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Dynamic Behaviour of Footbridges

Comportement dynamique des passerelles

Dynamisches Verhalten von Fussgängerbrücken

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SUMMARY

The dynamic behaviour of footbridges is briefly reviewed. Effects due to pedestrian loading are considered. The main factors that influence response are construction material, stiffness and frequency. Methods to alleviate unduly lively structures are summarised.

RÉSUMÉ

Le comportement dynamique des passerelles est examiné en considérant les effets des charges imposées par les piétons. Les facteurs principaux qui influencent la réponse de l'ouvrage sont les matériaux de construction, la rigidité et la fréquence. Les méthodes employées pour neutraliser le comportement d'ouvrages trop sensibles aux oscillations sont passées en revue.

ZUSAMMENFASSUNG

Es wird ein kurzer Überblick über das dynamische Verhalten von Fussgängerbrücken gegeben. Einflüsse infolge Fussgängerbelastungen werden berücksichtigt. Die wichtigsten Faktoren, welche das Antwortspektrum beeinflussen, sind Baustoff, Steifigkeit und Frequenz. Verbesserungsmethoden für übermässig schwingungsempfindliche Bauwerke werden zusammengefasst.

1. INTRODUCTION

Vibration of footbridges is a problem that designers have had to take seriously since the introduction of cast iron construction in the 18th century. One of the earliest cases of catastrophic vibration was in 1831 when 60 soldiers, marching in step across a cast iron bridge at Broughton, brought about its collapse. This was almost certainly the cause of the instruction still posted on some bridges, for soldiers to break step when crossing, Figure 1. Many of the early iron bridges were susceptible to wind, for example the Brighton Chain Pier was damaged in 1833 and 1836. At that time it was recognised that suspended footbridges were lively in wind and there were many examples which were stabilised by tie-down ropes.

In more recent years there have been examples of footbridges that have proved to be unacceptably lively to pedestrians and have been modified by one means or another. Such cases rarely receive any publicity which is regrettable because there is still a lot to be learned from experience. Moreover there is no discredit to designers because until the recent publications of BS 5400 *[1]* and the Ontario Code *[2]* there has been no guidance from Design Standards.

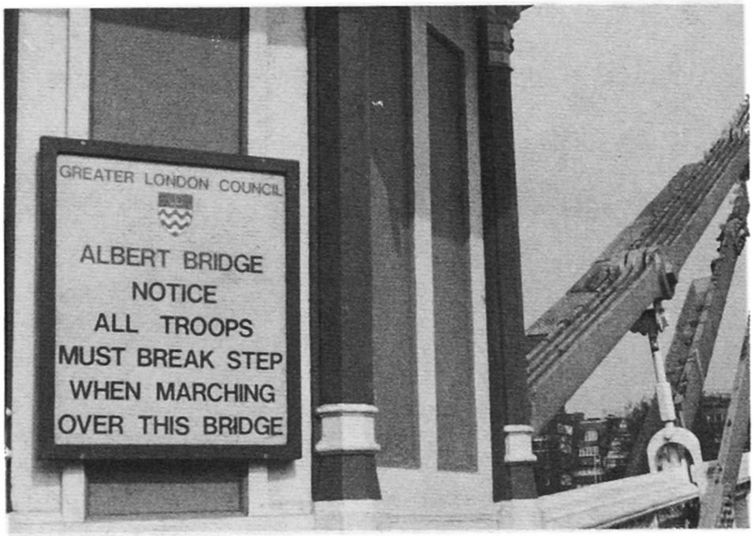

Fig.1 Notice on the Albert Bridge London

Modern footbridges are usually constructed with continuous superstructures having up to five or so spans. Construction materials are commonly reinforced or prestressed concrete, composite (concrete deck connected to steel beams) and wholly steel. Other materials such as aluminium and timber are less common. For long spans, cable-stayed support is tending to supplant catenary suspension because of its economy and elegance. Many footbridges are system built to a low cost and because of the competitive nature of the market it is necessary to have as efficient a design as possible. In consequence designs are continually being pared down and are edging towards having lower stiffness and more lively behaviour.

Pedestrian loading is the principal cause of footbridge vibration problems and is the main concern of this article. Nevertheless aerodynamic loading cannot be ignored and there have been a number of instances where it has been necessary to modify the design of footbridges because of excessive movements of cables or superstructures. These cases have generally involved long and flexible structures and 'typical' footbridges are not normally considered to present an aerodynamic problem.

2. ACCEPTABLE LEVELS OF VIBRATION

The human body is sensitive to vibration, and discomfort is felt by most people long before the footbridge is at risk. Consequently the level of dynamic response is usually dictated by human acceptance.

A number of authors have studied or reviewed human tolerance of vibration in different environments [3,4,5]. In most cases adverse reaction is psychological rather than purely physiological in origin and the criterion of acceptance is a subjective matter depending on a range of considerations not the least of which is the person in question. Some people are disturbed and frightened by a level of vibration to which others are indifferent. It is not uncommon for a new footbridge to attract public misgiving when it is first used but to become quickly accepted as people become accustomed to it.

It is not practical to fix an absolute limit on vibration amplitude. Either the limit would be too high to be effective or some perfectly acceptable bridges would not conform as a result of their behaviour during short isolated periods of high-amplitude vibration. In BS 5400 it is required to calculate response to a dynamic load representing a notional pedestrian [3]. The limiting acceleration is $\pm \frac{1}{2} \sqrt{f_o}$ m/s^2 (f_o is the first bending frequency) for frequencies up to 4 Hz which gives a value of $\pm$ 0.70 m/s^2 ($\pm$ 4.5 mm) at the median walking frequency of about 2 Hz. The choice of $\pm \frac{1}{2} \sqrt{f_o}$ is based on an assessment of the available data on human response and is matched with the loading function so that acceptable but known-to-be-lively bridges just conform [3]. This procedure restricts liveliness and ensures that vibrations during normal use are acceptable for all but a few structures. A lower limit would probably be better for regular wind-induced movements [5].

3. FACTORS AFFECTING DYNAMIC RESPONSE

For pedestrian loading the principal factors affecting response of a bridge are the force-time function for foot contact, the bridge stiffness, damping, mass, span length, and the mode shape. In general it is necessary to solve the equation of motion to calculate response (in BS 5400, using the notional pedestrian loading function) but for some simple structures it is convenient to apply the expression [1]

$$a = 4 \pi^2 f_o^2 y K\psi$$

where a = the maximum vertical acceleration
y = the static deflection for a load of one pedestrian at mid-span
ψ = a dynamic response factor which increases with increase in span length and decrease in damping [1,3].
K = a configuration factor which lies within the range 1.0 to 0.6 for simple footbridges having up to three spans [1,3].

It is assumed that the pedestrian walks at a steady pace rate equal to the frequency of the footbridge, and the input force is independent of pace rate.

4. LOADING

There have been several investigations of the characteristics of pedestrians and the dynamic response of footbridges [3,6,7,8]. Force-time curves have been established for foot contact during walking and running [3,9,10].

In a survey of walking characteristics it has been found that over 95 per cent of a sample of pedestrians passing an observation point have pace rates between 1.5 and 2.5 Hz. About one per cent of the sample have pace rates above 2.5 Hz and 2.8 to 3.0 Hz is the most common jogging range. Frequencies up to 5 Hz are possible but running above 3.5 Hz rarely occurs on public footpaths. Measurements made on 44 footbridges have shown that running causes responses which are about twice as high as for walking at the same rate [11].

About 5 per cent of observations involved pairs of people walking in step for at least ten paces. A pair of pedestrians in step causes up to twice the response of a single person, but it does not follow that every incidence causes this effect because a small mismatch between them reduces the loading.

In practice loading will depend on the characteristics of individual pedestrians, the flow rate and arrival pattern. In addition the spectrum of response amplitudes depends on the frequency of the footbridge in relation to the spectrum of pedestrian pacing rates.

A further complication occurs because a footbridge can be excited by walking or running at $\frac{1}{2} f_o$.

Maximum response for a single pedestrian occurs when the pace frequency is very close to the bridge frequency but the proportion of the maximum that occurs when the bridge is excited 'off frequency' depends on the degree of mismatch and the bridge damping. The response of a lightly damped bridge is very sensitive to frequency because the peak in response has a very narrow band width, for example the measured response of a footbridge excited at a pacing frequency of 0.07 Hz below f_o (1.93 Hz) was one seventh of the value at f_o. For this footbridge, f_o is close to the median pedestrian pacing frequency so that there is a high probability that it will be excited at resonance during normal usage. Another footbridge was found to have a 60 per cent bigger response at resonance but f_o was 2.62 Hz which is outside the normal walking range. At this frequency it is calculated that there is a forty times lower likelihood of it being excited above $\pm \frac{1}{2} \sqrt{f_o}$.

It may be concluded that a typical footbridge is likely to be excited regularly at or above the calculated response if its frequency lies within the range 1.7 to 2.2 Hz. This is unfortunate because many footbridges have first bending frequencies in this range. A footbridge having a frequency of about 3 Hz is likely to be excited occasionally by joggers at an amplitude of about three or four times the response calculated by the simple formula.

In BS 5400 it is assumed that footbridges having first bending frequencies above 5 Hz are too difficult to excite and their vibration can safely be ignored. Between 4 Hz and 5 Hz a reduction factor is applied to the response calculated under matched-frequency loading in order to allow for the attenuation to acceleration when typical bridges are excited 'off frequency'. Wheeler *[6]* proposes a similar procedure but in this case the reduction factor adjusts to the lower incidence of resonance when f_o is above 2 Hz. Another approach *[8]* seeks to calculate amplitude and return periods on a stochastic basis.

There may be scope for the application of 'Monte Carlo' methods of analysis *[12]* for calculating the response of a footbridge when a normal population, including joggers and people in pairs, cross the structure at different pace rates. The sophistication of this approach might be justified if reliable data could be introduced relating human tolerance to amplitude and duration over the normal population. In the meantime the notional pedestrian calculation remains a useful design tool.

Some consideration needs to be given to vandal loading but it is normally sufficient to ignore pedestrian comfort in this case. BS 5400 requires that the bearings should resist upward and lateral movement and that prestressed structures should be capable of sustaining a 10 per cent reversal of live-load moment.

5. DYNAMIC CHARACTERISTICS

5.1 Damping

A lot of work has been done on damping particularly for highway bridges and the state-of-the-art up to 1977 has been reviewed [13]. Damping is most commonly represented by the logarithmic decrement, δ, measured from a free decay and calculated over several cycles to minimise errors.

$$\delta = \frac{1}{n} \operatorname{Log}_e \frac{A_o}{A_n}$$

where n is the number of cycles, A_o is initial amplitude and A_n is the amplitude after n cycles. This implicitly assumes that the damping is viscous and that δ has the same value throughout the decay. In practice it increases with amplitude of movement, usually in a sigmoidal manner with so called upper and lower values [14]. The lower value is comparable with the material damping which would be exhibited by a homogenous beam with frictionless supports, see Table 1.

Table 1

Typical values of damping (δ)

	Material	Beam	Bridge
Steel	0.002 to 0.008	0.004 to 0.03	0.02 to 0.06
Concrete	0.01 to 0.06	0.02 to 0.06	0.02 to 0.2

The upper value occurs at bigger amplitudes of movement when energy is mainly absorbed by friction at joints, bearings and between the substructure and ground. In most investigations to date, these effects have not been recognised and damping values have been reported without reference to amplitude of movement so that they are meaningless.

It is necessary to define a single representative value of damping that has some meaning in relation to the performance of footbridges and which can be used as a comparison between bridges. This is referred to as the standardised damping, δ_{st}. It is the value of damping occurring at the maximum displacement amplitude excited by a pedestrian walking across at the fundamental bending frequency. The value of standardised damping is usually close to upper damping.

In tests on footbridges values of standardised damping were measured and considered in relation to the design parameters [11]. It was found that there is no relationship with span length, or stiffness. Although it has been suggested by some investigators that damping increases with frequency of vibration mode, this is not supported by the available data. Damping is influenced by the construction material. Steel footbridges exhibit the lowest values and reinforced concrete the highest, see Figure 2. Although this is in line with values of the material damping, it is almost certainly influenced by the differing forms of construction and connections. Ranges of values of damping for different connections in steel are given in Table 2.

In BS 5400 values recommended for design calculations are: steel 0.03, composite (steel beams with concrete decks) 0.04 and concrete (including both reinforced and prestressed structures) 0.05. Quite a few of the measured values of standardised damping are lower (see Figure 2) but in most cases this resulted in relatively low differences in the calculated response under pedestrian loading. It is not possible to calculate damping at the design stage or do better than use the code values.

5.2 Frequencies and stiffness

Frequencies and stiffness can be fixed at the design stage by suitable choice of structural configuration to meet design requirements. Calculation of frequencies can be carried out by use of computer programs but for many structures similar accuracy can be obtained using simplified methods [15]. In practice values of frequencies are influenced by features, such as mode of articulation, which have to be assumed for calculation. Because of the difference between assumed and actual material properties and articulation, calculated values of frequency and stiffness usually differ from measured values, sometimes by quite large factors. This is particularly so for concrete structures.

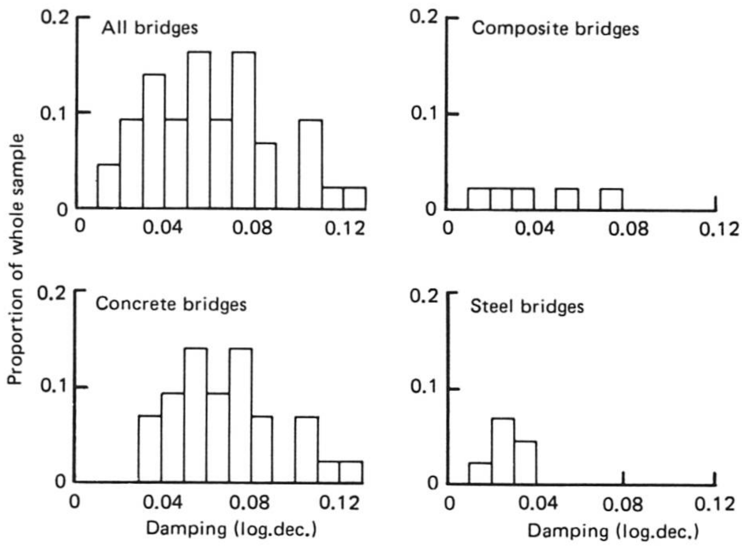

Fig.2 Standardised damping

Table 2

Values of damping for common types of steel connection (δ)

Rolled I-beams	Joints			
	Bolted	Riveted	Welded	Shear connection
0.003 to 0.007	Loose 0.01 to 0.07 Tight 0.005 to 0.03	0.006 to 0.02	0.004 to 0.008	0.04 to 0.11

Values of frequency decrease with increased span length but the relationship is blurred by the range of design considerations, see Figure 3. For a given span length, steel structures, which are generally lighter in weight, exhibit the highest frequencies. Values of f_0 within the sensitive range of frequencies (1.7 to 2.2 Hz) can occur for concrete spans exceeding about 20m and steel spans exceeding 35m. Measured stiffness (force per unit displacement at mid-span) are typically in the range 2 to 30 kN/mm. Steel plated structures are the most flexible and values measured were in the range 2 to 8 kN/mm.

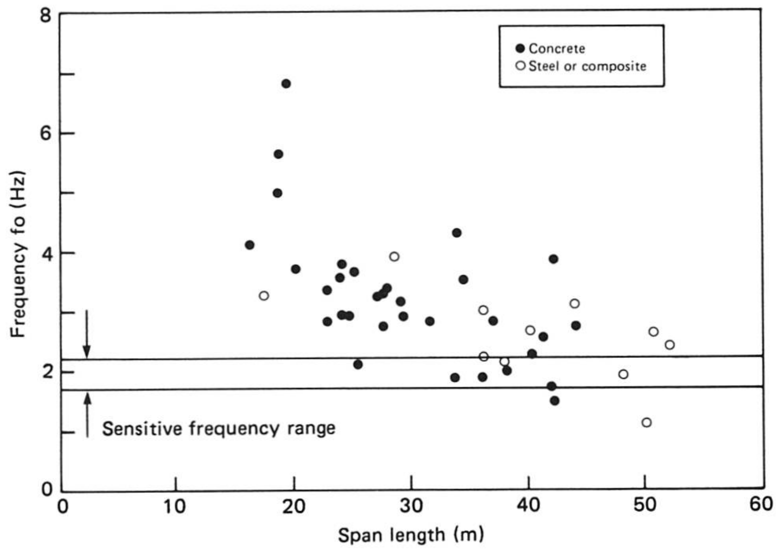

Fig.3 First bending frequencies for different span lengths

6. DYNAMIC RESPONSE

In studies of footbridge response and public acceptance, attention has been given almost exclusively to first bending frequencies. Movements at other frequencies and in lateral and torsional modes are not normally considered but occasionally give rise to comment [16].

Generalisations about the influence on response of single factors, such as damping or span length, are difficult to make with confidence from measured results because of interdependences observed in practice. Use of the simple formula for response enables a theoretical assessment to be made of the effect of the principal variables.

Low damping and low stiffness both contribute to lively behaviour but stiffness is relatively more important. For example if two footbridges of average length have the same frequency and damping but one is five times stiffer than the other the less stiff bridge will have a maximum acceleration of five times the other. If two otherwise identical footbridges have damping values of 0.03 and 0.15 the less damped structure will have a maximum acceleration of only twice the other when a notional pedestrian crosses the structure. On the other hand, the number of cycles of vibration above a given value are inversely proportional to damping, and in general a footbridge is more likely to be lively if it has a damping value below 0.03.

The responses of footbridges measured when excited by investigators walking at pacing rates equal to f_o are given in Figure 4 in relation to damping. Generally speaking the highest responses are exhibited by steel and composite structures because these tend to have the lowest values of damping and stiffness. Concrete structures do not usually have responses of more than 0.6 m/s^2. The dynamic response factor ψ increases with span length but the longer-span footbridges are not always the most responsive because their stiffness and damping are not necessarily low.

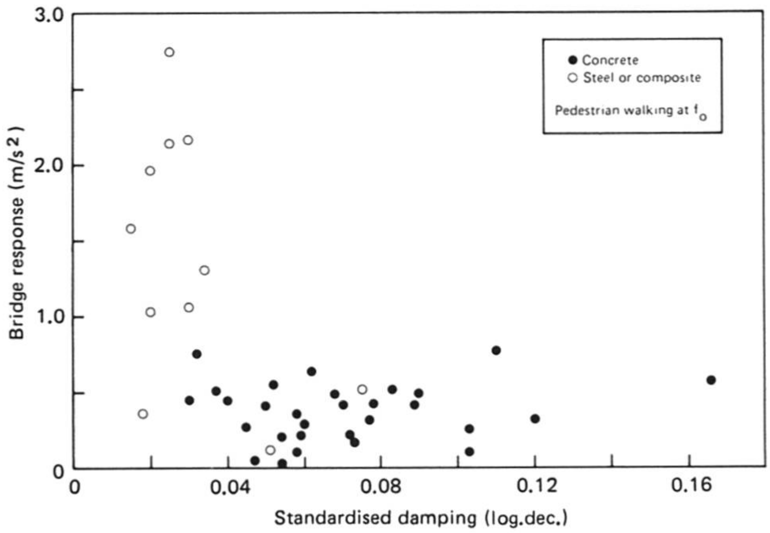

Fig.4 Bridge response to a pedestrian for different values of damping

Response is shown in relation to frequency in Figure 5. The liveliest footbridges are steel and composite and their frequencies are in the range 2 to 3.5 Hz. The only concrete footbridge to have a high response was a prestressed concrete ribbon structure.

Response decreases with increased stiffness in the manner shown in Figure 6. It is evident from the envelope of the results that footbridges having responses which exceed the limiting acceleration are those having stiffnesses less than about 8 kN/mm. The manner in which design factors influence response can be summarised as follows:

Stiffness	- lively behaviour may occur if less than 8 kN/mm
Damping	- " " " " " " 0.03 log. dec.
Frequency	- no systematic effect but structures in the range 1.7 to 2.2 Hz are most likely to respond to normal pedestrians.
Span length	- no systematic effect can be measured, even though the response factor increases with span length; but concrete spans less than 20m and steel spans less than 35m are unlikely to have values of f_o in the range 1.7 to 2.2 Hz.

A footbridge having either low stiffness or damping will not necessarily be too lively, but if it has both, it has a high likelihood.

Inherently lively bridges having frequencies around 3 to 4 Hz do occasionally arouse comment, in the first case from excitation by jogging and in the second case from excitation by walking at $\frac{1}{2}$ f_o; but these structures do not present as serious a problem. There are few joggers in the normal population of pedestrians and the incidence of this form of excitation is comparatively rare (except possibly in parks or at railway stations). Excitation is more difficult at $\frac{1}{2}$ f_o and responses are lower around 4 Hz.

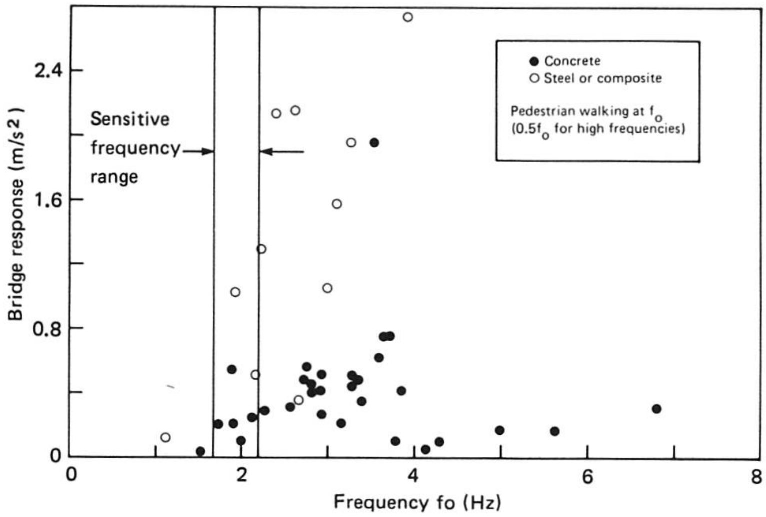

Fig.5 Bridge response to a pedestrian in relation to first bending frequency

To achieve a reasonable expectation of eliminating all comment it would be necessary to lower the acceleration limit for the notional pedestrian by a very large factor because reaction to vibration is recognised to be related to the logarithm of amplitude. A reduction factor of about four would probably be necessary over the excitable frequency range. This would require considerable increases in stiffness and damping in many steel or composite bridges, and some concrete bridges.

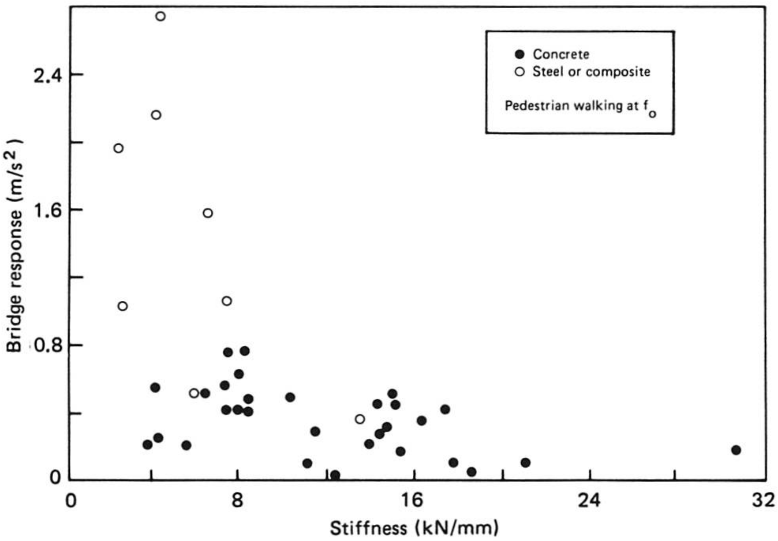

Fig.6 Bridge response to a pedestrian in relation to stiffness

Using the principle of notional pedestrian loading and limiting acceleration it is possible to restrict the potential liveliness of footbridges but designing to the response formula can be misleading. There is an apparent advantage to be gained by reducing the frequency of a bridge which is just above the permitted acceleration, possibly by adding mass. In this way the structure may be made acceptable to the code but the new frequency might be closer to the walking range and could actually result in livelier behaviour in use.

7. METHODS TO REDUCE RESPONSE

7.1 Increased stiffness

In the past, a popular method of reducing response has been to avoid specific frequency ranges, usually by increasing stiffness [17,18]. This improves the situation in two ways; increased stiffness makes the bridge more difficult to deflect and changing the frequency shifts it away from the normal walking range. The changes can be made by stiffening the superstructure as in the St. James's Park footbridge where f_o was raised from a calculated value of 2.0 Hz to 3.0 Hz [19]. When constructed the measured value was 3.5 Hz. If the required increase in stiffness is small it might be sufficient to substitute continuous heavy gauge steel with cross-bracing for lightweight aluminium hand railings. Another method which has been used is to prop the superstructure so that span lengths are reduced. There have been cases where the superstructure has been tied down by a steel cable anchored in the central reservation between carriageways. This method increases the resistance to movements and introduces damping due to hysteresis of the steel cable but it is only fully effective if the cable is taut at all times.

7.2 Added damping

An attractive and efficient method of reducing response is by added damping. This increases the potential for absorbing energy. Added damping has been successfully engineered on a cable-stayed footbridge in Tasmania in 1969. The cables had exhibited unacceptable oscillations believed to be due to galloping in comparatively light winds [20]. In wind induced vibration the response is more sensitive to damping than in pedestrian induced vibration. Viscous dampers of the type used on sports cars were fitted between the cables and the superstructure. This worked very well and the damping was increased to between 0.04 and 0.05. A similar scheme has been used on the long-span Brettone cable-stayed highway bridge.

In another example, a steel box girder footbridge having a main span of 57m, and a value of f_o of 1.63 Hz, was found to have a damping value of only 0.005. At the design stage it had been anticipated that there might be unduly lively behaviour and that it might be necessary to arrange for added damping before the bridge was completed. Friction devices were fitted to the hand rails and damping in the working range was increased to 0.01. More friction devices, fitted to the abutments made a further increase in damping, to between 0.04 and 0.05. At this level of damping the bridge was difficult to excite intentionally and its maximum response to a pedestrian walking at f_o was 1.0 m/s^2; although this is greater than $\frac{1}{2}\sqrt{f_o}$, it has been found by 'rule of thumb' that if the response to controlled walking tests is less than $\sqrt{f_o}$ and f_o is outside the range 1.7 to 2.2 Hz, the bridge will be acceptable in practice. This is because controlled walking tests made against a metronome, with a degree of bounce in the walk, exert a bigger dynamic load than would normally occur during pedestrian usage.

7.3 Dynamic absorbers

Dynamic absorption essentially involves a tuned mass-spring-damper system. Unlike added damping it is free-standing and does not depend on reacting between two parts having relative movements. In simplistic terms it 'splits' the critical frequency, usually f_o, and also absorbs energy. Whereas a 'frequency splitter' requires a mass which is about 5% of that of the superstructure, a dynamic absorber requires a mass which is only 0.5%. This means that for a typical footbridge it can be designed to about the size of a small beer barrel.

In a laboratory experiment at TRRL, a flexible and lightly damped steel experimental box girder of 30m length was constructed. The frequency was 3 Hz, stiffness was 3.53 kN/mm and damping was 0.008. The response to a pedestrian walking at a pacing rate of f_o was 2 m/s^2 which is over twice the limiting acceleration. When a tuned dynamic absorber was fitted damping was raised to 0.36 and the response was reduced to 0.5 m/s^2.

The first recorded example of a dynamic absorber was an installation fitted to the 213m main span of the Cleddau Bridge at Milford Haven to reduce the response to wind *[20]*. A type of dynamic absorber was fitted to a footbridge in Japan.

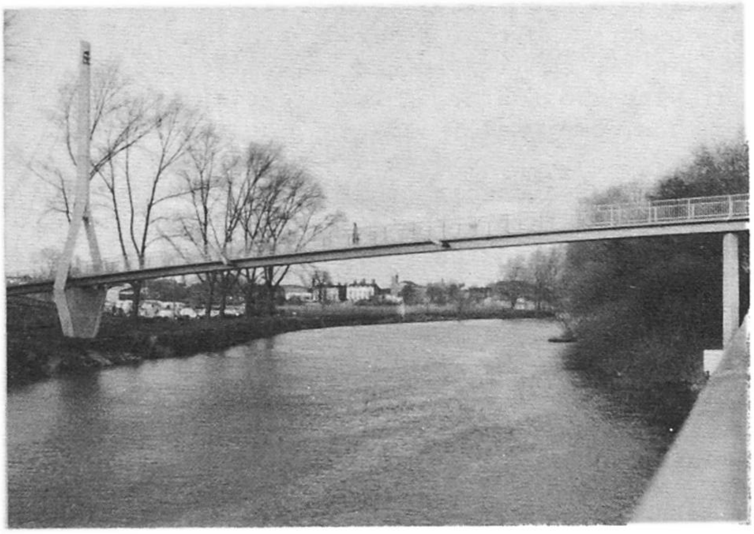

Fig.7 Frankwell footbridge

Tuned dynamic absorbers *[22]* have been fitted to two footbridges in England. In the first, it was calculated during design that it might be unduly responsive to pedestrians. It was a steel box girder cable-stayed footbridge having a main span of 48m, a stiffness of 2.5 kN/mm and a value of f_o of 1.92 Hz, Figure 7. The response could not be calculated accurately until a value of damping was known and it was believed that there might be a significant contribution due to the hysteresis of the six stay-cables. In the event the damping was only 0.024 for vertical superstructure movements of up to ± 22 mm (standardised damping was 0.02) and the cables contributed little or nothing. The response to a person walking at f_o exceeded the acceleration limit, there were complaints, and it was necessary to take action. A tuned dynamic absorber was fitted inside the box at the centre of the main span. Overall damping was increased to 0.25 and the response was reduced from 1.03 m/s^2 to 0.25 m/s^2, which is well within the limit.

The second footbridge was composed of a steel box girder with a concrete deck, Figure 8. It had a single span of 36m, a value of f_o of 2.23 Hz and a standardised damping of 0.034. It was to be used mainly by school children who it was feared would quickly learn to vibrate it so that it was necessary to reduce its response. A dynamic absorber was tuned and fitted at the centre of the span. The damping was increased to 0.3 and the response to a pedestrian walking at f_o was reduced from 1.3 m/s^2 to 0.3 m/s^2 see Figure 9.

8. CONCLUDING COMMENTS

- Steel and composite footbridges are generally more flexible, have lower damping and are more responsive to vibration than concrete footbridges.
- The majority of people walk at pacing rates around 2 Hz. It follows that footbridges having low values of damping (less than 0.03 logarithmic decrement) and stiffness (less than 8 kN/mm) are less likely to be acceptable if the fundamental bending frequency (f_o) is within the range 1.7 to 2.2 Hz.

- Structures can be assessed by controlled walking tests at f_o. If the response is less than $\sqrt{f_o}$ and f_o is outside the range 1.7 to 2.2 Hz, the footbridge will be acceptable in practice.
- Errors in calculating first bending frequencies can be significant in relation to the narrow shape of the typical bridge response curve and the distribution of walking frequencies. Consequently, care must be exercised in any design approach which is sensitive to frequency.
- When footbridges are found to have excessive responses, they can be modified by a variety of means. The most attractive and cost effective methods are by introducing extra damping through friction, hydraulic dampers or tuned dynamic absorbers.

Fig.8 Stoke Lane footbridge

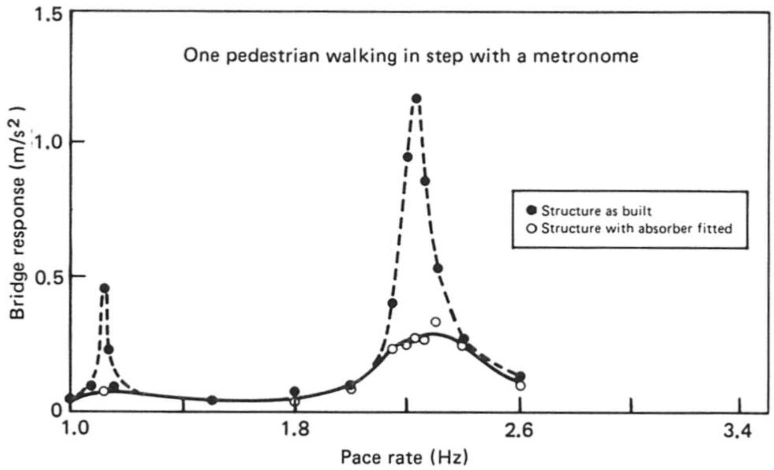

Fig.9 Effect of dynamic absorber on response of Stoke Lane footbridge at different pace rates

9. ACKNOWLEDGEMENTS

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