Behaviour and design of steel plated structures

Behaviour and Design of Steel Plated Structures

Comportement et dimensionnement des structures formées de tôles d'acier

Tragverhalten dünnwandiger Stahlkonstruktionen und deren Bemessung

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SUMMARY

In 1974, the ECCS Technical Committee «Stability» appointed a task working group. Its goal was to gather knowledge about the behaviour up to collapse of thin-walled structures and to develop simple and sufficiently accurate design methods. The group is planning to publish by the end of 1985 a book bearing the same title as the present paper. The present survey is only aimed at giving a preview of the forthcoming book, while trying to maintain a high level engineering analysis, supported by experimental and numerical data and leading to practical design rules.

RÉSUMÉ

En 1974, le Comité Technique «Stabilité» de la CECM a créé un groupe de travail. Il s'agissait de rassembler les connaissances dans le domaine du comportement à la ruine des structures en tôles minces et d'établir un ensemble cohérent de règles de dimensionnement à la fois simples et précises. Le groupe se prépare à publier fin 1985 un ouvrage portant le même titre que le présent article. Ce dernier n'a d'autre but que de donner un aperçu du livre en présentant des résultats de haut niveau scientifique, étayés tant par des essais de laboratoire que par des simulations sur ordinateur et conduisant à des règles pratiques de dimensionnement.

ZUSAMMENFASSUNG

Eine 1974 von der Technischen Kommission «Stabilität» der EKS gegründete Arbeitsgruppe soll die Kenntnisse auf dem Gebiete des Tragverhaltens dünnwandiger Stahlkonstruktionen zusammenfassen und daraus genügend genaue, aber doch einfache Bemessungsregeln ableiten. Als Ergebnis dieser Arbeit soll Ende 1985 ein Buch erscheinen, dessen Titel mit demjenigen des vorliegenden Berichts übereinstimmt. Hier soll das zukünftige Werk vorgestellt werden; es handelt sich um wissenschaftlich begründete Ergebnisse, die durch Laborversuche und Computersimulationen bestätigt wurden und zu praktischen Bemessungsregeln geführt haben.

1. INTRODUCTION

1.1. General

The timeliness of the present survey is basically due to the change which has oc-
curred, at the international level, in the philosophy of structural safety. Al-
most everywhere, the theory of safety embodied by the concept of a global factor
of safety applied to the stresses (allowable stresses) has been abandoned. It is
now replaced by the semi-probabilistic theory of limit states, for which the safe-
ty factor is partly applied to the stresses and partly to the loads. This change
occurred first in Western Europe in the late sixties [1.1.] and afterwards in
North America [1.2.] as well as in East European Countries [1.3.] IABSE participa-
ted actively in this change by organizing a successful Symposium in London [1.4.],
that had a substantial impact on the community of structural engineers.

The reader is supposed to be acquainted with the new concept, according to which
any structure must comply with ultimate limit states, on the one hand, and with
serviceability limit states on the other one. In the field of plate structures,
several research works have been launched since 1970 as a result of several acci-
dents that occurred to several large steel box girder bridges [1.5.] ; their aim
was to improve our knowledge of the behaviour up to collapse of steel plates and
plate structures.

As a consequence of these bridge collapses provisional rules were drafted by ECCS
and in some countries of Western Europe. After a tremendous research effort,
they were followed by design rules belonging to official codes and standards, es-
pecially in the U.K. [1.6.], in West Germany [1.7.], in Switzerland [1.8.] and in
the United States [1.9.]. Several symposia and colloquia have been held all over
the world during the last decade ; proceedings and books have also appeared during
this period [1.10. to 1.13.]. Nevertheless, it was clear that during the trouble-
some period that has extended through the last fifteen years the difficulty of
finding simple but realistic design rules was reflected by the successive changes
that occurred in the main specifications.

The Task Working Group 8/3 of ECCS has spent much time and effort in the drafting
of a book, entitled as the present survey, that is expected to appear at the end
of 1985 [1.14.]. The writers, who all belong to this task group, wish to present
hereafter the leading physical ideas of the aforementioned book, while avoiding,
as much as possible, the mathematical developments. They believe that the forth-
coming book will present suitable solutions to most of the plate problems that
occur in steel buildings, bridges and, to a certain extent, in ship building.
The aforementioned book will contain around 300 réferences. The present survey
will only refer to some of them, according to the main ideas developed.

1.2. Evolution of the design of plate structures

From the viewpoint of structural safety the introduction of the concept of ulti-
mate limit states had tremendous consequences. The small displacement elastic
model, that was the background of the linear plate buckling theory, became unsui-
table and had to be replaced by large displacement elastoplastic models with ac-
count being taken of geometrical and structural imperfections. That means that
these models, to be realistic, had to involve as well the out-of-flatness of the
plates due to the residual stresses that are always present because of the kind
of fabrication and manufacturing process. As a consequence, the magnitudes and
the statistical distributions of these imperfections had to be investigated on
actual structures. An IABSE Survey published in 1980 [1.15.] dealt with the to-
lerances in steel plate structures.

The rigorous analysis of the behaviour of a steel plate structure in the collapse
stage is usually beyond the ability of analytical theories. Therefore, most of the
progress in this field was reached by means of tests on plates, plate girders and

box girders and by numerical simulation using finite difference - and finite ele-
ment programs ; to be useful, these programs have to cope with large displace-
ments, residual stresses, geometrical imperfections and progressive yielding.
The confidence in such programs requires first a good agreement between numeri-
cal results and experimental ones. As the use of the above computer programs is
generally too costly for practical purposes and is therefore restricted to re-
search work, the development of several simplified - but nonlinear - models for
design aims has been and will still be welcome.

1.2.1. From the linear plate buckling theory...

The linear theory of buckling yields the concept of *buckling by bifurcation*.
That means that a perfect bar or plate - i.e. perfectly straight or flat and
without structural imperfections - and perfectly loaded - i.e. subject to loads
acting axially or in the middle plane - will remain in its original configura-
tion till a certain load, called *critical load*, for which it can suddenly de-
flect with an indeterminate amplitude. It is well known since EULER (1744) that
the critical load of a pin-ended column axially compressed (fig. 1.1.a) and
buckling elastically is :

$$N_{cr} = \frac{\pi^2 . EI}{l^2} \tag{1.1.}$$

EI being the flexural rigidity associated to the buckling direction.

(a)   (b)
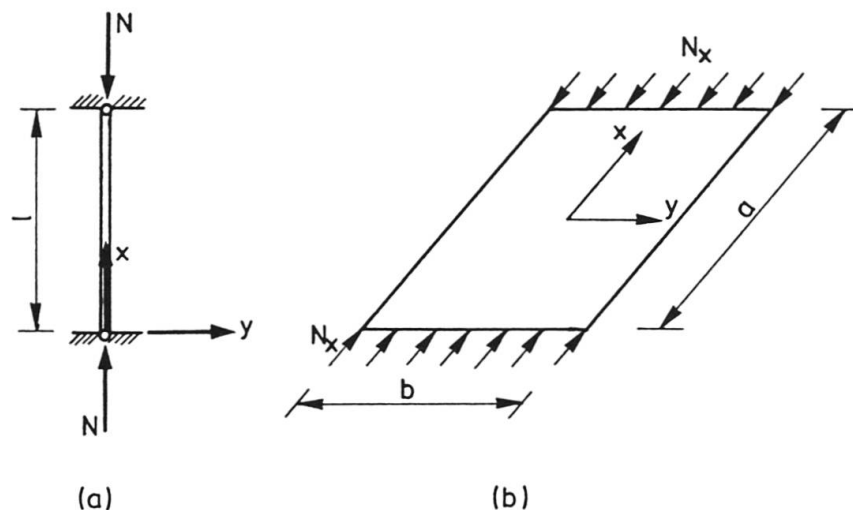
Figure 1.1. - Column and plate buckling.

In a similar way, the critical load of a rectangular simply supported plate of
dimensions a x b and thickness t and uniformly compressed in the direction x
(fig. 1.1.b) is :

$$N_{x,cr} = k \frac{\pi^2 . D}{b^2} \tag{1.2.}$$

where $D = Et^3/12.(1 - \nu^2)$ is the flexural rigidity and the buckling coefficient
k is given by :

$$k = (\frac{a}{b} + \frac{b}{a})^2 \tag{1.3.}$$

Because of unavoidable geometrical imperfections - out-of-straightness of the
bar and out-of-flatness of the plate -, the structural elements do not buckle
by bifurcation, but rather by *divergence*. That means that from the beginning of
the loading the bar deflects transversaly, so that the final stage of the beha-
viour is simply asymptotic to the critical load.   Also, especially

Fig. 1.2. - Equilibrium paths for a bar
and for a plate.

because of residual stresses, the
critical load cannot be reached
(fig. 1.2. - curve OCB).

BRYAN [1.16.], TIMOSHENKO [1.17.]
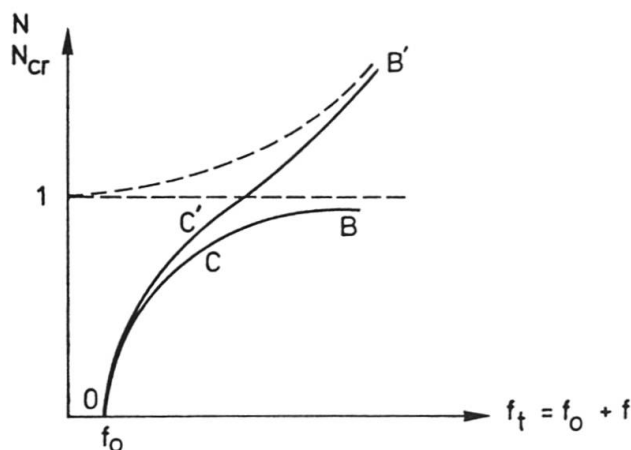and afterwards many distin-
quished scientists developed the
linear plate buckling theory for
the case of elastic unstiffened-
and stiffened isotropic plates,
orthotropic plates,... for seve-
ral boundary conditions and loa-
dings. Different authors develo-
ped charts as design aids for
plate buckling calculations ;
those established by KLÖPPEL et
al [1.18.] are the most complete
and were used to design nearly
all big steel plate bridges till
recently.

1.2.2. ... To the postcritical behaviour.

Actually, there is a fundamental difference between bars and plates regar-
ding instability. Indeed, because of imperfections, a bar cannot reach its
critical load. On the contrary, a plate is able to present an ultimate load
that can be much higher than its critical load and thus shows a considerable
postbuckling strength (fig. 1.2. - curve O C'B'). The latter was exploited as
early as the second half of the 19th century, when several railway bridges
were built in Great Britain and elsewhere according to the rules of the Mecha-
nics of Materials and to the results of tests on large models for investi-
gating the behaviour against buckling. At that time, however, the linear plate
buckling theory was-non existent and so a fortiori there was no nonlinear theory,
which alone is able to take account of the postcritical behaviour.

As a result, one can say that most bridges built during the present century
were, in regard to plate buckling, less advanced than, for instance, the Menaï
straits bridges !

As shown above, the fundamental weakness of the linear plate buckling theory is
that it ignores the postbuckling strength. This shortcoming has been espe-
cially emphasized at the colloquium held in Liège in 1962 [1.19.], stressed
again several times theselast twenty years and especially at the 1968 IABSE Con-
gress in New-York where the senior author concluded that the linear plate buck-
ling theory was not an adequate basis for design and gave an unrealistic pictu-
re of the actual behaviour of steel plated structures.

The correct nonlinear theory of elastic flat plates is due to von KARMAN [1.20.];
it was later extended by MARGUERRE [1.21.] to plates having a slight initial
curvature. Accordingly, the first numerical results were obtained from the 19th
century onwards by hand methods. Presently, numerical procedures - finite diffe-
rences and finite elements - are used to solve the nonlinear equations that go-
vern plate buckling problems.
The accidents already mentioned that occurred during the period 1969 to 1971
demonstrated, on the one hand, the importance of detailing and, on the other one,
the oversimplified character of the linear plate buckling theory. The senior
writer emphasized these facts at the 1976 Tokyo Congress of IABSE [1.22.]; his
paper already contains more than one hundred references dealing with the buckling
design of plate - and box - girders.

1.3. Present knowledge of ultimate limit strength of plate structures

Task Working Group 8/3 of ECCS is concerned with plate structures. The above mentioned forthcoming book [1.14.] prepared by this T.W.G. will attempt at a physical representation of the main results collected over 15 years of research. Its scope is first the *behaviour* of unstiffened and unstiffened plates, in the light of most recent research work, and then the *analysis and design* of two kinds of plate structures, i.e. slender plate girders and large box girders; the interaction between general buckling and local plate buckling is studied too in the restricted field of hollow sections. The following chapters are presented accordingly.

The book will not be concerned at all with nonlinear numerical techniques, the use of which is too costly to be envisaged daily for designing civil engineering structures, that are mostly non repetitive objects.   These techniques are only devoted to research purpose, to the simulation of tests, to the calibration of simplified design methods.   These are derived from ultimate strength models of structural elements that are subjected to quasi-static non-cyclic loadings ; the ultimate limit state is reached when a failure mechanism or when sufficient yielding occurs. Thus, these models take account of the plastic properties of the material, allowing plastic adaptation and stress redistribution. For this purpose, lower bound solutions are preferred because of safety reasons; ideally, they have to comply everywhere with equilibrium conditions and the yield criterion. Such requirements cannot be fulfilled strictly,so that models must be calibrated with regard to test results or/and with numerical simulations. It must also be stressed that the elaboration of existing physical models has been largely due to   experimental observation of the postcritical behaviour and of the failure mode.

One must confess that ultimate strength models have the disadvantage at present of being specific to elementary loading cases and thus covering only a part of all the possible combinations of these loading cases.

1.4. Connection with Eurocode n° 3

At the request of the Commission of the European Communities, several sets of specifications and recommendations, called EUROCODES, were prepared recently. More especially, EUROCODE 3, that is devoted to steel construction [1.23.], was prepared by an international drafting panel chaired by Professor P.J. DOWLING but subject to the guidance of ECCS Technical Committee 8 regarding stability problems.

A first draft is presently being submitted for consideration by the national groups. Because of the work in progress within TWG 8/3, the part devoted to plate buckling has to be improved and/or completed. There is no doubt that the forthcoming book will help the drafting panel to work accordingly in the near future.

2. BEHAVIOUR OF UNSTIFFENED PLATE COMPONENTS

2.1. Compression plates

Let us consider a thin rectangular plate that is simply supported along its four edges and subjected to uniform compression across its width b (fig. 2.1.). In the postcritical range, the distribution of longitudinal stresses is non uniform because of the bowing effect in the central zone of the panel. As a result, only the longitudinal - unloaded - edges will yield. For design purpose, the non uniform stress distribution is replaced by a uniform one over an *effective* width b_e (< b) with an amplitude f_y.
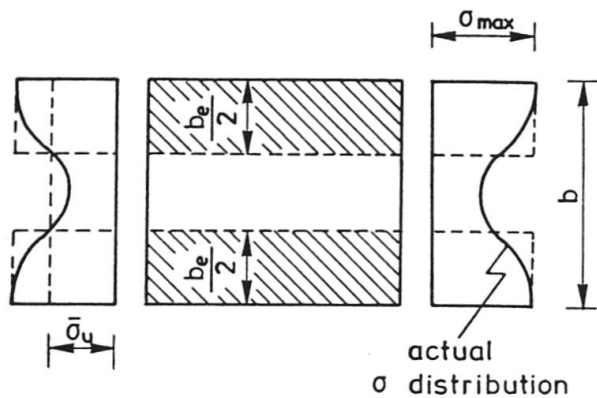
Fig. 2.1. - von KARMAN assumption.

and the average ultimate stress $\bar{\sigma}_u$:

$$\bar{\sigma}_u \equiv \frac{b_e}{b} f_y = \sqrt{\sigma_{cr} \cdot f_y} \tag{2.5.}$$

By introducing the slenderness ratio :

$$\bar{\lambda}_p \equiv \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{b}{t} \frac{1.05}{\sqrt{k}} \sqrt{\frac{f_y}{E}} \tag{2.6.}$$

equation (2.4.) becomes :

$$\frac{b_e}{b} = \frac{1}{\bar{\lambda}_p} \not> 1 \tag{2.7.}$$

The von KARMAN approach[2.1.] equates the critical plate buckling stress $\sigma_{cr,e}$ of the fictitious plate of width b_e to the yield stress :

$$\sigma_{cr,e} = f_y \tag{2.1.}$$

with :

$$\sigma_{cr,e} \equiv k.0,9E \left(\frac{t}{b_e}\right)^2 = k.\sigma_{cr}\left(\frac{b}{b_e}\right)^2 \tag{2.2.}$$

σ_{cr} being the critical stress of the actual plate of width b :

$$\sigma_{cr} = k. 0,9 E \left(\frac{t}{b}\right)^2 \tag{2.3.}$$

This gives :

$$\frac{b_e}{b} = \sqrt{\frac{\sigma_{cr}}{f_y}} \tag{2.4.}$$

Initial out-of-flatness of the plate has a deleterious effect on b_e and thus on $\bar{\sigma}_u$; this effect is especially large in the range $2/3 < \bar{\lambda}_p < 3/2$. Residual stresses have a similar effect. To take account of both kinds of imperfections, several authors suggested modifications to the von KARMAN formula; for example :

WINTER [2.2.] $b_e/b = (1 - \frac{0.22}{\bar{\lambda}_p})/\bar{\lambda}_p \not> 1$ \hfill (2.8.)

FAULKNER [2.3.] $b_e/b = 1.05 (1 - \frac{0.26}{\bar{\lambda}_p})/\bar{\lambda}_p \not> 1$ \hfill (2.9.)

For combined compression and bending, the stresses vary linearly across the width b. The above formulae can be extended to this case provided the full width b be replaced by the depth b_c of the compression zone (fig. 4.1.). The latter may be measured from the neutral axis of the full section. However, it is generally accepted that bending is less affected by imperfections than pure compression, so that the value 0.22 in formula (2.8.) can be reduced to 0.10 for pure bending.

2.2. Plates subject to shear

Plates bounded by stringers and subject to shear exhibit an ultimate load that can be far beyond the critical shear load. First, WAGNER [2.4.] predicted the ultimate load by means of the concept of a *complete diagonal tension field* that occurs after shear buckling has taken place. This approach may be satisfactory

for aeronautical structures because of the very large slenderness of the plate. For civil engineering structures, however, it was found than an *incomplete diagonal tension field* would be better, because of the limited rigidity of the surrounding stiffeners. KUHN [2.5.] modified WAGNER's approach accordingly. The behaviour of a girder panel subject to shear goes through two stages : a beam action up to the critical load and a tension field in the postbuckled range.

The aforementioned works did not succeed in influencing substantially the everyday design of civil engineering structures. One had to wait until around 1960 before the idea was exploited again; a survey of recent attempts will be given later (see section 4).

2.3. Combination of stresses

The ultimate carrying capacity of plates subject to combined normal and shear stresses was mainly investigated using numerical simulations. The most extensive parametric study is due to HARDING et al [2.6.]. It confirmed an interaction formula originally suggested by HORNE et al that is quite similar to the well known one for elastic plate buckling but is modified to conform to the results of elasto-plastic numerical analysis :

$$\frac{\sigma_c}{S_c\, f_y} + (\frac{\sigma_b}{S_b\, f_y})^2 + (\frac{\tau\, \sqrt{3}}{S_s\, f_y})^2 \leqslant 1 \qquad (2.10.)$$

σ_c is the pure compression stress component, σ_b the maximum value of the bending component and τ the co-existent shear stress. σ_c has to be taken as negative for tensile stress. The factors S_c, S_b, S_s are drawn from numerical results and are presented as charts or analytical expressions ; the main governing parameters are the slenderness of the plate, the restrained or unrestrained character of the unloaded edges and, for shear, the aspect ratio.

Biaxial compression can be incorporated by modifying the first term of (2.10.) as follows :

$$[(\frac{\sigma_{c,x}}{S_{cx}\, f_y})^2 + (\frac{\sigma_{c,y}}{S_{cy}\, f_y})^2\,]^{1/2}$$

where $\sigma_{c,x}$ and $\sigma_{c,y}$ are the pure compression components in directions x and y respectively and S_{cx} and S_{cy} are the corresponding factors.
The tension field mechanism approach (section 4) is a possible alternative for the interaction of bending and shear.

2.4. Effects of lateral loads

The effects of loads acting transversally to the plate are not investigated in detail here because they are not very common in civil engineering structures. If interested the reader is referred to the book [1.13.] that devotes two chapters to this problem, which is still being investigated experimentally and numerically at Imperial College, London.

3. BEHAVIOUR OF STIFFENED PLATE COMPONENTS

3.1. General

The buckling coefficient k of a stiffened plate component depends on the same parameters as for the unstiffened one but, in addition, on some other coefficients that define the kind of stiffening : the relative rigidities of the ribs : flexural γ, torsional θ and extensional δ.

For numerical values of k, the reader is referred to the two volumes of charts prepared by KLÖPPEL et al [1.18.] for simply supported plates and open section stiffeners whose torsional rigidity is neglected. MASSONNET et al [3.1.] took account of the torsional rigidity and established a limited amount of results pertaining to closed section stiffeners.

3.2. The concept of optimum rigidity γ^*

Let us restrict ourselves to the case of open section stiffeners, the torsional rigidity of which is negligible and let us consider for instance a square plate subjected to uniform compression in its plane and provided with a longitudinal stiffener at mid-depth (fig. 3.1.a).
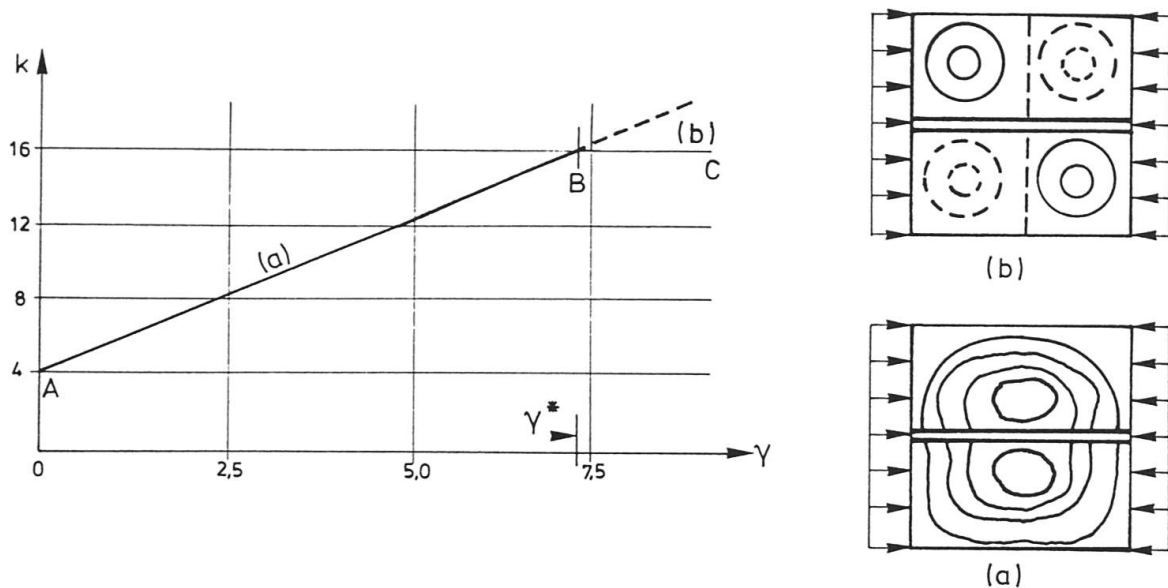

Fig. 3.1. - Buckling coefficient k in function of the relative flexural rigidity γ for a square plate.

When the flexural rigidity of the sole rib is zero, the critical stress is that of the unstiffened plate. The buckling coefficient k will increase with the flexural rigidity, because the rib tends to stabilize the plate the larger the rigidity of the rib. However, the increase of k is limited because no gain of critical stress can be expected as soon as the plate buckles in the second mode (fig. 3.1.b), the attachment of the rib being a nodal line for the buckling pattern. The relative flexural rigidity that corresponds to this change in buckling patterns and is associated with the maximum value of the buckling coefficient is called *optimum* and designated as γ^*. This concept of optimum rigidity can be generalized to other loading cases.

Because of imperfections, and especially initial out-of-flatness, the actual behaviour of the plate is quite different from the previous one, as demonstrated by experimental studies for a long time [3.2.]. It is indeed observed that the stiffener with $\gamma = \gamma^*$ bends considerably in the postcritical range and that a rigidity γ^{**} equal to several times γ^* is required in order that the stiffeners remain rigid up to collapse. The increase of γ beyond γ^* can yield an appreciable increase of ultimate strength, even for compression flanges [3.6.].

The inadequacy of the concept of optimum rigidity was also clearly shown by ROUVE [3.3.], who investigated the question numerically in the elastic range ; for instance, for the square plate considered above , the value γ/γ^* that must be adopted to enforce the first buckling mode is a function of the relative initial imperfection w_0/t (fig. 3.2.). To be realistic, a computer program

must also take account of material nonlinearities ; such programs are available and have been used for several simulations of tests [3.4.] [3.5.].
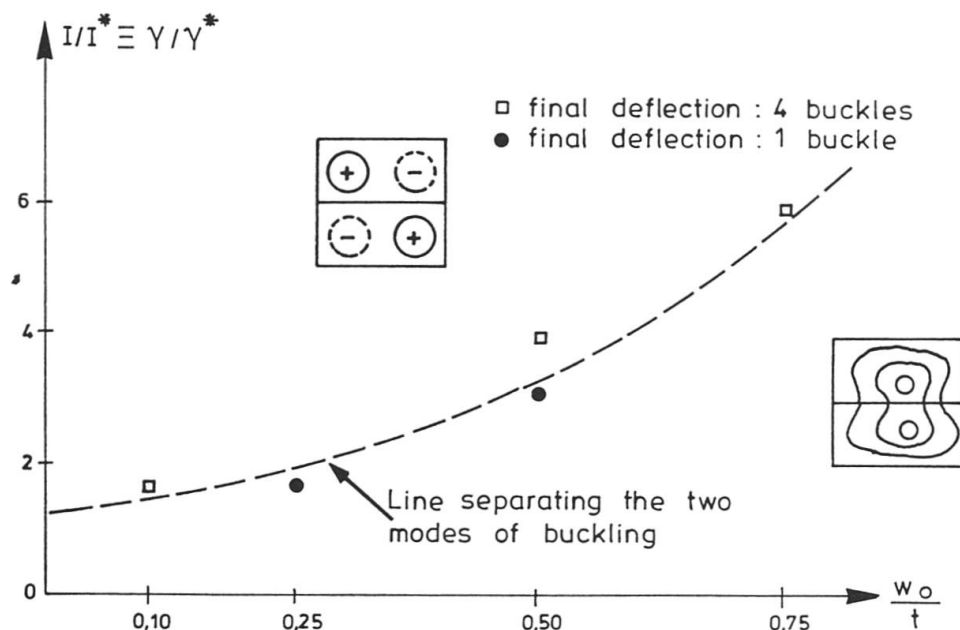

Fig. 3.2. - Magnification factor I/I^* in function of the relative
            initial imperfection.

3.3. Stiffened flanges in uniform compression

Bottom flanges of large box girders are stiffened longitudinally and transversally and are generally considered as simply supported. The longitudinal stiffeners are to be designed so that they do not perish prematurely by local or torsional buckling.   Regarding the transverse stiffeners,they are mostly designed to be rigid up to collapse ; thus, the buckling check is usually made for longitudinally stiffened panels located between two adjacent webs and two consecutive transverse stiffeners. Because of their large torsional rigidity, closed section stiffeners, and especially the trapezoidal box shape, can provide an appreciable increase in the carrying capacity [3.7.].

Numerical simulations performed by WEBB [3.8.] allowed definition of the several basic failure modes which can occur mainly with respect to the slenderness ratio L/r of the stiffener and the b/t ratio of the plate.
Three main zones can be distinguished. The first one is defined by yielding across the plating and a subsequent extensive buckling, when b/t < 60 and L/r < 65. Failure occurs in an overall mode following failure of the relatively slender stiffeners when roughly L/r > 65 for any b/t value in the practical range (30 < b/t < 80). When the yield stress exceeds the critical panel buckling stress, the behaviour is controlled by plate buckling with the corresponding third zone.

3.3.1. The strut approach.

The most approximate approach to idealize the behaviour of such a stiffened compression flange is to imagine that the latter is replaced by a series of unconnected struts consisting of the stiffener proper and of an associated plate effective width. This procedure is the more realistic, the plate has more stocky longitudinal stiffeners and a breadth larger than the panel length ; indeed. in this case, the buckling shape is nearly cylindrical and unless close to the unloaded edges, the struts all behave similarly like compressed columns. The stabilizing membrane effect in the transverse direction is nearly non-existent

and cannot provide the plate with a substantial increase of strength. The ultimate load of the compression flange is obtained as the sum of the ultimate load of all the struts. This approach is the simplest but is not suitable at all to take account of an unequal distribution of compression stress, due to shear lag for instance.

MOOLANI et al [3.9.] devoted much work to the strut approach and produced load-shortening results by inelastic computer analysis, that were used as data to generate a moment-curvature-thrust relationship. This study covered geometrical and structural imperfections and the continuity of the strut. Provided the longitudinal stiffeners do not collapse by local instability, failure can occur either by plate in compression due to squashing or yielding or by compressive yielding of the stiffeners ; a third kind of failure - by tensile yielding of the stiffener outstand - can sometimes occur for large slenderness when the centroïd is closer to the plate than to the outstand.

Many researchers have tried to derive simple design rules of compression stiffened plates by the strut approach ; a sophisticated model was adopted in BS 5400 [1.6.].

3.3.2. The orthotropic plate approach.

Here, the stiffened compression flange is analyzed as an equivalent orthotropic plate by a nonlinear theory. Some realistic simplifications are made in order to make the results useful for practical applications.

The models developed according to this approach refer mainly to a two-step procedure. In the first step, the flange is treated as a materially orthotropic plate and the results are corrected, in a second step, to take account of the discrete character of the stiffeners and thus of the actual behaviour of the plate subpanels between the stiffeners. Such an approach was initiated around 1970 [3.10.]; improvements were brought afterwards by several researchers concerning the direction of the plate buckling, the collapse criterion, the closed shape of stiffener cross-section,...Latest developments in the field are due to RUBIN [3.11.] and JETTEUR [3.12.] and subsequent theoretical investigations for the interaction between plate buckling and shear-lag were developed jointly in Liège and Prague [3.13.].

3.3.3. The discretely stiffened plate approach.

Such an approach can only be developed by numerical nonlinear analysis using the finite elements or finite differences procedure. Very interesting research work was done in that way, but this kind of analysis is long, tedious and expensive and is not suitable for everyday design.

4. DESIGN OF ONLY TRANSVERSALLY STIFFENED PLATE GIRDERS

The design of plate girders is examined mainly with respect to instability phenomena.

4.1. Bending capacity

Because of the large slenderness of the web, the distribution of the normal bending stresses in a girder subject to bending, becomes nonlinear in the postcritical range (fig. 4.1.). The web behaves as if only a part of the compressive zone is effective. The effective width concept allows for a consideration of the influence of the resulting stress redistribution on the bending capacity.

The effective width b_e of the compressive zone of depth b_c can be computed according to section 2.1. Once the "hole" in the web is introduced, the properties of the reduced section are calculated and the ultimate bending capacity is evaluated accordingly ; the ultimate strength of the compressive zone being the determining one.
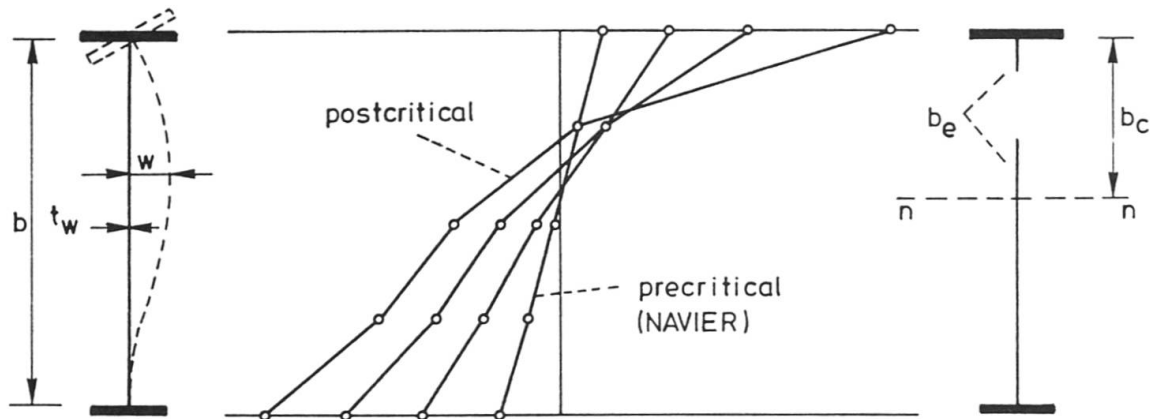
Figure 4.1. - Stress distribution for a plate girder with slender web.

It is important that no local instability occurs in the compression flange befo-
re the ultimate load of the web is reached, otherwise the yield stress would not
be reached in this flange.Firstly, the torsional buckling of the compression
flange can occur if the thinness of the flange is too large ; when this is avoided
the flange is fully effective in compression. By noting c the half-breadth of
the flange, the limiting thinness is adopted as :

$$\frac{c}{t_f} = 0.45 \sqrt{\frac{E}{f_y}}$$

where t_f is the flange thickness.

In addition, the web must be thick enough to resist safely to the vertical
buckling of the compression flange in the plane of the web. That implies a
limiting thinness of the web :

$$\frac{b}{t_w} = \sqrt{\frac{\pi^2}{36(1 - v^2)} \frac{A_w}{A_f} \frac{E}{f_y}}$$

where t_w is the web thickness, A_w and A_f the cross sectional area of the web
and of a single flange respectively.

Lastly, the lateral buckling of the girder as a whole must be checked by consi-
dering the efficiency of the web. If the latter is fully effective, the lateral
buckling load is taken equal to the buckling load of a section composed of the
compression flange and a depth $b_c/3$ of the adjacent web. When, on the contrary,
the compression zone of the web is only $b_e < b_c$, only a part $0.4\ b_e$ of the web is
supposed to act with the compression flange. As an alternative,European curves
for lateral buckling may be used ; because of the slenderness of the girder,
however,one must refer to the elastic shape factor for major axis bending. For
welded plate girders, it is suggested to take n = 2 in the equation of the la-
teral buckling curve to take account of higher residual stresses.

4.2. Shear capacity

BASLER [4.1.] was the first who produced an ultimate shear model for plate gir-
ders as used in civil engineering. He assumed that the flanges were too flexi-
ble to allow for an anchorage of the tension field; thus, only a tension band
develops that anchors only on the transverse stiffeners. Some test results
were demonstrated to be in a pronounced disagreement with BASLER's predictions
and several attempts were then made, especially in Great Britain, in Tscheckoslo-
vakia, in the U.S. and in Japan, in order to improve the BASLER model by taking
account of the flexural rigidity of the flanges. An IABSE Colloquium was orga-
nized in 1971 on this subject [4.2.] and a review of the physical background of

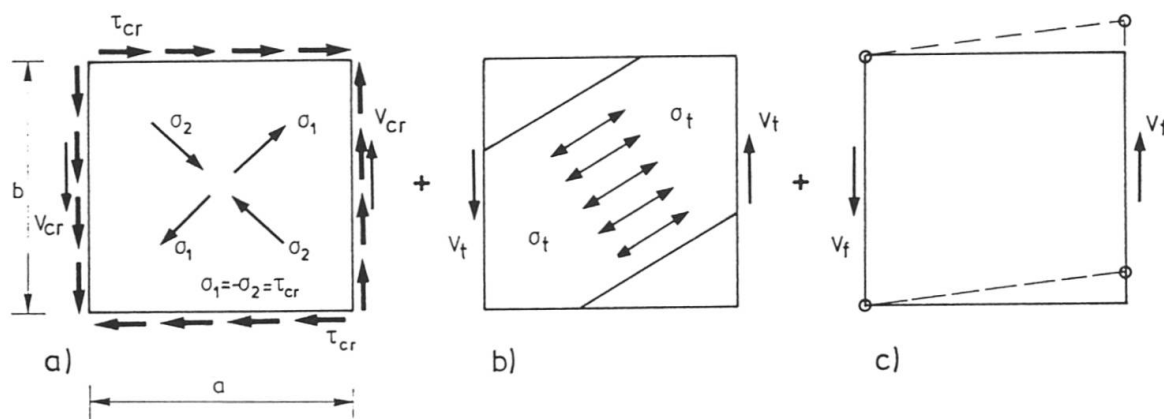most of the models was prepared subsequently [4.3.] by ECCS.

Figure 4.2. - The three parts of the ultimate shear load V_u.

As long as the critical shear stress is not exceeded, the girder is assumed to behave according to the classical theory of Mechanics of Materials ; when τ_{cr} is reached, the web buckles diagonally and the compressive principal stress is blocked at $\sigma_2 = - \tau_{cr}$ whilst the tensile principal stress is allowed to increase further until a certain diagonal band is yielded.Finally, a plastic mechanism can occur in the frame surrounding the web consisting of the flanges and the transverse stiffeners. All the tension band theories developed by different authors after BASLER take advantage of the three parts :
- the critical shear load : V_{cr} ;

- the shear load V_t due to tension band action ;

- possibly ,the shear component V_f due to the frame mechanism.

Thus the ultimate shear load is :

$$V_u = V_{cr} + V_t + V_f \qquad\qquad (4.1.)$$

It is generally assumed that at the ultimate shear load the bending moments and normal forces are transmitted by the flanges only ; indeed, the web yields under $(V_{cr} + V_t)$ and is no longer available for any normal stresses.

The apparently most convenient tension band model is the Cardiff one [4.4.][4.5.]. The inclination ϕ of the tension band is unknown and is to be searched by an iterative procedure in order to maximize V_t ; as an approximation, ϕ can be taken as 2/3 θ where θ is the inclination of the geometric diagonal. Anyway, the maximum of V_t is very flat with respect to θ so that the value of V_t is not very sensitive to an error on ϕ .

An interesting alternative to the Cardiff model was suggested by DUBAS [4.6.], who refers to a tension band as diagonal of a PRATT truss. This approach is very simple and gives direct expressions of the ultimate shear stress as a function of the critical shear stress and of the yield shear stress.

Numerical simulations [3.4.] of the behaviour up to collapse of girders subject to shear have confirmed the general assumptions made in all the ultimate strength models. They revealed that plastic hinges in the frame occur well after the maximum load is reached and that they affect an appreciable length, in contradiction with the concept of concentrated plastic hinges.

4.3. Interaction between bending and shear

The interaction between shear and bending depends on the kind of model used for evaluating the shear capacity. If no contribution of the flanges is considered in the evaluation of the shear capacity, the plastic moment $M_{y,f}$ of the flanges can be reached though the shear force attains V_u.
The second part of the diagram (fig. 4.3.) is given by :

$$\frac{M}{M_{y,f}} = 1 + (\frac{M_y}{M_{y,f}} - 1) [1 - (\frac{V}{V_u})^2] \nleq \frac{M_u}{M_{y,f}} \qquad (4.2.)$$
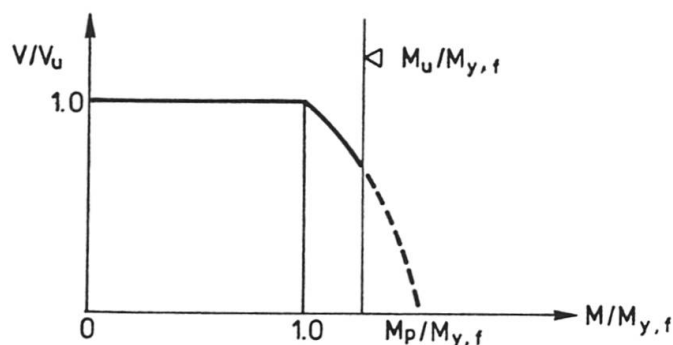
where M_y is the yield moment of the full section and M_u the ultimate bending capacity. If, on the contrary, the model takes account of the flange rigidity, the interaction is more pronounced ; a conservative solution is to assume that the bending moments and axial forces are supported by the flanges only.

Fig. 4.3. - Interaction diagram M - V.

4.4. Design of the transverse stiffeners

Because the arrangement of transverse stiffeners influences only the shear capacity, it is generally economical to limit their number and possibly increase the web thickness accordingly. Transverse stiffeners can also form the posts of transverse bracing or framing.

Transverse stiffeners are subject to the following forces :
- those transmitted by web and flanges in the postcritical range ;
- the external concentrated loads acting on them.

The analysis of postcritical forces acting on an intermediate stiffener is carried out in [4.7.]. As an alternative simplified procedure, the compressive load is obtained as :

$$V_{t,stiff} = V - 0,8 \ V_{cr}$$

where V is the shear force and V_{cr} the shear critical buckling load. Concentrated loads are to be added to V_t, especially for load bearing stiffeners.

The stiffener is then controlled as a beam column, in accordance with the well-known interaction formulae. Because the compressive load is not constant along the stiffener height, an effective length of 0.9 b would appear to be adequate. For the calculation of the section properties, the stiffener is assigned an effective web width. Moreover, the dimensions of the stiffener components must prevent local buckling (see also 6.1.3.).

4.5. Other topics

The forthcoming book considers in addition some special topics, such as :
- the design of end posts and transverse stiffeners at inner supports ;
- the patch loading ;
- the design of unstiffened webs and of webs fitted with vertical undulations ;
- the effect of holes in webs on the ultimate carrying capacity ;
- the effects of curvature in plane or in elevation ;
- the design of hybrid girders ;
- the design of tapered webs.

The investigation of some of these topics are still in progress and more practi-
cal proposals may be expected in a near future.

4.6. Serviceability and fatigue

The criterion for the serviceability of thin webs is not easily   be quanti-
fied because it is more governed by psychological considerations than by
a limiting stress state. In order to define it one could, for instance, either
limit the magnitude of the largest transverse deflection of the web or pre-
vent sudden changes in the buckling pattern (snap-through phenomena).

An elementary analysis   shows that, at least as long as the working loads
encountered currently in ordinary steel girders are considered, the question of
deflection magnitude does not seem to be of much importance. For possible snap-
through phenomena, it is suggested [4.8.] one should comply with a servi-
ceability criterion that is derived from a modified interaction equation between
plate critical buckling stresses :

$$\frac{\overline{\sigma}_c}{K\,\sigma^0_{c,cr}} + (\frac{\overline{\sigma}_b}{K\,\sigma^0_{b,cr}})^2 + (\frac{\overline{\tau}}{K\,\tau^0_{cr}})^2 \leqslant 1 \qquad\qquad (4.3.)$$

The numerators are the stress components under service loads for pure compres-
sion, bending and shear respectively. The denominators are the corresponding
critical buckling stresses affected by a coefficient K that can be much lower
than the load factor, so that the girder may function the postcritical range un-
der service loads. It is suggested to be more liberal for structures subject to
quasi-static loads (K = 1,1) than for those submitted to repeated loads (K=1,5).
In the above interaction formula the buckling coefficients are chosen in accor-
dance with some restraints along the boundaries of the plate panel. The servi-
ceability is then found to be governing when the b/t ratio of the web reaches
230 for static loaded buildings and 180 for bridges.

For thin-walled plate girders the main concern with regard to fatigue is the
out-of-plane deflections that may generate high local bending stresses and a
breathing of the web at repeated loads. That may initiate fatigue cracks, espe-
cially at the web to flange welded connection. This problem is a rather diffi-
cult one and no satisfactory proposal can be made at present, for one has to
know the magnitude and the time distribution of the frequently repeated loads;
in addition, for fatigue, the load factor, which would probably be lower than
unity, is not well defined. Research work is still needed in this field ; in
the meantime, some authors suggest limiting the b/t value of the webs.

5. LONGITUDINALLY STIFFENED PLATE GIRDERS

When the b/t web thinness exceeds the limits imposed by serviceability or fati-
gue criteria, the web is usually fitted with longitudinal stiffeners for that
is generally more economical than to increase the web thickness. The postcriti-
cal behaviour of longitudinally stiffened plate girders is not fundamentally
different from that of girders with transverse stiffeners only. Therefore, only
the new aspects will be dealt with.

5.1. Bending capacity

Measurements of strains during tests show (fig. 5.1.) that the distribution of
in plane longitudinal stresses of a slender thin-walled plate girder does not
obey the elementary rules of the strength of materials. Though at the stiffe-
ner location the stress magnitude corresponds more or less to a straight distri-
bution between the flange stresses, according to the beam theory, a pocket ap-
pears in the compression zone, already for a moderate load intensity. The ef-
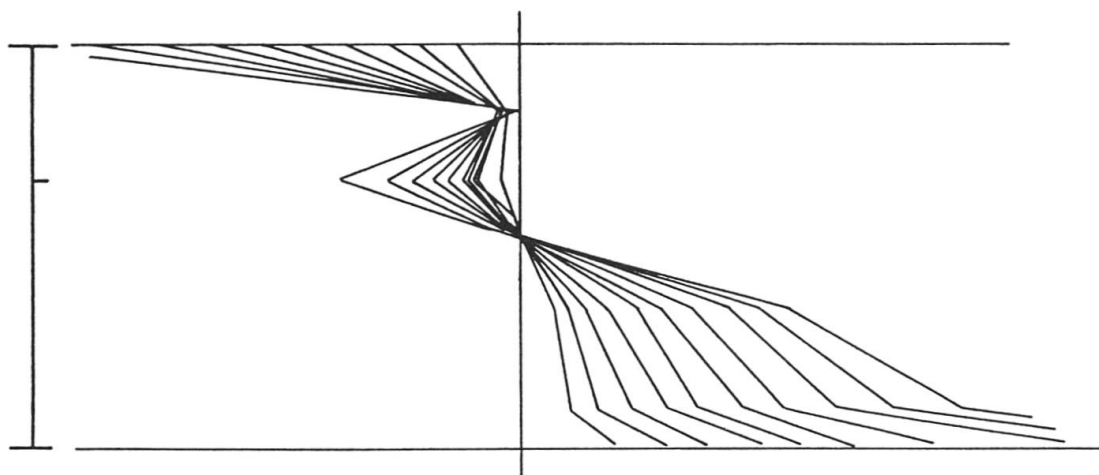fective width concept can be extended to this case.

Figure 5.1. - Stress distribution for a longitudinally stiffened
plate girder.

For a longitudinally stiffened plate girder one has to introduce effective
widths for each compressed subpanel (fig. 5.2.), provided the longitudinal ribs
are stiff enough to generate nodal lines of the buckling pattern up to the post-
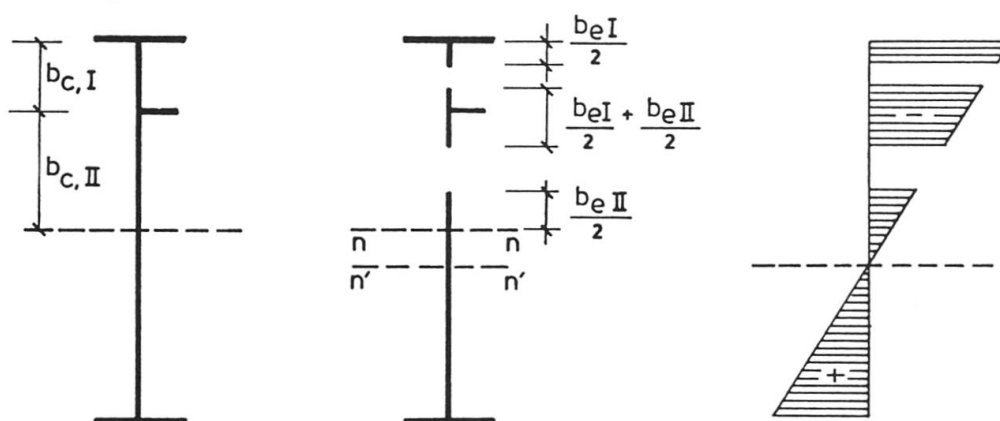critical range.

Figure 5.2. - Effective width model for a longitudinally
stiffened plate girder.

The use of the effective width concept does not seem to be appropriate for
flexible stiffeners, but this is not too important as the solution for stiffe-
ners rigid up to collapse is in most cases very economical.

The presence of rigid longitudinal stiffeners makes the subpanels independent
of   each other ; that yields a decrease of the thinness to be considered for
fatigue conditions (breathing) and of the magnitude of the web buckling pattern,
so that an appreciable increase of the serviceability can be expected. In addi-
tion, longitudinal stiffeners increase the strength against vertical buckling
of the compression flange in the plane of the web.

5.2. Shear capacity

Longitudinal stiffeners, provided they are rigid,   reduce   the fields sub-
ject to shear buckling. When all the subpanels are of the same dimensions,
they have theoretically the same ultimate strength ; when they are different,
the larger subpanel first buckles and loses its stiffness so that the adjacent
ones have to take a greater share of any additional shear load. The postbuck-
ling behaviour of a longitudinally stiffened web is rather more complex than that

of an unstiffened one. The initiation of buckling in the weakest panel corres-
ponds to the end of the first stage corresponding to shear only. As can be ex-
pected, the ultimate shear load is increased, similarly to the critical shear
buckling load, by the presence of longitudinal stiffeners.

Several ultimate models for longitudinally stiffened plates subject to shear
were suggested; their results were compared [5.1.] with those of a rather res-
tricted number of experiments. For practical design procedures, the Cardiff
proposal seems quite appropriate [4.5.] [5.2.] : it assumes (fig. 5.3.) that a
tension band develops independently of the longitudinal
stiffeners, except to the extent that the stiffeners de-
termine the buckling strength that initiates the tension
band. That means that the panel shear strength V_t can be
computed with the same formula as for an unstiffened pa-
nel, whereas the value V_{cr} of the stiffened web is gover-
ned by the minimum value $\min \tau_{cr,i}$ of the weakest subpa-
nel, which is also used to determine the tension field
stress σ_t.
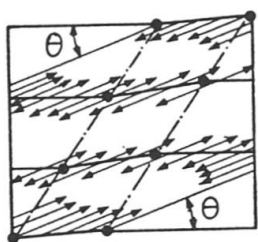
Fig. 5.3. - Tension
   field for a longitudinally
   stiffened web.

5.3. Interaction between bending and shear

There are no substantial differences between the present problem and that discus-
sed in section 4.3.

5.4. Alternative design method

The ultimate stress concept already mentioned in section 2.3. is applied to the
subpanels of longitudinally stiffened plate girders. That implies that the
stiffeners are designed to be rigid up to collapse. Let us recall that the in-
teraction formula is :

$$\frac{\sigma_c}{S_c\, f_y} + \left(\frac{\sigma_b}{S_b\, f_y}\right)^2 + \left(\frac{\tau\, \sqrt{3}}{S_s\, f_y}\right)^2 \leqslant 1 \tag{5.1.}$$

between the effective stresses in compression σ_c, in bending σ_b and in shear τ.
The values of S_c, S_b and S_s take account of the actual behaviour up to collapse.
Though this formula is quite similar to the interaction formula of the linear
plate buckling theory, the meaning is quite different because it corresponds to
an ultimate limit state. This equation is established for a certain range of
residual stresses and initial deflections in line with the customary fabrica-
tion tolerances [5.3.].

For webs under bending, the stress $(\sigma_c + \sigma_b)$ that fulfils the above interaction
equation in the top subpanel will generally be less than the ultimate stress
of the flange. The bending moment and/or axial load resulting from this diffe-
rence in the stress value between web and flange may be redistributed within a
certain amount - no more than 60 % - in certain conditions.

The ultimate stress concept yields results that are similar to those corresponding
to the effective width concept, provided the assumptions related to the postcri-
ticale behaviour are the same.

5.5. Design of longitudinal stiffeners

The advantage of rigid stiffeners has already been clearly demonstrated. Several experiments and/or numerical simulations show that stiffeners having the relative optimum rigidity γ_L^*, defined in accordance with the linear plate buckling theory, do not actually remain rigid up to collapse. Such stiffeners constitute nodal lines for the buckling pattern if and only if their rigidity γ_L is at least m_L times the optimum rigidity γ_L^*.

The magnification factor m_L depends mainly on the web thinness b/t_w. One has for open section stiffeners :

$$m_L = 0,023 \ b/t_w - 1,5 \quad (\not> 4 \text{ and } \not< 1.25) \tag{5.2.}$$

and, for closed section stiffeners, because of the beneficial effect of the large torsional rigidity :

$$m_L = 0.0105 \ b/t_w \quad (\not> 2,5 \text{ and } \not< 1.25) \tag{5.3.}$$

For the calculation of γ_L, the stiffener is provided with an effective web width b_e, equal to $b_e = t_w \ \sqrt{E/f_y}$, that, for closed section stiffeners, is to be added to the distance between the stiffener webs. Because of the approximative character of m_L, a rough determination of γ_L^* is usually sufficient and can be made simply for the determining stress case (bending or shear).

Some additional strength requirements may have to be examined in some cases, especially when some lateral bending occurs.

5.6. Design of transverse stiffeners

Because of the presence of longitudinal stiffeners, both rigidity and strength conditions are required for the transverse stiffeners. For the rigidity condition, one has to define an equivalent web thickness t_{equ}, so that an unstiffened web having such a thickness would have the same ultimate strength as the longitudinally stiffened web. Concerning the strength viewpoint, it is assumed that the longitudinal stiffeners develop lateral forces on the transverse stiffeners, equal to 1 % of those existing in the stiffeners. Transverse stiffeners are thus designed as beam-columns. For more details reference can be made to [4.8.].

Secondary transverse stiffeners are sometimes necessary ; their sole purpose is to reduce the unsupported length of the longitudinal stiffeners and therefore their required relative flexural rigidity γ_L. In these stiffeners only a stiffness requirement is needed :

$$I_{T,sec} \geq 1,5 \ I_L \tag{5.4.}$$

5.7. Other topics

The strength with respect to patch loading is helped by the presence of stiffeners. Some recommendations are given in the forthcoming book [1.14.] with regard to design.

Because longitudinal stiffeners reduce the out-of-plane deflections of the web, psychological effects are less relevant. Usually, the serviceability limit state needs no special check. The problem of snap-through is restricted to the subpanels provided the longitudinal stiffeners can be regarded as rigid.

The breathing of the web under repeated loading and the corresponding fatigue cracks are substantially reduced by the presence of longitudinal stiffeners. Nevertheless, fatigue may occur especially, at the ends of longitudinal stiffeners; the best solution is to arrange these continuously along the girder length, through cuts made in the transverse stiffeners.

6. BOX GIRDERS

Small box girders with longitudinally unstiffened webs and flanges are covered by section 4. Large box girders with longitudinally stiffened webs and flanges require special design rules.

Box girders differ from plate girders in mainly three ways :
- the slenderness and the width of the flange, generating shear lag and plate buckling of the flange ;
- the amount of edge restraint, which is usually less than in plate girders ;
- the existence of plate diaphragms that transmit the reactions to the supports.

6.1. Flange buckling

Only two approaches are of practical use : the strut approach and the orthotropic plate approach. For both of them, some failure modes must be eliminated ; especially the torsional buckling of stiffener outstands and overall buckling of the orthotropically stiffened panel between main girder webs.

6.1.1. The strut approach

In the strut approach, the stiffened flange is idealised by a series of unconnected column sections consisting of a stiffener and an associated plate width. The load-deformation behaviour of the flange plate is nonlinear ; it was studied mainly in the U.K. [6.1.] [6.2.] and yields plate strength curves that incorporate the effect of initial imperfections - the amplitude of which is $(b/165)$ $\sqrt{f_y/355}$ - and compressive welding residual stresses of 10% of the yield stress f_y. To correctly incorporate plate panel data into column design, both stiffness- and strength factors would be needed ; however, for simplicity in design, only the lowest factor is kept, so as to provide a conservative approach and is identified with the buckling coefficient S_c used for web plates under compressive loading [5.3.]. Thus, the strut section is taken as the stiffener plus S_c times the stiffener spacing. The limiting stress σ_u for the strut is then given by the analytical expression describing the European column buckling curves [6.3.]:

$$\overline{N} = \frac{1 + \eta + \overline{\lambda}^2}{2\,\overline{\lambda}^2} + \frac{1}{2\overline{\lambda}^2} \sqrt{[1 + \eta + \overline{\lambda}^2]^2 - 4\,\overline{\lambda}^2} \not> 1 \qquad (6.1.)$$

with

$$\overline{N} = \sigma_u/f_y' \quad \text{and} \quad \overline{\lambda} = \sqrt{\sigma_E/f_y'} \ .$$

In above expression, σ_E is the Eulerian column buckling stress; η is an imperfection parameter taking account of the initial imperfection and of the eccentricity of the compressive load.   Regarding the fictitious yield stress f_y', it allows possibly for other coincident stresses. Because the strut can fail by compressive yielding either of the plate or of the stiffener outstand, the buckling equation must be applied twice with relevant values of η and f_y'.

When the strut buckles towards the plate, the outstand is governing and $f_y' = f_y$. For buckling towards the outstand, the plate is determining; because of the possible presence of shear stresses in the flange, which reduce the compressive buckling stress, f_y' is derived from the von MISES yield criterion so that:

$$f_y' = \sqrt{f_y^2 - 3\tau^2} \ .$$

6.1.2. The orthotropic plate approach

The present approach takes account of the effect of transverse continuity of the plating of the stiffened flange that provides a better stability for the longitudinal stiffeners than is allowed for in the strut approach. The orthotropic effect is double. First, the transverse tensile stresses that occur when the flange deflects out-of-plane reduce the transverse deflection ; however, the

effect on the deflection of the central stiffeners is only significant when the flange is not too strongly stiffened by multiple ribs. Secondly, the out-of-plane deflection induces a non-uniform direct stress distribution, that must not be confused with stress variations due to shear lag ; this exerts an influence on failure load as stiffeners having higher deflections are subject to lower stress levels.

Design rules based on the orthotropic plate approach can only be derived from the generalized von KARMAN - MARGUERRE nonlinear equations. In a first step, the discretely stiffened compression flange is idealized by an orthotropic plate with uniformly smeared rigidities and the ultimate strength is assumed to be reached when a certain collapse criterion is satisfied ; that yields the efficiency of the orthotropic plate ρ_g as the ratio between the actual force transmitted by the flange and the squash load of the latter. In a second step, one takes care of possible local buckling of the plate between adjacent stiffeners, by defining a relative effective width $\rho'=b_e/b$ according to (2.9.) and then a local efficiency ρ_1 :

$$\rho_1 = \frac{\rho' + (m-1)\ \delta}{1 + (m-1)\ \delta} \tag{6.2.}$$

where $(m-1)$ and δ are the number and the relative area of the longitudinal stiffeners. Thus, the effective efficiency of the actual compression flange is then given by :

$$\rho = \rho_g\ \rho_1 \tag{6.3.}$$

This basic idea was improved recently, especially in [3.11.] [3.12.].

Three criteria may be governing for a design. When shear lag is important, onset of yielding at the web-flange junction is often decisive. On the other hand, according to the position of the centroid with respect to the extreme fibres, onset of compressive yielding is to be considered in the centre of the flange panel, either in the sheet, or in the stiffener ; to allow for some plastic redistribution, the onset of yielding is considered at mid-depth of the plate and at the centroid of the stiffener proper. By combining each of these conditions with the two nonlinear equations, one can derive the values of the longitudinal membrane stress σ_0 at the unloaded edges, the stress loss σ_1 due to buckling, and the additional deflection w at the centre of the flange.

The average longitudinal stress $\bar{\sigma}_{orth}$ in the sheet plus stiffeners system is :

$$\bar{\sigma}_{orth} = \rho_s\cdot\sigma_0 - \frac{\sigma_1}{2} \tag{6.4.}$$

where ρ_s is the shear lag efficiency.
It was observed that, provided the stress σ_0 is the governing criterion, the average collapse stress is given with a sufficient accuracy by a FAULKNER like equation :

$$\frac{\bar{\sigma}_{orth}}{f_y} = \rho_s \quad 1,05[\ \sqrt{\frac{\sigma_{cr}}{\rho_s\cdot f_y}} - 0.26\ \frac{\sigma_{cr}}{\rho_s\ f_y}]\ , \tag{6.5.}$$

σ_{cr} being the critical plate buckling stress of the flange. For both other criteria, charts are calculated and are useful design aids for everyday practice.

The efficiency of the plating between the stiffeners is taken according to (2.9.), i.e. :

$$\rho' = 1,05[\ \sqrt{\frac{\sigma'_{cr}}{\bar{\sigma}_{sh}}} - 0.26\ \frac{\sigma'_{cr}}{\bar{\sigma}_{sh}}] \tag{6.6.}$$

where σ'_{cr} is the critical plate buckling stress of the plate subpanel.

Then the mean collapse stress of the stiffened plate, with account taken of local plate buckling is :

$$\overline{\sigma}_u = \frac{\rho' \, bt \, \overline{\sigma}_{sh} + A \, \overline{\sigma}_{stiff}}{bt + A} \qquad (6.7.)$$

in which A is the total cross-sectional area of the longitudinal stiffeners. In the above expressions, $\overline{\sigma}_{sh}$ and $\overline{\sigma}_{stiff}$ are the average stresses in the sheet and in the stiffeners respectively; they are obtained from $\overline{\sigma}_{orth}$ by adding the bending stresses due to buckling. Numerical calculations show that a very satisfactory approximation of ρ_1 is obtained by replacing both $\overline{\sigma}_{sh}$ and $\overline{\sigma}_{stiff}$ by $\overline{\sigma}_{orth}$, which makes the calculations more simple, so that :

$$\overline{\sigma}_u \simeq \frac{\rho' \, bt + A}{bt + A} \, \overline{\sigma}_{orth} \qquad (6.8.)$$

Orthotropic action can contribute significantly to strength when stiffening arrangements reduce the overall plate deformation or provide the transverse tensile forces with a better edge restraint ; such cases are especially encountered for closed section stiffeners, for narrow flanges stiffened by only a restricted number of stiffeners and for very lightly stiffened plates.

Higher collapse loads can be observed when imperfections are in the same sense in adjacent spans of flanges continuous over cross-frames. From a design viewpoint, it is nevertheless recommended to choose conservatively supported conditions.

6.1.3. Torsional buckling of stiffeners

Design rules are generally formulated so that local torsional buckling of stiffener outstand is not governing the limiting stresses ; this local buckling mode is indeed highly unstable and can give catastrophic collapse. It is thus quite important to preclude this mode of buckling ; this goal is reached by specifying stockiness requirements in the form of minimum thinness ratios [6.4.]:

for flat stiffeners : $c/t \leqslant 0.41 \sqrt{\dfrac{E}{f_y}}$ \hfill (6.9.a)

for equal leg angles : $c/t \leqslant 0,50 + 0.28 \sqrt{\dfrac{E}{f_y}}$ \hfill (6.9.b)

for closed section stiffeners : bottom flange : $b/t \leqslant 1,2 \sqrt{\dfrac{E}{f_y}}$ \hfill (6.9.c)

$$\text{walls} : \quad b/t \leqslant 1,7 \sqrt{\frac{E}{f_y + \sigma_a}} \qquad (6.9.d)$$

where σ_a is the average compressive stress on the effective stiffener section at collapse.

The above limitations are obtained by requiring that the secant stiffness does not fall, due to nonlinear effects, by more than 2,5 % for open section stiffeners and 5 % for closed section ones.

6.1.4. Design of transverse stiffeners.

The design basis for transverse stiffeners is normally to ensure that stiffener deflections are limited so that they do not affect the support conditions assumed in the design of the longitudinally stiffened panels. This results in a stiffness requirement.

To the analytical solution of the problem [6.5.], which is rather laborious, some approximate design approaches are frequently preferred. Both approaches based on the orthotropic plate concept [6.6.] and on the bar on spring supports [1.9.] lead respectively to expressions of the necessary moment of inertia of the transverse stiffener. The values obtained are conservative compared to the analytical solution ; however, as they are derived from the linear plate buckling theory and therefore do not take account of the detrimental effect of

imperfections, these values must be multiplied by an amplification factor m_T.

While the above approaches are based on rigidity considerations, another design concept considers the question of strength. The transverse stiffener must sustain a vertical lateral load q, which is a certain percentage of the flange compressive load and is uniformly distributed over the transverse stiffener length. In this respect, quite different proposals were made in the past, but it is believed that the considerations in the analysis of a lateral load q of 1 % of the compressive flange force is also appropriate.

6.1.5. Shear lag and flange behaviour

Guidance on shear lag effective breadth ratios is given in [6.7.]; some attempts were made recently in order to give rather simple formulae [6.8.] [6.9.].

The presence of shear lag yields uniform stress distribution in the compression flange. The interaction between shear lag and plate buckling is a complex problem; its importance depends mainly on the definition of the limit state under consideration. Tests and numerical simulations show that shear lag tends to disappear near to the actual collapse load and that interaction occurs generally only when shear lag is very pronounced. Interaction is very sensible to the degree of redistribution that is allowed for [3.13.]; the larger the latter, the less the interaction. A large level of redistribution requires a high ductility.

6.2. Web buckling

The opinions of different authorities are diverging on the applicability of the design criteria for webs of plate girders to webs of box girders. Contradictions may be found in current specifications.

In the forthcoming book [1.14.], only girders with stiffened wide flanges are considered.

6.2.1. Bending capacity

For girders subjected to bending, the web behaviour should not depend on the flange shape ; models for webs of plate girders thus remain applicable for box girders. Some minor alterations are nevertheless needed. For instance, an effective width of flange is taken into account for calculating the stress distribution in the webs.

6.2.2. Shear capacity

The low bending stiffness of box girder flanges can render questionable the applicability of the tension field concept to webs of such girders. Anyway, the tension field concept should be extended with caution to box girders. Especially, the strength contribution due to the frame mechanism must be neglected.

The shear collapse load for webs of box girders without longitudinal stiffeners can be conservatively adopted according to the "true" BASLER solution as :

$$V_u = bt_w [\tau_{cr} + \frac{f_y - \sqrt{3} \, \tau_{cr}}{2(\sqrt{1+\alpha^2} + \alpha)}]$$ (6.10.)

where α is the aspect ratio of the web.

For longitudinally stiffened webs, as already mentioned in section 4, the ultimate shear load is the sum of the critical shear buckling load V_{cr} - calculated from the minimum τ_{cr} of the weakest subpanel - and of the tension band force V_t determined for the entire web panel between transverse stiffeners with longitudinal stiffeners disregarded. Thus the flange contribution is neglected.

The ultimate stress approach, mentioned in section 2.3, may be used as an alternative for pure shear.

6.2.3. Combined bending and shear

As for webs of plate girders, either an interaction between V/V_u and M/M_u or the ultimate stress concept can be used.

6.3. Other topics

Among the other problems discussed in [1.14.], let us mention: the torsion of box girders, some considerations on the diaphragms, the influence of lateral loads and the effect of curvature as well in elevation as in planview.

7. INTERACTIVE BUCKLING

7.1. General

The scope is restricted to thin-walled hollow bars with rectangular cross section used for centrally   - or eccentrically compression.

Two modes of instability can be present : column buckling and plate buckling of the walls. According to BLEICH [7.1.], the optimum design is obtained when the critical plate buckling stress of the walls is just equal to the critical column buckling stress. Such a requirement yields the maximum allowable b/t ratio of the walls, with regard to the loading, the boundary conditions, the material,..etc. The above condition is based on the concept of linear buckling and is not at all representative of actual structures ; these are indeed affected by structural and geometrical imperfections. Owing to the effect of initial imperfections, column and plate buckling occur by divergence of the equilibrium and not by bifurcation ; the average ultimate strength of the column is always lower while that of the wall is larger than the corresponding critical buckling stress. Therefore BLEICH's approach is not satisfactory.

7.2. Plate buckling curves in pure compression

In the frame of an extensive experimental and theoretical research programme [7.2.][7.3.], an attempt was made to develop plate buckling curves in uniform compression for simply supported plates of width b and thickness t. For the sake of similarity with the non well-known European column buckling curves, non-dimensional coordinates are used :
reduced slenderness :

$$\bar{\lambda}_v = \sqrt{\frac{f_y}{\sigma_{cr}}}$$
$$= \frac{b}{1,9t} \sqrt{\frac{f_y}{E}} \qquad (7.1.)$$

reduced load : $\quad \bar{N}_v = \dfrac{\sigma_v}{f_y} \qquad\qquad (7.2.)$

where σ_v is the average compression ultimate stress.
In accordance with ECCS Recommendations [1.1.], the curves $\bar{N}_v = f(\bar{\lambda}_v)$ present a plateau $\bar{N}_v = 1$ in the range $\bar{\lambda}_v \leqslant 0.8$. (the limiting b/t ratio is thus 1.52 $\sqrt{E/f_y}$). The analytical expressions are derived from a physical model and test calibration :

$$\bar{N}_v = \frac{1 + \beta \,(\bar{\lambda}_v - 0.8) + \bar{\lambda}_v}{2\,\bar{\lambda}_v} - \frac{1}{2\bar{\lambda}_v} \sqrt{[1 + \beta \,(\bar{\lambda}_v - 0.8) + \bar{\lambda}_v]^2 - 4\,\bar{\lambda}_v} \not\leq 1 \qquad (7.3.)$$

with $\beta = 0.35$ for hot finished-and $\beta = 0.67$ for cold finished sections ; the latter have larger residual stresses.
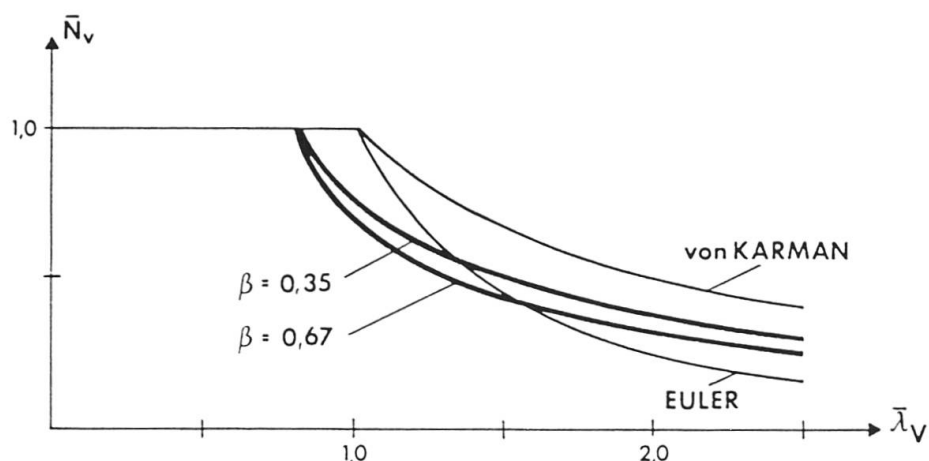
Figure 7.1. - Reduced load in function of the reduced slenderness.

7.3. Collapse load of a stub column

A stub column is a part of hollow section column, the slenderness of which prevents any column buckling. The collapse load N_v of a short coupon is obtained by adding the collapse loads of the four walls, the latter being obtained from plate buckling curves.

For this calculation account is taken of the restraint provided by the narrower walls on the wider ones.

7.4. Collapse load of centrally loaded column

Because, for a short coupon, the ultimate load is represented by N_v, the ultimate load of the slender column cannot exceed N_v. Thus, the value N_v plays for the thin-walled sections the same role as the squash load N_{pl} does for compact sections.

It was observed that the characteristic values of the experimental results comply with modified European column curves (fig. 7.2.), the analytical expression of which is :

$$\overline{N}' = \frac{1 + \alpha(\overline{\lambda}'-0.2) + \overline{\lambda}'^2}{2\,\overline{\lambda}'^2} - \frac{1}{2\overline{\lambda}'^2}\sqrt{[1 + \alpha(\overline{\lambda}'-0.2) + \overline{\lambda}'^2]^2 - 4\,\overline{\lambda}'^2} \not> 1 \qquad (7.4.)$$

with $\alpha = 0.21$ for hot finished sections and $\alpha = 0.49$ for cold finished ones. The non-dimensional coordinates are :

$$\overline{N}' = \frac{N_K}{N_V} = \frac{\sigma_K}{f_V} \qquad\qquad\qquad (7.5.)$$

$$\overline{\lambda}' = \frac{\lambda}{\pi}\sqrt{\frac{f_V}{E}} \qquad\qquad\qquad (7.6.)$$

σ_K is the average ultimate stress of the column and $f_V = N_V/A$ is the average ultimate stress of the stub column.
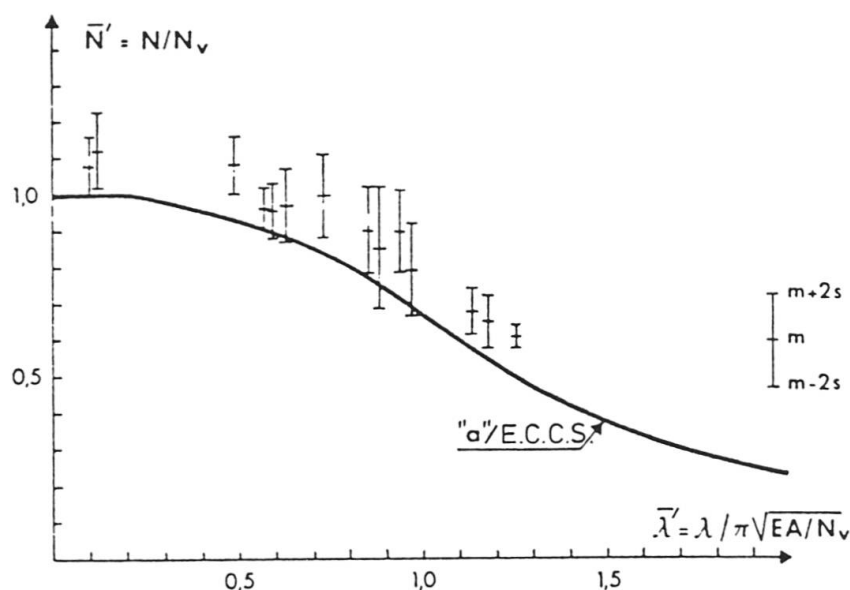
Figure 7.2. - Comparison between theory and test results.

7.5. Collapse load of an eccentrically loaded column

The analysis of thin-walled beam-columns, i.e.columns subjected to axial load and bending moment, is also based on the use of an AYRTON-PERRY formulation.

In [7.4.], both monoaxial bending and biaxial bending are considered. It is beyond our scope to give here the detailed expressions allowing for an actual design. We can just draw the attention to the fact that reduced section moduli are to be used in the terms dealing with bending; they are calculated for the reduced section, i.e. the section composed by effective widths of webs and flange, since buckling can occur in the walls provided the latter are thin.
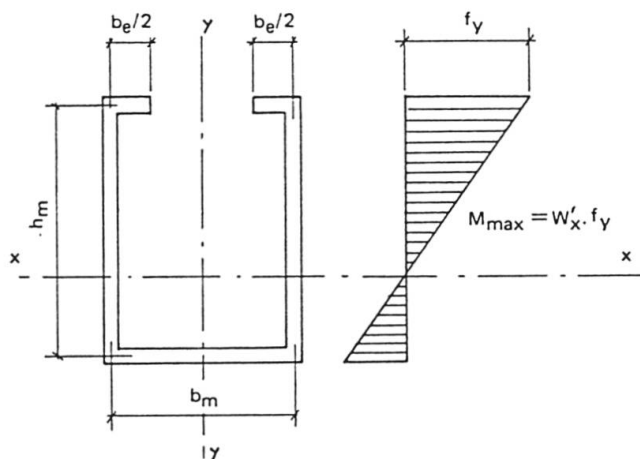
It is worth noting that, for the hollow sections belonging to current production, the webs - i.e. the walls that are bent - are not thin; that means that they cannot buckle prior to the yielding of the most compressed fibre. As the tension flange will be fully effective, only an effective width in the compression flange need be considered. (fig. 7.3.).

Figure 7.3. - Effective section for an eccentrically loaded column.

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