mainly from the analysis of rigid-jointed frames, and their sophistication often reflects that of the original approach.

Several of the methods include noteworthy refinements in the representration of the joints. That due to Tautschnig [1, 20, 21] allows shear deformation of the panel zone to be included in the moment-rotation relationship for the joint. Pilvin [23] models the joint as an elastic-plastic structural element (Figure 4a) which allows for interaction between different forms of end force. Stutzki [14] simulates joint behaviour by means of a trussed assemblage which enables both flexural and shear stiffness to be allowed for (Figure 4b). This has been included in a general second-order method for the analysis of space frames which also takes account of the spreading of plastic zones [24, 25].

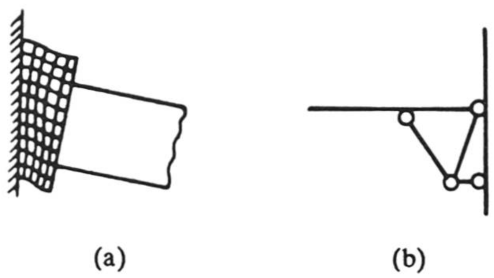


Fig. 4. Models for joint

3.5 Conclusion

The review given above enables two general conclusions to be made:
- Sophisticated methods of analysis are already available. These enable all significant influences on frame behaviour to be taken into account, including joint flexibility. Moreover, it is to be expected that further highly refined approaches will be developed. Such methods of analysis make possible a thorough investigation of the behaviour of semi-rigid construction.
- Joint flexibility may in principle be incorporated into several common computer programs for frame analysis. This would permit joint flexibility to be dealt with in practice, at whatever level of accuracy is required by the designer, provided that obstacles such as the prediction of connection behaviour (see Chapter 5) can be overcome.

However, there are still a number of topics related to analysis which require further investigation.

3.6 Need for further studies

These principally concern the following areas:
- Verification of analytical procedures against tests on full scale semi-rigid frames. Some test results have recently been published [26] and other tests are in progress.
- Comparison of the various methods of analysis, to define their range of application and suitability for the different limit states.
- Definition of the range of joint rigidity, within which the behaviour of the connections has to be considered in the analysis. Such information is necessary to avoid undue complication in design.

- Definition of those parameters which govern frame behaviour. In particular, the influence of joint dimensions, structural imperfections, loading path and accuracy in representing the joint's moment-rotation characteristics should all be studied.

Research presently underway should considerably deepen knowledge of these topics in the near future.

4. COLUMN STABILITY

4.1 General

In recent years extensive studies of the behaviour of axially loaded, pin-ended steel columns have led to a clearer understanding of the roles played by the various factors, such as residual stresses and lack of straightness, which influence column strength. However, the most important single factor in the case of real columns - effective slenderness - still has to be determined in a far from scientific manner. For "rigid" framing, methods based on elastic critical load theory [27, 28] permit the effects of end restraint to be assessed on a consistent basis, although most design methods still make the tacit assumption that the effect of such restraint on column ultimate strength will be equivalent to its effect on the same column's elastic critical load. Even this degree of rationality is absent from the design of columns in "simple construction", for which effective length factors based on a designer's interpretation of terms such as "partially restrained" are used. In order that column design be placed on a more consistent basis the emphasis should shift from considerations of pin-ended columns to the study of columns as part of a structure. In the case of "simple" framing this means recognising the end restraint supplied by the combined action of the semi-rigid connections and the surrounding beams, taking into account the actual M-φ behaviour of practical forms of connection.

4.2 Existing studies

Early attempts [29, 30] to study the effect of semi-rigid connections on column strength were based on elastic stability theory (bifurcation approach) and as such utilised only the initial rotational stiffness of the connections. Because connection stiffness tends to reduce with increasing deformation it is questionable whether such an approach closely represents the real situation.

Authors	Ref	Method used for column analysis	M-φ representation
Stutzki	14	Incremental with quasi-Newton iteration with load-deflection states	Piecewise linear
Jones, Kirby and Nethercot	31	Incremental finite element with Newton-Raphson iteration	B-spline
Chapuis and Galambos	32	Numerical integration of differential equation using moment-thrust-curvature relationships	Linear
Sugimoto and Chen	33	Tangent stiffness solution of governing differential equations assuming deflected forms	Bilinear
Shen and Lu	34	Iterative numerical solution of governing equations	Linear
Vinnakota	35	Finite difference solution of governing differential equations	Piecewise linear
Opperman Matthey	36 37	Incremental finite element	Bilinear

Table 1 Contributions to the end-restrained column problem

More recently several authors [14, 31–37] have used a variety of numerical approaches to study isolated columns, the semi-rigid connections being modelled as non-linear springs. Table 1 lists these contributions, noting the type of M–φ representation used. In the past, little opportunity existed to verify these solutions against experimental data since only one test on a column with semi-rigid end restraint had been published [38]. However, recent tests [39] on column sub-assemblages of the type illustrated in Figure 5 will assist in rectifying this situation.

4.3 Results

Whilst some of the studies listed in Table 1 have produced results which serve only to provide a better general understanding of the subject, others have attempted to use the results of systematically organised parametric studies as a basis for suggesting tentative design proposals. An extended summary of the position in late 1983 is available [40]. Some of the more general findings of this are as follows:

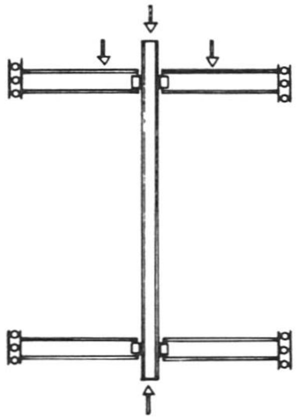


Fig. 5. Column sub-assemblage

- Some of the studies of column ultimate strength conducted so far, which are based on attainment of the peak load, suggest that because this occurs at comparatively low end rotations, great accuracy in modelling the connection M–φ curve is unwarranted.
- Some evidence exists to suggest that the presence of end restraint reduces the spread in column strengths caused by variations in residual stress, initial lack of straightness, etc.
- The relationship between the strength of an end-restrained column and a similar pin-ended column is as shown in Figure 6 with the upper curve plotting progressively higher for stiffer forms of end restraint.
- Design studies [41] for a series of typical bay spacings, storey heights and column loadings suggest weight savings for columns of up to 11 per cent if restraint sufficient to justify effective length factors of 0.9 (rather than 1.0) is present. An alternative study [42] using a more advanced column design method which is relatively insensitive to the level of moment in the member suggested savings in columns and beams together of around 15 per cent – almost as much as was possible using rigid joints.

The results of the more extensive parametric studies have formed the basis for a redefinition of the effective length concept along the lines shown in Figure 6. When considering the maximum strength of an end-restrained column this becomes "That length which when used in conjunction with the column curve for pinned ends gives the same strength as the failure load for the end-restrained columns" [31, 40]. Based on a study of 83 combinations of column type and modest degrees of end restraint, as provided by single and double web angles, header plates and top and seat angle connections, Sugimoto and Chen [33] have suggested that the effective length factor k be determined from a knowledge of the initial slope α of the connection M–φ curve (expressed in terms of M/M_p) using

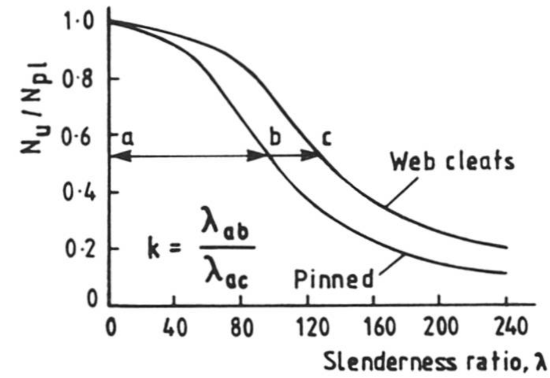


Fig. 6. Definition of effective length

$$k = 1.0 - 0.017\alpha \nless 0.60 \qquad (2)$$

This assumes that k does not depend on λ, a hypothesis that is also supported by the studies of Jones et al [31]. It also excludes any consideration of beam flexibility; whilst this is probably insignificant in cases of stiff beams and very flexible connections, it is likely to become increasingly unconservative as connection stiffness increases. Although this problem has not been studied at the fundamental level, an approximate allowance may be made in the manner first suggested by Galambos and developed in [40].

Alternatively, recourse may be made to a more substantial analysis [43] using the subassemblage representation of Figure 5. Although such studies are not yet complete, it would appear that even comparatively flexible connections are capable of transmitting a sufficient proportion of the beam restraint for quite low column effective lengths to be justified. They also permit the variation in column end moment with increasing applied column load to be monitored; although the exact variation will, of course, depend upon the load path, it is clear from both tests [39] and analysis [43] that columns provided with semi-rigid end connections exhibit the same type of moment shedding previously observed in rigidly jointed frames [44].

4.4 Need for further studies

Considerable efforts have been made within a comparatively short time to develop an appreciation of the influence of semi-rigid joint action on column behaviour. However, further work remains to be done:

- Results are still needed for columns provided with realistic representations of connections which frame into the column web; this first requires the availability of suitable joint data.
- Further experimental data on the behaviour of columns provided with semi-rigid connections are needed, against which analysis procedures can be verified. This should include columns in unbraced frames.
- An analysis should be developed which considers the full range of column behaviour, including the region beyond the attainment of maximum load. In addition to studying unloading characteristics and deformation capacity, this should be used to check carefully the amounts of connection rotation required at various stages and hence the exact needs for experimental $M-\varphi$ data and its mathematical representation.
- The analytical approaches that have now been extended to the consideration of subassemblages, thereby permitting the full interaction of beams, connections and the column, need to be utilised in parametric studies. Two items in particular require attention:
  - (i) The interaction of beam flexibility and joint flexibility in providing end restraint to the column.
  - (ii) The transfer of beam loading through the connection into the column as the relative stiffness of each part of the assembly changes during the loading history.
- The sensitivity of column strength to the variations in stiffness of nominally identical connections should be assessed by a carefully planned programme of study.
- Studies are needed on column stability utilising recent investigations into base effects [1].
- The three-dimensional behaviour of end-restrained columns should be investigated.

It is reasonable to expect that significant progress will be made with each of these items (except the last) within the near future, since each is known to be currently under investigation. Studies on three-dimensional behaviour may be expected to be carried out in the longer term. Clearly it is difficult to speculate on the outcome of such research but the objective should be to produce rationally based guidance that will enable designers to exploit the benefits of the semi-rigid nature of steelwork connections. For the immediate future, problems related to use of the proposals already developed are discussed below.

5. DESIGN OF BRACED FRAMES

5.1 General

This section examines how far existing methods can be applied to the design of braced frames with semi-rigid connections. Both elastic and plastic approaches are reviewed, stating the design philosophy on which the methods are based. For elastic design, areas where knowledge is lacking are identified and suggestions are made for the development of guidelines in the immediate future. Plastic design methods are already well-established.

5.2 Elastic design

In general, there are two alternative approaches to semi-rigid elastic design. The first is applicable to simple beam and column structures, in which an approximate allowance may be made for the stiffness of the connections by a limited redistribution of moment. In the second, the stiffness is represented by M–φ relationships based on experimental evidence. These relationships are taken into account during analysis and the frame's components are sized on the basis of the resulting moments and forces. Each approach is considered below.

5.2.1 Approximate design by limited redistribution of moment

Allowance is made for the stiffness of nominally pinned connections by assuming an end restraint moment calculated as a percentage of the free moment applied to the beam. It follows that when designing the columns account must be taken of any out-of-balance resulting from the end moments in the beams. This approach is described in a recent set of national regulations [45], with the end restraint moment restricted to 10% of the free moment. The beam-to-column connections must be specifically designed to transmit the assumed restraining moments. In addition, welds and fasteners are designed for the actual moment capacity resulting from the other components of the connection, not the assumed moment.

The advantages of this method are as follows:
- The structure is rendered statically determinate and the internal actions are therefore easily calculated.
- A smaller beam section will be required, compared with "simple" design. This could lead to a lower overall height to the structure, with consequent savings.
- Following the usual practice in "simple" design, the designer will not consider the effects of pattern loading on column design. Thus with equal beam spans and the same intensity of loading, the net moment to be resisted by an internal column will be zero. It follows that the same section will be required as for "simple" design.

There are some disadvantages though, in comparison with "simple" design:
- The requirements that connections be designed to transmit the assumed restraining moment may lead to increased sizes within the connection.
- The requirement that welds and fasteners be designed for the actual moment capacity of the connection results from a desire to avoid brittle forms of failure. This is likely to lead to larger welds and fasteners.
- Additional calculations are required, particularly for connection design.

It would appear that this approach is rarely used when first sizing the members. This is because a reduction of only 10% in the bending moments within the beams is usually regarded as insufficient justification for the method to be adopted, particularly when the possible disadvantages in connection design are considered. This method is at present of most use when increase of loading has to be accommodated late in the design process. In view of the simplicity of the method though, it is appropriate to consider what improvements could be made:
- As the various types of connection possess different degrees of stiffness, analyses should be made to determine the maximum degree of redistribution that could be relied upon for each type of connection. The connections examined should be substantially the same as those used in "simple" design.
- The requirement that welds and fasteners be designed for the actual moment capacity resulting from other components in the connection (rather than the assumed moment) could be unduly onerous. Premature brittle failure of the connection is to be avoided, but could such failure be prevented in a less conservative manner?

5.2.2 Design based on moment-rotation characteristics

Whilst redistribution has the advantage of simplicity in analysis, it will inevitably underestimate the restraint supplied by many connections. Fuller account can be taken of this by using M–φ characteristics appropriate to the particular connections being proposed. Such relationships are based on experimental evidence, and are included in the analysis of the structure as advocated in the recent AISC LRFD Specification [46].

In comparison with 5.2.1, this approach has the following advantages:
- A closer representation can be made of the real behaviour.
- Greater economy can usually be achieved in beam design because the restraining moments need not be lower bound values.
- The designer can retain the same joints as those required by "simple" design.
- Alternatively, the connections can be arranged to achieve such end restraint that pre-arranged beam sections can be used.
- As the column moments are known with greater certainty, the designer is encouraged to use advanced methods of column design [42] which lead to greater economy.

Although design based on $M-\varphi$ characteristics was suggested many years ago [8], little use seems to have been made of such proposals. The disadvantages are as follows:
- Designers lack ready access to reliable information concerning the moment-rotation characteristics of connections.
- The structure is not statically determinate and the determination of the action effects may necessitate specialized analysis procedures. Patterned loading may need to be considered to determine the critical loading condition for column design.
- Information is lacking concerning the behaviour of semi-rigid connections made into the webs of H-section columns, which prevents an accurate evaluation of effective length for minor axis buckling.

These difficulties are now examined in more detail.

5.2.3 Moment-rotation characteristics for major-axis beam-column connections

Current knowledge available to researchers has been summarized elsewhere [1-4]. This information needs to be presented in a form which can be used by designers. This should be in the form of agreed $M-\varphi$ curves for various standard forms of connection, published by an authoritative international body. The curves should be accompanied by algebraic expressions. Guidance should be given on fabrication tolerances and erection procedures needed to ensure that the behaviour assumed in design is achieved in practice.

5.2.4 Structural analysis

Once $M-\varphi$ curves have been agreed, researchers will no doubt include these in existing analysis programs. The existence of such programs should be publicised to designers.
The development of simplified methods should also be considered. The use of "beam lines" in semi-rigid design has been demonstrated recently by Nethercot [47]. In Figure 7 PQ and QR correspond to the attainment of yielding at the supports and at mid-span respectively, for a given beam section under uniformly distributed loading. When the $M-\varphi$ curve for the proposed connection is superimposed on these lines, the intersection B defines the maximum value of end moment that can be developed. Once this is know the load capacity can be calculated.

It may also be necessary to consider the rotation of the column, for example when the load capacity of the beam is governed by the attainment of yield in the mid-span region. If the rotation occurs in the same direction as that of the connection a loss of stiffness results and the analysis of the beam should be based on the point B'.

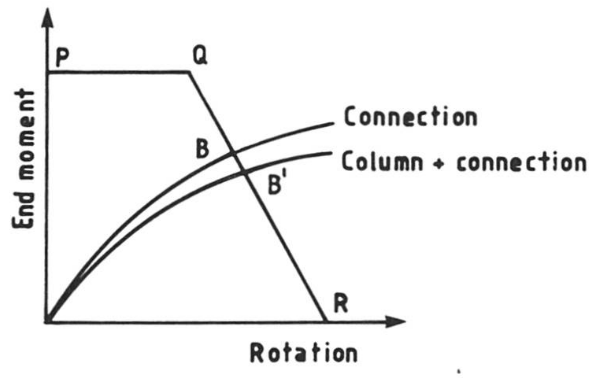


Fig. 7. Beam and connection lines

In the past, design charts such as Figure 8 have been presented [48]. These can be developed from the beam-line approach, and their ease of use has been demonstrated by Roberts [42]. No account need be taken of column flexibility provided that the beam stiffness does not exceed the total stiffness of the upper and lower column lengths by more than 50%. The effect of this simplification should be re-assessed using the agreed M-φ characteristics for present-day connections. If acceptable, its use will greatly ease the analysis.

5.2.5 Effective length of columns

The studies and design proposals described in Chapter 4 make it possible to calculate to various degrees of accuracy the effective length about the major axis. However, lack of information concerning the behaviour of connections which frame into the column web prevents these proposals from being applied to the minor axis. As this case usually controls the design of columns in multi-storey structures, full semi-rigid design about both axes is not possible at present. However, if rigid joints are employed for the minor axis connections then advanced methods of restrained column design can be employed, and sufficient economies can still be achieved over "simple" design [42]. In view of the complex nature of the column problem, a problem which also has implications for beam and connection design [49], it may be desirable to give interim design recommendations, applicable when semi-rigid design is confined to framing about the major axis of the columns.

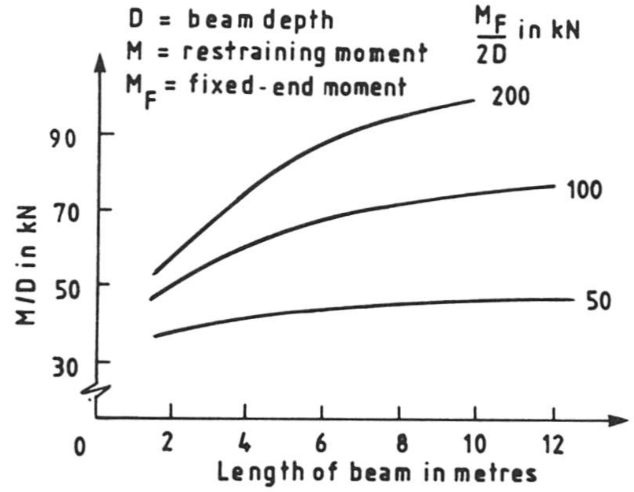


Fig. 8. Design using Class C connections [48]

5.3 Plastic Design

5.3.1 Introduction

Plastic design of braced frames is effectively the plastic design of the beams, taking account of the moment capacity of the connections. The cost of steel framing is strongly influenced by the beam-to-column connections. Designing these to have the full moment capacity of the beams will in most cases lead to fully stiffened and therefore expensive connections. Economy can be achieved by using unstiffened semi-rigid connections which cannot transmit the full moment capacity of the beams. When a beam fails, a mechanism is formed with plastic hinges in the middle section of the beam and at the supports. If the moment capacity of the connection, M_V, is smaller than that of the beam, M_p, the plastic hinge at the support will form in the connection itself. Otherwise, the plastic hinge will form in the beam, just aside the connection.

At knee-connections, the plastic hinge may form in the beam, the connection or in the column, depending on the relative moment capacity [50]. However, if the column is continuous, it is assumed that the combined moment capacity of the upper and lower column lengths is such that plastic hinges will not develop in these members.

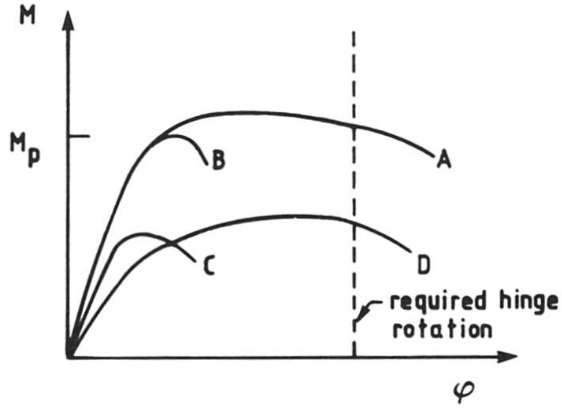


Fig. 9. M-φ curves for various connections

If a beam-type plastic hinge mechanism is to form, then redistribution of moments must take place. Such redistribution can only occur if the components which yield first have sufficient rotation capacity. This requirement imposes restraints on the choice of connections. Consider, for example, the types of behaviour shown schematically in Figure 9. Connection A meets the requirements of plastic design because it possesses sufficient rotation capacity. However, as its moment capacity exceeds that of the connected beam, it behaves as a rigid connection and sufficient rotation capacity is therefore required of the beam. Connection D also has sufficient rotation capacity, but as it has a smaller moment capacity and stiffness than the beam it behaves as a semi-rigid partial-strength connection. The joints corresponding to the characteristics B and C have to be rejected when redistribution of moments is required because of lack of rotation capacity.

5.3.2 Design criteria for semi-rigid connections

The need for rotation capacity at the connections depends on whether they yield first. It follows that the design criteria for the joints depend on their moment capacity and stiffness relative to those of the beam.

With relatively stiff connections, the first plastic hinges usually form in the connections followed, after some rotation, by a hinge also in the midspan section. Therefore rotation is required at the connection. On the other hand, for connections with relatively low stiffness, a plastic hinge first forms in the midspan section of the beam and no rotation capacity is needed at the connection. However, in order to meet the limits for deflection of the beam, a minimum stiffness has to be provided.

If plastic hinges form first at the supports, then the minimum rotation capacity φ required at the connection can be shown [51] to be:

$$\varphi \geqslant \frac{L}{6EI}\,(2M_p + 2M_v - 3M) \tag{3}$$

where L and EI are the span and flexural rigidity of the beam, respectively, and M is the actual moment in the connection.

If a plastic hinge forms first at mid-span, a requirement for minimum stiffness at failure may be derived:

$$\varphi \leqslant \frac{L}{8EI}\,(M_p - M) + \frac{2}{\delta_{\text{limit (failure)}}} \tag{4}$$

where $\delta_{\text{limit (failure)}}$ is the deflection of the beam (expressed as a fraction of its span) at which this member is considered to be in the ultimate limit state, even when a mechanism has not yet formed.

The limiting value of φ given by (3) is plotted in Figure 10, along with representative characteristics of unstiffened beam-to-column connections. Connection A is relatively stiff so that the first plastic hinges will develop in the connections. In this case Figure 10 shows that the connection has adequate rotational capacity. Connection B is relatively flexible, so that the first plastic hinge will form at the midspan section of the beam. In this case no rotational capacity is needed. If the first plastic hinges develop in the connections then C is unsatisfactory because of lack of rotational capacity.

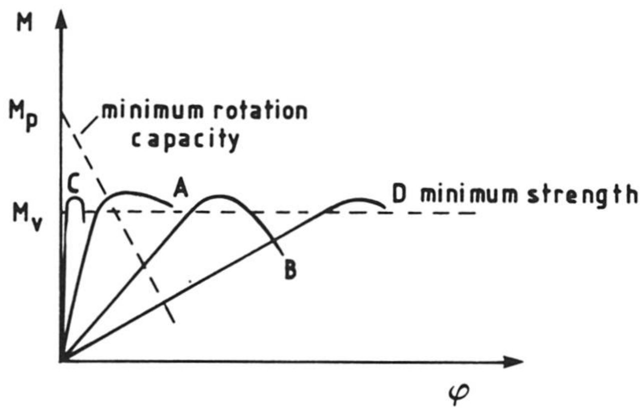


Fig. 10. M–φ curves for unstiffened connections

Based on the foregoing considerations, it is important to have design rules at one's disposal to calculate the moment capacity and the stiffness of a semi-rigid beam-to-column connection [52, 53]. Tests [54] have shown that whilst bolted connections fulfil the requirement for rotation capacity, this has to be checked for welded connections. Information is given in reference [1] on how to determine the behaviour of semi-rigid connections.

5.3.3 Design of the columns

When a column is deprived of rotational restraint from the beams by the development of plasticity then the effective length factor should be taken as unity. However, when not all the beams form mechanisms, the effective length can be calculated taking account of the remaining elastic parts of the structure. In this case though, account should be taken of the changes in the bending moments in the elastic beams which occur when they are called on to provide elastic rotational restraint to the columns [49, 55].

In checking the column stability, various load patterns need to be considered. The columns are subjected to normal forces and moments transferred from the beams to the columns. When a beam forms a plastic hinge mechanism, this moment is the moment capacity of the beam-to-column connection. For external columns full loading on the frame is the determining load case, whilst for internal columns checkerboard loading is the most severe condition [56].

6. DESIGN OF UNBRACED FRAMES

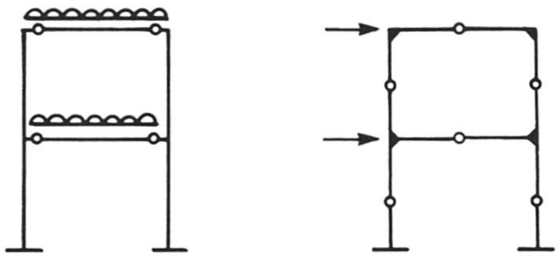
6.1 General

Modern buildings usually include stiff cores and these often brace the steel structure against sway. However, situations arise in which the layout restricts the effectiveness of the stiff components such that the steel frame must be treated as unbraced, at least about one axis of bending. In these circumstances an established technique in Britain and North America is to rely on the stiffness of connections to provide resistance to wind, even though such restraint is ignored under the action of gravity loads. The effectiveness of this approach has been the subject of studies which are outlined below. The chapter concludes with a discussion concerning the basis for rational design of unbraced frames for the ultimate limit state.

6.2 Wind connection method

This method has been described in a number of design guides [57–59] and has the virtue of simplicity. It is currently used in conjunction with allowable stress design philosophy.

Under vertical loading the connections are assumed to act as pins, and members are proportioned accordingly, to satisfy limits on working stress and deflection (Figure 11a).



(a) (b)

Fig. 11. Wind connection method

The effects of wind loading are examined separately, the connections now being assumed to act rigidly. The resulting moments and forces are usually determined by assuming points of contraflexure which renders the structure statically determinate (Figure 11b). These action effects are superimposed on those calculated under vertical loading. The design of the members is then completed by amending the proposed sections as necessary to withstand the combined effects. Sway deflections due to wind are also calculated assuming the connections are rigid.

It is usual to increase allowable stresses by 25%–33% under combined loading [57, 59]. This accounts for the reduced probability of full wind loading arising at the same time as full superimposed load. It has also been argued that the increase is justified in view of the intermittent nature of wind loading.

Recent discussion of the method [60] prompted Nethercot to summarize previous research into its adequacy [47]. An extensive study has been made by Ackroyd and Gerstle [61], using frames with realistic proportions designed using the AISC version of the method [57, 58] referred to as "Type 2 Construction". For comparison the frames were also designed to the AISC Specification [57] assuming "Type 3 Construction". This calls for elastic frame analysis with realistic account taken of the behaviour of semi-rigid connections under both gravity and wind loading. As these "exact" analyses were carried out at working load level, no attempt was made at first to predict the ultimate strength of frames designed by the wind connection method. Subsequently Ackroyd and Gerstle have developed an elasto-plastic analysis procedure [62] and this has been used to determine the behaviour of small subassemblages up to collapse.

The following general conclusions have been drawn from these investigations:
- The analysis procedure employed in the wind connection method consistently overestimates the critical values of moment in the beams, whilst underestimating the column moments. It follows that a frame designed by the wind connection method will typically have beam sections which are too large and columns that are too small, when checked by "exact" analysis.
- The axial forces in the columns are predicted closely by the wind connection method. It follows that the degree to which the columns are underdesigned may not be so great as the overdesign of the beams.
- In certain cases, an under-estimate of connection stiffness in design can cause a reduction in the ultimate load capacity of the frame. Reductions of up to 8% have been computed on long-span structures, only a few storeys tall, in which lateral load effects are small relative to the gravity loads on the beams. In such frames column sizes are usually governed by gravity loads and so small column sections result from the wind-connection method. However, as the reduction in capacity is only small, and may be reduced further if account is taken of patterned loading, this effect can probably be neglected in practice.
- As the method assumes that connections act rigidly under wind loading, the calculated sway deflections are likely to be much less than those given by "exact" analysis.

6.3 Further studies related to the wind connection method

In view of its simplicity, further studies should be undertaken to provide a firm justification for the use of the method. As a result of its empirical nature, it cannot be assumed that the encouraging conclusions reached by Ackroyd and Gerstle will necessarily be repeated if design is based on a limit state code [45, 52].

The strength of frames designed using the wind connection method in conjunction with such codes should therefore studied. The analysis should use agreed $M-\varphi$ characteristics for common types of joint. The earlier study of ultimate strength [62] was concerned with in-plane behaviour; the possibility of out-of-plane member instability should be considered.

Studies should also be carried out at the serviceability limit state, to provide guidance on the extent to which sway deflections calculated assuming rigid connections are likely to underestimate those given by "exact" analysis accounting for the flexibilities expected from practical connections.

To undertake such studies it is necessary to use efficient programs for analysis. That described in [62] is a sophisticated tool, including residual stresses, which causes large demands on storage and time. As a result, the study on ultimate strength was restricted to single storey, centre bay subassemblages extracted from complete multi-storey frames. As it is desirable that studies of strength should be on complete frames, it will be necessary to use a simpler program, for example one based on plastic hinge theory [63].

6.4 Limit state design

Analytical techniques already permit allowable stress design, taking account of joint flexibility under both gravity and wind loading. A rational method for ultimate load design would have to account for overall frame instability as the connections reduce in stiffness [64] and plasticity develops in the members. A necessary step is therefore the development of both efficient programs for analysis of ultimate strength and computer-orientated design procedures of the type proposed by Sedlacek [65].

It may in fact be best to confine attention to elastic design of unbraced frames, for the reasons given by Wood [66]:
- It should then be possible to produce tables of effective length factors for limited substitute frames free to sway, including allowances for the characteristics of common types of joint.
- It is possible that partial plastic design may be inappropriate because the serviceability limit state may control design.

It should also be investigated whether simplified second-order analysis and criteria for the use of first-order analysis, proposed for elastic rigid-jointed frames [67], are applicable to semi-rigid structures.

7. CONCLUSION

Analytical studies on the behaviour of flexibly-connected steel frames depend on knowledge of the moment-rotation characteristics of the joints. Provided these are known, a wide range of methods is available. These can be verified against tests on full-scale frames and are being used in thorough investigations of semi-rigid construction.

For braced frames, semi-rigid joint action reduces sagging bending moments within the beams. Advantage can be taken of this, employing either elastic or plastic approaches to design. Account must be taken of the beam end moments when designing the columns, but investigations show that comparatively flexible connections justify favourable values of effective length. An assessment of the value can be made from empirical equations or, more accurately, from analysis of column-beam subassemblages. However, lack of information concerning minor axis connections currently prevents full semi-rigid design about both axes of the column section.

When a frame is unbraced it has been established practice in some countries to rely on the stiffness of practical connections to provide resistance to wind, even though such restraint is ignored under gravity load. This approach has been found to give generally satisfactory results when used in allowable stress design, although excessive sway is only avoided by stray composite action. The development of rational methods for ultimate load design will result from current analytical investigations into the behaviour of unbraced frames.

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REFERENCES

1. BIJLAARD, F.S.K., NETHERCOT, D.A., STARK, J.W.B., TSCHEMMERNEGG, F., and ZOETEMEIJER, P., Structural Properties of Semi-rigid Joints in Steel Frames (submitted to IABSE).
2. NETHERCOT, D.A., Steel Beam to Column Connections - A Review of Test Data and their Application to the Evaluation of Joint Behaviour on the Performance of Steel Frames, CIRIA Project Record 338, London, 1985.
3. GOVERDHAN, A.V., A Collection of Experimental Moment-Rotation Curves and Evaluation of Predicting Equations for Semi-Rigid Connections, Doctoral Dissertation, Vanderbilt University, Nashville, Tennessee, 1984.
4. KISHI, N. and CHEN, W.F., Data Base on Steel Beam-to-Column Connections, CE-STR-86-26, School of Civil Engineering, Purdue University, 1986.

5. JONES, S.W., KIRBY, P.A., and NETHERCOT, D.A., The Analysis of Frames with Semi-Rigid Connections - A State-of-the-Art Report, J. Construct. Steel Research, Vol.3, No.2, 1983, pp. 2-13.
6. PIPPARD, A.J.S. and BAKER, J.F., The Analysis of Engineering Structures, 3rd ed., Arnold, London, 1936.
7. JOHNSTON, B.G. and MOUNT, E.H., Analysis of Building Frames with Semi-Rigid Connections, Transactions, ASCE, Vol. 107, 1942, pp. 993-1019.
8. BAKER, J.F., The Steel Skeleton, Volume One, Elastic Behaviour and Design, Cambridge University Press, 1960.
9. JOHNSTON, B.G. and HECHTMAN, R.A., Economical Design through Restraint in Beam Connections, Engineering News-Record, Vol. 125, No. 15, 1940, pp. 484-487.
10. MONFORTON, G.R., and WU, T.S., Matrix Analysis of Semi-Rigidly Connected Frames, J. Struct. Div., ASCE, Vol. 89, No.6, 1963, pp. 13-39.
11. LIGHTFOOT, E. and LE MESSURIER, A.P., Elastic Analysis of Frameworks with Elastic Connections, J. Struct. Div., ASCE, Vol. 100, No.6, 1974, pp. 1297-1309.
12. EDINGER, J.A., Nonlinear Frame Analysis Using the ANSYS General Purpose Computer Program, AISC, Research Report No. 83-1, 1983.
13. CONSENZA, E., DE LUCA, A., FAELLA, C., Nonlinear Behaviour of Framed Structures with Semi Rigid Joints, Costruzioni Metalliche, No. 4, 1984.
14. STUTZKI, C., Traglastberechnung raumlicher Stabwerke unter Berucksichtigung verformbarer Anschlusse, Doctoral Dissertation, University of Aachen, West Germany, 1982.
15. FIELDING, D.J., and CHEN, W.F., Steel Frame Analysis and Connection Shear Deformation, J. Struct. Div., ASCE, Vol. 99, No. 1, 1973, pp. 1-18.
16. ROMSTAD, K.M. and SUBRAMANIAM, C.V., Analysis of Frames with Partial Connection Rigidity, J. Struct. Div., ASCE, Vol. 96, No. 11, 1970, pp. 2283-2300.
17. ANG, K.M. and MORRIS, G.A., Analysis of Three-dimensional Frames with Flexible Beam-Column Connections, Canadian J. Civ. Engr., Vol. 11, No.2, 1984, pp. 245-254.
18. LOPETEGUI, J., Verfahren der orthogonalisierten Last-Verformungs-Zustände zur Lösung nichtlinearer Probleme der Stabstatik, Dissertation, University of Aachen, 1983.
19. GERSTLE, K.H., Flexibly-connected Steel Frames, in Steel Framed Structures - Stability and Strength, ed. NARAYANAN, R., Elsevier Applied Science Pub., London, 1985, pp. 205-239.
20. TAUTSCHNIG, A., Entwicklung eines neuen, makromechanischen Knotenmodels und Erstellung eines darauf aufbauenden EDV-programmes zur Berechnung von Stahskelettragwerken unter Berücksichtigung nichtlinearer Nachgiebigkeiten der Verbindungselemente insbesondere bei "steifenloser" Bauweise, Dissertation, University of Innsbruck, 1983.
21. KLEIN, H., Das elastisch-plastische Last-Verformungsverhalten M-φ steifenloser, geschweisster Knoten für die Berechnung von Stahlrahmen mit HEB-Stützen, Dissertation, University of Innsbruck, 1985.
22. POGGI, C. and ZANDONINI, R., Behaviour and Strength of Steel Frames with Semi-rigid Connections, Session on Connection Flexibility and Steel Frames, ASCE Convention, Detroit, 1985.
23. PILVIN, P., Modele de comportement statique de liaison de structures a barres, Construction Metallique, No.2, 1984, pp. 43-50.
24. SALEH, A., Traglastberechnung von räumlichen Stabwerken mit grossen Verformungen und Plastizierung, Dissertation, University of Aachen, 1982.
25. LOPETEGUI, J., SALEH, A., STÜTZKI, C., and SEDLACEK, G., PLAN-T: Ein Stabwerksprogramm für räumliche Stabwerke mit Berücksichtigung nichlinearer Einflüsse, User Manual, Report of the Institute for Steel Structures, University of Aachen, 1987.
26. STELMACK, T.W.., MARLEY, M.J. and GERSTLE, K.H., Analysis and Tests of Flexibly Connected Steel Frames, J. Struct. Div., ASCE, Vol. 112, No.7, 1986, pp. 1573-1588.
27. WOOD, R.H., Effective Lengths of Columns in Multi-Storey Buildings, Structural Engineer, Vol. 52, Nos. 7, 8 and 9, 1974, pp. 235-247, 291-302, 341-346.
28. JOHNSTON, B.G., Guide to Stability Design Criteria for Metal Structures, 3rd edition, John Wiley and Sons, New York, 1976.
29. DE FALCO, F. and MARINO, F.J., Column Stability in Type 2 Construction, Engineering Journal, AISC, Vol. 3, No. 3, 1966, pp. 67-71.
30. DRISCOLL, G.C., Effective Length of Columns with Semi-Rigid Connections, Engineering

Journal, AISC, Vol. 13, No. 4, 1976, pp. 108–115.
31. JONES, S.W., KIRBY, P.A. and NETHERCOT, D.A., Effect of Semi-Rigid Connections on Steel Column Strength, J. Construct. Steel Research, Vol. 1, No. 1, 1981, pp. 38–46.
32. CHAPUIS, J., and GALAMBOS, T.V., Restrained Crooked Aluminium Columns. J. Struct. Div., ASCE, Vol. 108, No. ST3, March 1982, pp. 511–524.
33. SUGIMOTO, H., and CHEN, W.F., Small End Restraint Effects on the Strength of H-Columns. J. Struct. Div., ASCE, Vol. 108, No. 3, 1982, pp. 661–681.
34. SHEN, Z.Y., and LU, L.W., Analysis of Initially Crooked End Restrained Steel Columns, J. Construct. Steel Research, Vol. 3, No. 1, 1983, pp. 10–18.
35. VINNAKOTA, S., Planar Strength of Restrained Beam-Columns. J. Struct. Div. ASCE, Vol. 108, No.11, 1982, pp. 2496–2515.
36. OPPERMAN, G.H.P., Refined Beam-Column Analysis, Third International Colloquium on the Stability of Metal Structures, Paris, Nov. 1983.
37. MATTHEY, P.A., Comportement des Poutres-Colonnes Métalliques en Double Te, Thesis No. 434, ICOM, Ecole Polytechnique Féderale, Lausanne, 1985.
38. BERGQUIST, D.J., Test on Columns, Restrained by Beams with Simple Connections. Dept. of Civil Engineering, University of Texas at Austin, January 1977.
39. DAVISON, J.B., KIRBY, P.A. and NETHERCOT, D.A., Column Behaviour in P.R. Construction – Experimental Behaviour, J. Struct. Eng., ASCE, August 1987.
40. NETHERCOT, D.A., and CHEN, W.F., Effect of Connections on Columns, Steel Beam to Column Connections, Special issue, J. Construct. Steel Research, to be published in 1988.
41. BJORHOVDE, R., Effect of End Restraint on Column Strength, Engineering Journal, AISC, Vol. 21, No. 1, 1984, pp. 1–13.
42. ROBERTS, E.H., Semi-Rigid Design using the Variable Stiffness Methods of Column Design, Joints in Structural Steelwork, ed., HOWLETT, JENKINS and STAINSBY, Pentech Press, Plymouth, 1981, pp. 5.35–5.49.
43. NETHERCOT, D.A., KIRBY, P.A., RIFAI, A.M., Behaviour of Columns in P.R. Construction-Analytical Studies, ASCE Convention, New Orleans, 1986.
44. SMITH, R.F., and ROBERTS, E.H., Test on a Fully Continuous Multi-Storey Frame of High Yield Steel, Structural Engineer, Vol. 49, No.10, 1971, pp. 451–466.
45. BS5950: Structural Use of Steelwork in Buildings : Part 1 : Code of Practice for Design in Simple and Continuous Construction : Hot-Rolled Sections. British Standards Institution, London, 1985.
46. Load and Resistance Factor Design Specification for Structural Steel Buildings, AISC, Chicago, 1986.
47. NETHERCOT, D.A., Joint Action and the Design of Steel Frames, Structural Engineer, Vol. 63A, No. 12, 1985, pp. 371–379.
48. PD3343 Recommendations for design (1959), published as Supplement No. 1 to BS 449: Part 1 : 1970, British Standards Institution, London.
49. SNIJDER, H.H., BIJLAARD, F.S.K. and STARK, J.W.B., Use of the Elastic Effective Length for Stability Checks of Columns and Consequences for Checks on Beams in Braced Frames, published in Instability and Plastic Collapse of Steel Structures, ed. MORRIS, L.J., Granada, London, 1983, pp. 152–159.
50. WITTEVEEN, J., Frame Analysis including Column Hinges, Stability of Steel Structures, Final Report, Second International Colloquium, Liége, April 13–15, 1977, ECCS, pp. 309–310.
51. WITTEVEEN, J. and BAKKER, C. Th. J., Beam-to-Column Connections in Non-sway Frames, Planning and Design of Tall Buildings, ASCE-IABSE Int. Conf., Lehigh University, 1972, Preprints Vol. DS-1972.
Also published in : A. SAWCZUK et al, Optimization in Structural Design, IUTAM Symposium, Warsaw, 1973, Springer Verlag, 1975, pp. 309–312.
52. EUROCODE 3: Common Unified Rules for Steel Structures. Directorate-General for Internal Market and Industrial Affairs, Commission of the European Communities, 1984.
53. ZOETEMEIJER, P., Summary of Research on Bolted Beam-to-Column Connections, Report 6-85-7 (English version), Technical University of Delft, Stevin Laboratory, 1983.
54. ZOETEMEIJER, P., Design Method for the Tension Side of Statically Loaded Beam-to-Column Connections, Heron, Vol. 20, No.1, 1974.
55. SNIJDER, H.H., BIJLAARD, F.S.K. and STARK, J.W.B., Strength Requirement for Beams in Braced Frames in relation to Column Stability, Stability of Metal Structures, Final

Report, Paris, November 16-17, 1983, pp. 247-251.
56. Manual on Stability of Steel Structures, European Convention for Constructional Steelwork, 1976, pp. 238-239.
57. Manual of Steel Construction, American Institute of Steel Construction, Chicago, 1980.
58. Type 2 Construction with Wind Moment Connections, American Institute of Steel Construction, Washington, 1976.
59. Steel Designers' Manual. Crosby Lockwood Staples, London, 4th edition, 1972, pp. 848-853.
60. VERULAM, Design of Multi-Storey Steel Frames, Structural Engineer, Vol. 62A, No. 6, 1984; No.7, 1984; No. 9, 1984, pp. 190-192, 218-219, 285-286.
61. ACKROYD, M.H. and GERSTLE, K.H., Behaviour of Type 2 Steel Frames, J. Struct. Div., ASCE, Vol. 108, No.7, 1982, pp. 1541-1556.
62. ACKROYD, M.H. and GERSTLE, K.H., Strength of Flexibly Connected Steel Frames, Eng. Struct., Vol. 5, 1983, pp. 31-38.
63. BRIDGE, R.Q., SPENCER, J.A. and ANTARAKIS, M.K., Acceptable Moment-Rotation Capacities for Semi-Rigid Connections, Joints in Structural Steelwork, ed. HOWLETT JENKINS and STAINSBY, Pentech Press, Plymouth, 1981, pp. 2.26-2.39.
64. PAU, A.U.Y., Effect of Semi-rigid Beam-to-Column Connections on the Structural Behaviour and Design of Multi-storey Frames, Thesis, Dept. of Civil Engineering, Monash University, 1983.
65. SEDLACEK, G., LOPETEGUI, J., SALEH, A. and STUTZKI, C., Ein computerorientiertes Verfahren zur statischen Berechnung räumlicher Stabwerke unter Berücksichtigung nichtlinearer Effekte, Bauingenieur, Vol. 60, 1985, pp. 297-305.
66. WOOD, R.H., Modern Frame Design and its Requirements for Research into Semi-Rigid Joints, Joints in Structural Steelwork, ed. HOWLETT, JENKINS and STAINSBY, Pentech Press, Plymouth, 1981, pp. 1.11-1.24.
67. U. VOGEL et al, Ultimate Limit State Calculation of Sway Frames with Rigid Joints, European Convention for Constructional Steelwork Publication No. 33, 1984, pp. 9-11.