

Zeitschrift: IABSE proceedings = Mémoires AIPC = IVBH Abhandlungen
Band: 14 (1990)
Heft: P-146: Generalization of the failure criterion for concrete

Artikel: Generalization of the failure criterion for concrete
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DOI: <https://doi.org/10.5169/seals-42838>

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Generalization of the Failure Criterion for Concrete

Généralisation du critère de rupture du béton

Verallgemeinerung des Bruchkriteriums für Beton

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SUMMARY

On the basis of the so-called elliptic failure criterion and of the distributions of the normal stresses in the compression zone of the rectangular cross-section of a reinforced concrete beam, assumed by CEB, a new formulation of the failure criterion for concrete is proposed. This failure criterion holds for a rectangular section subjected to combined bending, axial force and shear and states that the ultimate concrete strain must decrease when the influence of the shear force in the concrete increases in relation to the influence of the normal force.

RÉSUMÉ

Partant du critère elliptique de rupture et du tracé des contraintes de compression dans la section rectangulaire d'une poutre en béton armé, supposé par le CEB, une nouvelle formulation du critère de rupture du béton est proposée. Ce critère, valable pour une section rectangulaire soumise à un moment fléchissant, une force longitudinale et un effort tranchant, traduit l'idée que, à la limite, la contrainte longitudinale doit diminuer lorsque la contrainte de cisaillement augmente.

ZUSAMMENFASSUNG

Auf der Grundlage des sogenannten elliptischen Bruchkriteriums und des CEB-Ansatzes für die Druckspannungsverteilung im Rechteckquerschnitt eines Stahlbetonbalkens wird eine neue Formulierung des Kriteriums für Betonbruch vorgeschlagen. Dieses Bruchkriterium gilt für einen Rechteckquerschnitt mit kombinierter Beanspruchung auf Biegung, Normalkraft und Schub und besagt, dass die Grenzdehnung des Betons abnehmen muss, wenn Schubbeanspruchung im Beton gegenüber der Normalkraftbeanspruchung zunimmt.



1. INTRODUCTION

A section subjected to a bending moment M , combined with a normal force (N), fails due to rupture either of the concrete or the reinforcement.

According to the CEB Model-Code concrete failure occurs when the maximum strain in the concrete (ϵ_c) attains 3,5 ‰ ($= \epsilon_{cu}$) [1].

When, besides, the section is subjected to a shear force (V), the previous criterion seems to us too optimistic.

Without any calculation it may be assumed that ϵ_{cu} must decrease if the influence of the shear force in the concrete (V_c) increases in relation to the influence of the compressive force (N_c).

2. MAXIMUM FORCE IN THE COMPRESSION ZONE

Let us consider a beam with rectangular section, subjected to a bending moment (M) and a normal force (N).

Part of the concrete section is compressed (area equal to cb).

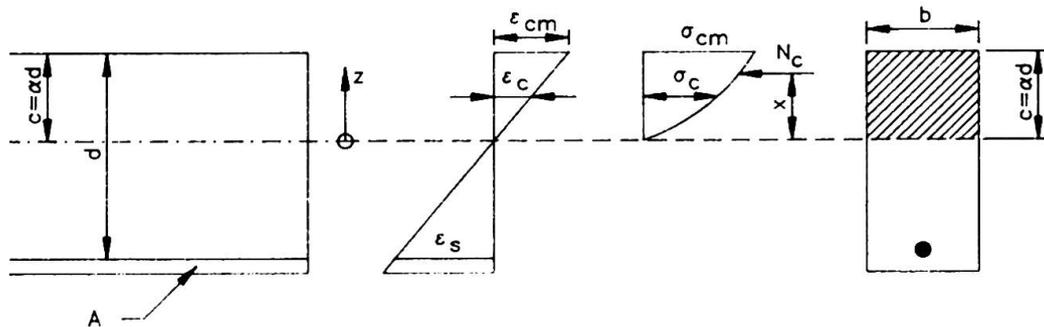


Fig.1 Reinforced concrete beam with rectangular section

In case of a uniaxial stress ($V = 0$) the stress-strain diagram (σ_c/ϵ_c) of the CEB Model-Code, generally accepted nowadays, is maintained :

- $0 < \epsilon_c \leq 2 \text{ ‰}$: parabolic course to f'_c (concrete compressive strength measured on cylinder)

$$\sigma_c = f'_c \frac{\epsilon_c}{2} \left(2 - \frac{\epsilon_c}{2} \right) \quad (1a)$$

- $2 \text{ ‰} < \epsilon_c \leq 3,5 \text{ ‰}$: constant stress f'_c

$$\sigma_c = f'_c \quad (1b)$$

In this formula ϵ_c is expressed in ‰.

If ϵ_{cm} represents the concrete strain (‰) on the upper side of the concrete section, the resultant (N_c) of the stresses σ_c and its location (x) with regard to the neutral axis (nn) may be calculated :

$$N_c = \int_0^c b \sigma_c dz \quad (2)$$

$$x = \frac{\int_0^c b \sigma_c z dz}{\int_0^c b \sigma_c dz} \quad . \quad (3)$$

The substitution of Eqs. (1a) and (1b) in Eqs. (2) and (3) gives (see [2] and [3]) :

- for $0 < \epsilon_{cm} \leq 2 \%$.

$$\frac{N_c}{bdf'_c} = \alpha \left(\frac{\epsilon_{cm}}{2} - \frac{\epsilon_{cm}^2}{12} \right) = \alpha n_c \quad (2a)$$

$$\frac{x}{d} = \alpha \frac{\frac{1}{3} - \frac{\epsilon_{cm}}{16}}{\frac{1}{2} - \frac{\epsilon_{cm}}{12}} = \alpha \xi \quad (3a)$$

- for $2 \% < \epsilon_{cm} \leq 3,5 \%$.

$$\frac{N_c}{bdf'_c} = \alpha \left(1 - \frac{2}{3\epsilon_{cm}} \right) = \alpha n_c \quad (2b)$$

$$\frac{x}{d} = \alpha \frac{3\epsilon_{cm}^2 - 2}{6\epsilon_{cm}^2 - 4\epsilon_{cm}} = \alpha \xi \quad (3b)$$

If also shear stresses act in addition to longitudinal stresses, the uniaxial stress changes into a biaxial stress. The criterion of failure under biaxial stress is represented schematically in Fig. 2; σ_{cI} and σ_{cII} are the principal stresses.

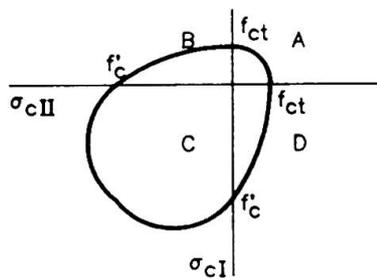


Fig. 2 Fracture criterion under biaxial stress



In Fig. 2 four typical areas may be distinguished :

- A : tension - tension
- B : compression - tension
- C : compression - compression
- D : tension - compression.

From Mohr's circle it follows that, when a compressive stress σ_c occurs simultaneously with a shear stress τ_c , the principal stresses σ_{cI} and σ_{cII} are tensile and compressive stresses respectively (areas B and D).

Different intrinsic curves were proposed for the areas compression/tension :

- straight line [4]
- criterion of Walther [5]
- criterion of Mohr [6]
- criterion of Beltrami - Hruban - Viergin [7]
- criterion of Bruggeling [6][9]
- criterion of Bresler - Pister [7].

For more information about these criteria see reference [6].

For the sake of simplicity a linear relation between σ_{cI} and σ_{cII} will be adopted further on. So the intrinsic curve is replaced in the compression/tension area B by a straight line (see Fig. 3) :

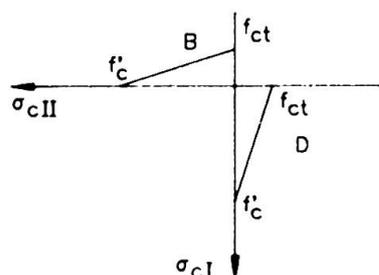


Fig.3 Failure criterion adopted in the zones B and D (σ_{cI} , σ_{cII})

$$\sigma_{cII} = \frac{f'_c}{f_{ct}} \sigma_{cI} + f'_c \quad (4)$$

σ_{cI} : tension
 σ_{cII} : compression .

The substitution of the expression of σ_{cI} and σ_{cII} as a function of σ_c and τ_c in the Eq. (4) gives :

$$\sqrt{\sigma_c^2 + 4 \tau_c^2} \left(1 + \frac{f'_c}{f_{ct}}\right) + \sigma_c \left(1 - \frac{f'_c}{f_{ct}}\right) - 2 f'_c = 0 . \quad (5)$$

Solved for τ_c , it gives :

$$\tau_c = \frac{1}{4} \left[\left(\frac{f'_c - f_{ct}}{f'_c + f_{ct}} \right)^2 - 1 \right] \sigma_c^2 + \left[\frac{(f'_c - f_{ct}) f'_c f_{ct}}{(f'_c + f_{ct})^2} \right] \sigma_c + \left[\frac{f'_c f_{ct}}{f'_c + f_{ct}} \right]^2 . \quad (6)$$

In a σ_c/τ_c system of coordinates Eq. (6) represents an ellipse.

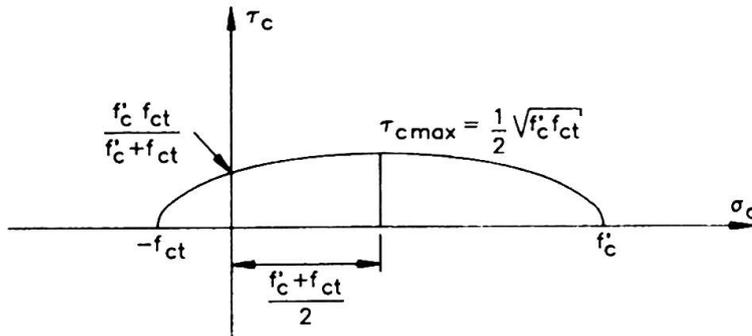


Fig. 4 Failure criterion as a function of σ_c and τ_c

With $\sigma_c = 0$ (at the neutral axis) corresponds :

$$\tau_{cu} = \frac{f'_c f_{ct}}{f'_c + f_{ct}} \quad (7)$$

If ϵ_{cm} is smaller than 2‰ , the σ_c -diagram shows a monotonically increasing course (see Fig. 5a). If, on the contrary, ϵ_{cm} is greater than 2‰ , the value of σ_c remains constant from $\epsilon_c = 2\text{‰}$ and upwards to ϵ_{cm} (see Fig. 5b).

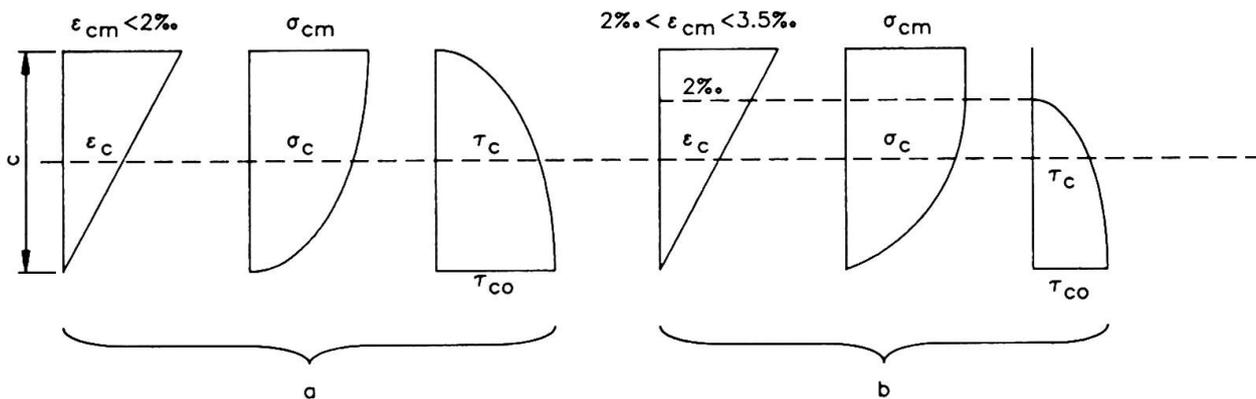


Fig. 5 Compression zone of a rectangular section

To every σ_c -diagram corresponds a τ_c -curve, which can be calculated from the equilibrium of the horizontal stresses [2],[10].

As, with ϵ_{cm} -values, smaller than 2‰ , σ_c is monotonically increasing, τ_c must be a continuous function of z .

For $z = 0$, then $\sigma_c = 0$; shear failure occurs when $\tau_{co} = \tau_{cu}$ (ultimate shear stress corresponding to $\sigma_c = 0$).

For $z = c$, then $\tau_c = 0$ on account of the reciprocity of the shear stresses on the upper side of the beam.

The τ_c -diagram is given schematically in Fig. 5a.

If ϵ_{cm} is greater than 2‰ , σ_c remains constant for $z > \frac{2}{\epsilon_{cm}} c$.

From the equilibrium of the horizontal stresses results that τ_c is zero from $z = \frac{2}{\epsilon_{cm}} c$ to $z = c$.

For $z = 0$, then $\sigma_c = 0$; shear failure occurs when $\tau_{co} = \tau_{cu}$.



The exact function $\tau_c(z)$ is theoretically calculable. The correctness of it, however, depends on the σ_c -function. As this is only an approximation, a correct deduction of $\tau_c(z)$ is questionable, to be sure, considering that, in order to find a realistic τ_c -curve, exact data have to be available, not only as to σ_c itself, but also as to the derivative of σ_c .

Therefore, we prefer to maintain a parabolic course for τ_c , according to the boundary conditions, indicated in Fig. 5. In the point $z = 0$, the tangent to the τ_c curve is vertical, as we assume that the longitudinal stress is zero below the neutral axis. The shear stress thus remains theoretically constant there as far as the bottom reinforcement.

Consequently, the resultant of the shear stresses at failure in the compression zone may be calculated :

$$\text{- for } 0 < \epsilon_{cm} \leq 2 \text{ ‰} \quad V_{cu} = \frac{2}{3} \alpha b d \frac{f'_c f_{ct}}{f'_c + f_{ct}} \quad (8a)$$

$$\text{- for } 2 < \epsilon_{cm} \leq 3,5 \text{ ‰} \quad V_{cu} = \frac{4}{3 \epsilon_{cm}} \alpha b d \frac{f'_c f_{ct}}{f'_c + f_{ct}} \quad (8b)$$

V_{cu} represents the limit shear force in the compression zone, corresponding to a given value of ϵ_{cm} .

As in the compression zone there is a strain gradient it seems logical to replace the tensile strength (f_{ct}) in the Eq. (8) by the bending tensile strength ($f_{ct,fl}$) :

$$f_{ct,fl} \approx \frac{f_{ct}}{0,6} \quad (9)$$

This adaptation is indeed also justified by test results (see 3).

According to Eqs. (2) and (8), N_{cu} and V_{cu} are functions of ϵ_{cm} for a given concrete quality.

Summarising, it gives :

$$\text{- for } \epsilon_{cm} \leq 2 \text{ ‰} \quad n_{cu} = \frac{N_{cu}}{\alpha b d f'_c} = \frac{\epsilon_{cm}}{2} - \frac{\epsilon_{cm}^2}{12} \quad (10a)$$

$$v_{cu} = \frac{V_{cu}}{\alpha b d f'_c K} = \frac{2}{3} \quad (11a)$$

$$\text{with } K = \frac{f_{ct,fl}}{f'_c + f_{ct,fl}} \quad (12)$$

$$\text{- for } 2 \text{ ‰} < \epsilon_{cm} \leq 3,5 \text{ ‰} \quad n_{cu} = \frac{N_{cu}}{\alpha b d f'_c} = 1 - \frac{2}{3 \epsilon_{cm}} \quad (10b)$$

$$v_{cu} = \frac{V_{cu}}{\alpha b d f'_c K} = \frac{4}{3 \epsilon_{cm}} \quad (11b)$$

So we may draw a $\frac{N_{cu}}{\alpha b d f'_c} / \frac{V_{cu}}{\alpha b d f'_c K}$ -diagram, which represents an

interaction N_{cu}/V_{cu} (see Fig. 6).

Each point corresponds to a certain value of ϵ_{cm} .
 From $\epsilon_{cm} = 0$ to 2 ‰ v_{cu} remains constant (0,667)(line AB).
 From $\epsilon_{cm} = 2$ to 3,5 ‰ the relation n_{cu}/v_{cu} is a linear function (line BC).
 Indeed, the elimination of ϵ_{cm} between Eqs. (10b) and (11b) gives :

$$n_{cu} = 1 - \frac{v_{cu}}{2} . \quad (12)$$

Along AB and BC, n_{cu} and v_{cu} can be considered ultimate values.
 For $\epsilon_{cm} = 3,5$ ‰ n_{cu} is constant (0,810) and, of course, an ultimate value;
 v_{cu} , however, has a maximum ultimate value (0,381) but can decrease to 0,
 while n_{cu} remains constant.
 Consequently, the line CD represents a limit state for n_{cu} (= 0,810) but not
 for v_{cu} (0,381). The values of ϵ_{cm} are to be considered ultimate values
 ($\epsilon_{cm} = \epsilon_{cu}$) under combined moment and shear force.

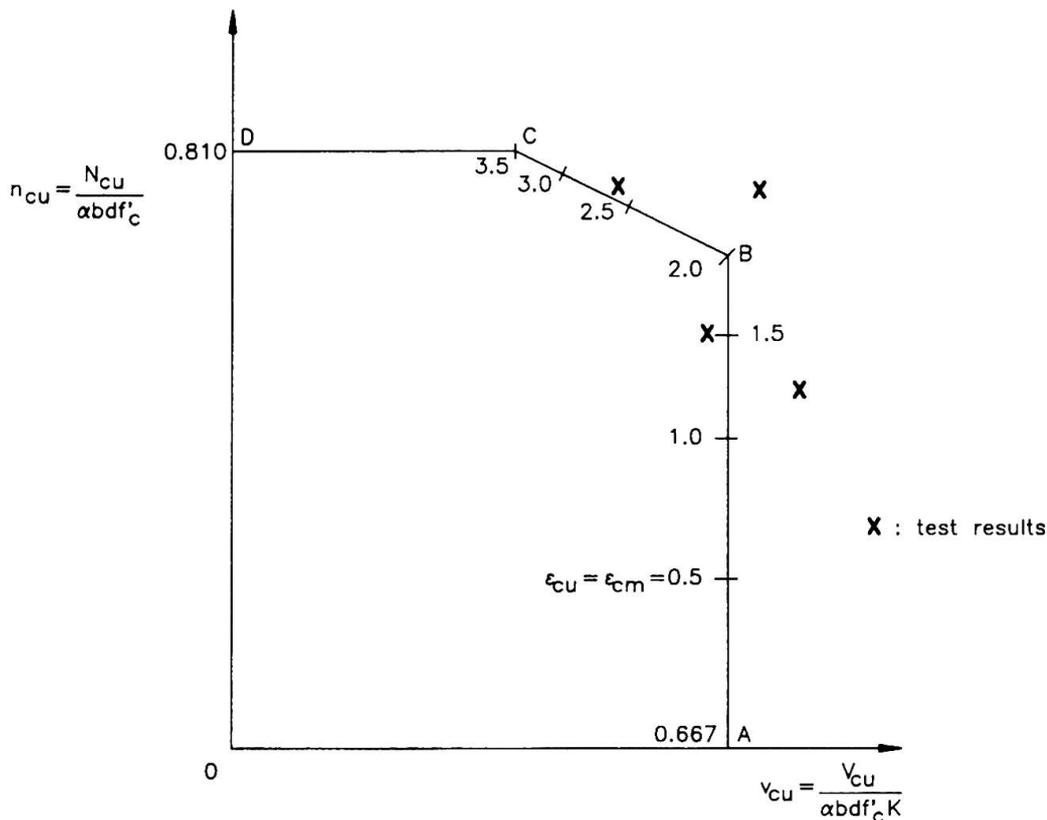


Fig.6 Shear/normal force-interaction diagram

3. CONFIRMATION BY EXPERIMENT OF THE INTERACTION DIAGRAM $N_{cu}K/V_{cu}$

In order to try out the Eqs. (10) and (11) in reality, a special test set-up was built [6] [11].

In the test beam, resting on two supports, the tensioned concrete is left out in a certain place ($\alpha\alpha$) so as to simulate the crack effect. Thus, the bottom reinforcement is not covered with concrete in that part of the beam as shown in Fig. 7. At both ends of the beam strengthened consoles are provided.

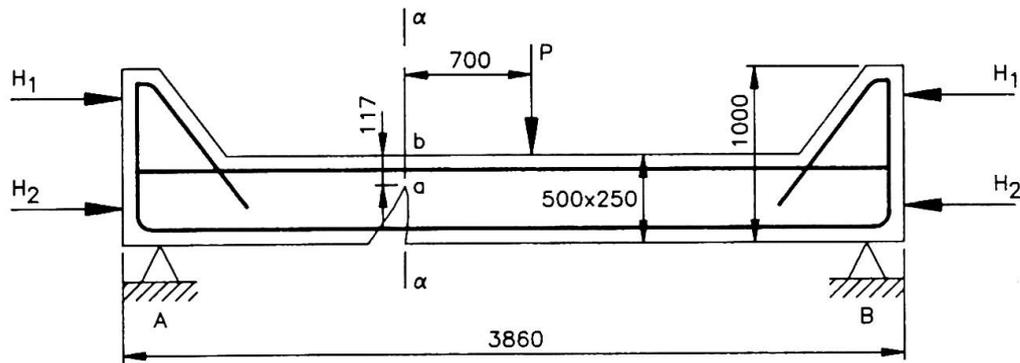


Fig.7 Test set-up

On the consoles act two adjustable prestressing forces (2 jacks of 750 kN capacity) : H_1 and H_2 .

The force P is gradually increased, always taking care to adjust the forces H_1 and H_2 in such a way that at the point a $\epsilon_c = 0$ always and at the point b ϵ_{cm} corresponds to the value intended for the test.

At a certain load P the beam collapses. At that moment N_{cu} is known with accuracy, and so is V_{cu} , belonging to it, acting in the cross-section $\alpha\alpha$ at the moment of failure.

Per ϵ_{cm} - i.e. per test - only one point can be determined in the $N_{cu}K/V_{cu}$ diagram.

Table 1 shows the results, obtained from the 4 tests performed.

$\epsilon_{cm} = \epsilon_{cu}$	N_{cu}	V_{cu}
	$\alpha b d f'_c$	$\alpha b d f'_c K$
1,20 ‰	0,485	0,763
1,50 ‰	0,562	0,639
2,74 ‰	0,757	0,709
2,80 ‰	0,762	0,517

Table 1 Test results.

This test programme was carried out by Dr.ir. L. Vandewalle in the laboratory for reinforced concrete of the Catholic University of Leuven.

The test set-up is very delicate and relatively dangerous as fracture is brought about explosively.

The values determined by experiment are indicated in Fig. 6. A comparison with the proposed Eqs. (10) and (11) shows that the replacement of f_{ct} by $f_{ct,f1} = f_{ct}/0,6$ is justified and that the Eqs. (10) and (11) describe the relation to be found quite well, as shown in Fig. 6.

4. PROPOSAL FOR CONCRETE FAILURE

Let us consider the Eqs. (10) and (11). Dividing the N_{cu} formula by the V_{cu} formula we obtain :

- for $0 < \epsilon_{cm} \leq 2 \text{ ‰}$.

$$\frac{N_{cu}K}{V_{cu}} = \frac{3}{4} \epsilon_{cm} - \frac{1}{8} \epsilon_{cm}^2 \quad (13a)$$

- for $2 \text{ ‰} < \epsilon_{cm} \leq 3,5 \text{ ‰}$.

$$\frac{N_{cu}K}{V_{cu}} = \frac{3}{4} \epsilon_{cm} - \frac{1}{2} . \quad (13b)$$

So $N_{cu}K/V_{cu}$ is only a function of ϵ_{cm} .

The quantity ϵ_{cm} , considered here, stands for the value of ϵ_{cm} at the rupture of the concrete. As a matter of fact, ϵ_{cm} may be considered to be the ultimate admissible strain of the concrete ($\epsilon_{cm} = \epsilon_{cu}$).

The Eq. (13) represents a generalization of the criterion for concrete failure under combined compressive and shear force (see Fig. 8).

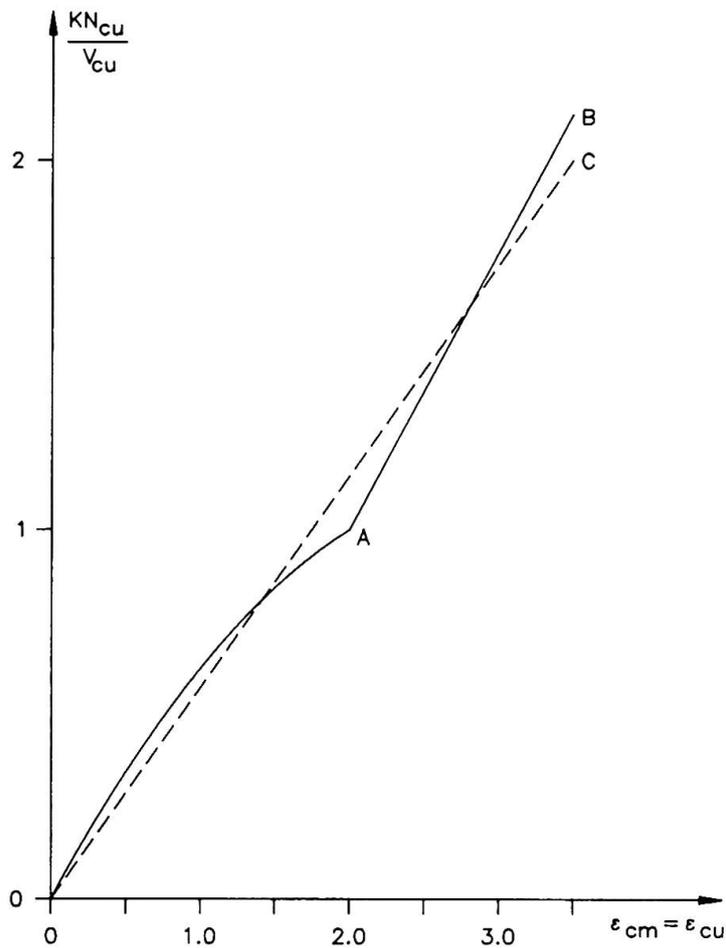


Fig.8 Proposed failure criterion under combined compressive and shear force

With a good approximation the relation $\frac{KN_{cu}}{V_{cu}} / \epsilon_{cu}$ may be given by :



$$\epsilon_{cu} = 1,75 \frac{KN_{cu}}{V_{cu}} . \quad (14)$$

The value of 1,75 has been calculated by means of the least-squares approximation.

This formula holds for $\frac{KN_{cu}}{V_{cu}} < 2$.

When $\frac{KN_{cu}}{V_{cu}}$ increases, N_{cu} remains constant as $\epsilon_{cu} = 3,5 \%$. (see Eq.(10b))

$$n_{cu} = 0,810.$$

In that case V_c is no longer an ultimate value because

$$v_c < (v_{cu})_{3,5 \%} = \frac{0,810}{2} = 0,405 .$$

The influence of V_{cu} is then rather negligible.

5. APPLICATION

What is the ultimate shear force in the compression zone of a rectangular section, subjected to a bending moment (M) and a shear force (V) ?

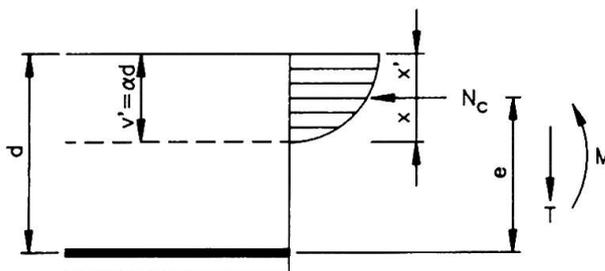


Fig.9 Reinforced concrete beam with rectangular section

From Fig. 9 it follows that :

$$e = d - v' + x = d(1 - \alpha + \frac{x}{d}) = \epsilon d$$

$\frac{x}{d}$ is given by the Eq. (3) :

$$\epsilon = 1 - \alpha(1 - \xi) . \quad (16)$$

Let us suppose that the stress-strain relation of the reinforcement is given as indicated in Fig. 10.

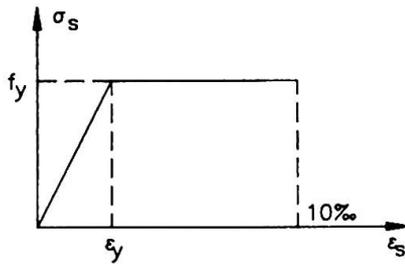


Fig.10 Stress-strain diagram of steel

$$\begin{aligned} \sigma_s &= E \epsilon_s & \text{for} & \epsilon_s < f_y/E \\ \sigma_s &= f_y & \text{for} & \epsilon_s \geq f_y/E . \end{aligned} \quad (17)$$

Expressing the equilibrium of the section, we find :

$$\begin{aligned} N_C &= A \sigma_s \\ M &= \epsilon d A \sigma_s \\ b d f'_c \alpha n_c (\epsilon_{cm}) &= A \sigma_s (\epsilon_s) \end{aligned} \quad (18)$$

$$M = \{1 - \alpha [1 - \zeta (\epsilon_{cm})]\} d A \sigma_s (\epsilon_s) . \quad (19)$$

Finally, we may write a relation between ϵ_{cm} , ϵ_s and α (see Fig. 1) :

$$\frac{\epsilon_{cm}}{\epsilon_s} = \frac{\alpha}{1 - \alpha} . \quad (20)$$

The problem now is, given a section, a reinforcement percentage, a bending moment, steel and concrete properties, to determine the ultimate shear force in the compression zone.

To this end the unknowns ϵ_s , ϵ_{cm} , α and σ_s have to be found.

Therefore, the system of the four Eqs. (17),(18),(19) and (20) has to be solved.

Unfortunately, an analytic solution is impossible so that a calculation by iteration is required.

Once $\epsilon_{cm} = \epsilon_{cu}$ is determined, $N_C = N_{cu}$ may be calculated (see Eq. (2)).

Consequently, V_{cu} follows from Eq. (14) :

$$V_{cu} = \frac{1,75 \text{ KN}_{cu}}{\epsilon_{cu}} .$$

The difference $V - V_{cu}$ must be taken up by shear reinforcement.

6. CONCLUSION

The following conclusions may be drawn from the present investigation :

- The failure criterion " $\epsilon_{cu} = 3,5 \text{‰}$ " is not a sufficient requirement when the cross-section is subjected to combined bending, tension and shear.



- The equation " $\epsilon_{cu} = 1,75 \frac{K N_{cu}}{V_{cu}}$ ", which is on the safe side, means an extension of the generally accepted failure criterion for concrete ($\epsilon_{cu} = 3,5 \%$).
- This proposed failure criterion states that " ϵ_{cu} " must decrease when the influence of V_c increases in relation to the influence of N_c .

REFERENCES

1. CEB-FIP Model Code 1978.
2. MORTELMANS F., Interaction diagram bending moment/shear force considering the absorbable forces in the cracked and the non-cracked concrete, Internal report Departement Bouwkunde K.U.Leuven, 20-ST-13/18-10-1984, 63 pp.
3. MORTELMANS F., Calculation of constructions - Part 5 : Reinforced concrete III, Acco (Leuven), 1985, 583 pp. (in Dutch).
4. PRUIJSERS A.F., Shear resistance of beams based on the effective shear depth, report 5-86-1, T.H. Delft 1986.
5. WALTHER, R., Über die Berechnung der Schubtragfähigkeit von Stahl- und Spannbetonbalken - Schubbruchtheorie, Beton- und Stahlbetonbau 11, 1962.
6. VANDEWALLE L., MORTELMANS F., New failure criterion for concrete in the compression zone of a beam, Rilem Workshop on "Applications of fracture mechanics to concrete structures", 28-30 June 1989, Abisko, Sweden.
7. GODYCKI-CWIRKO T., Le cisaillement dans le béton armé, Dunod, 1972.
8. BRUGGELING A.S.G., Theorie en praktijk van het gewapend beton deel 1, Stichting Professor Bakkersfonds, 1986.
9. BRUGGELING A.S.G., Het gedrag van betonconstructies deel B, Professor Bakkersfonds.
10. MORTELMANS F., The relation between bending and shear force, Cement 10 ('s Hertogenbosch), 1985, pp. 758-770 (in Dutch).
11. JANS G., MATTHE P., Absorbable shear force in the compression zone of beams in reinforced concrete, thesis K.U.Leuven, 1981 (promotor : Prof.dr.ir. F. Mortelmans), 110 pp. (in Dutch).
12. TAYLOR, H.P.J., The fundamental behaviour of reinforced concrete beams in bending and shear, ACI-Sp-42, Vol. 1 (1974).
13. BAZANT, Z.P., KIM, J.K., Size effect in shear failure of longitudinal reinforced beams, ACI-Journal Vol. 81 (1984).